Records Arithmetick,

OR,

THE GROUND OF ARTS:

TEACHING

The perfect work and practice of Arithmetick, both in whole Numbers and Fractions, after a more easie and exact form then in former time hath been set forth:

Made by M. *Robert Record*, D. in Physick.

Afterward augmented by M. John Dee.

And since enlarged with a third part of Rules of Practice, abridged into a briefer method then hitherto hath been published, with divers necessary Rules incident to the Trade of Merchandise; with Tables of the valuation of all Coyns, as they are current at this present time.

By JOHN MELLIS.

And now diligently perused, corrected, illustrated and enlarged; with an *Appendix* of figurative Numbers, and the extraction of their Roots, according to the method of *Christian Vrsitus*: with Tables of Board and Timber measure; and new Tables of Interest, after 10. and 8. *per* 100; with the true value of Annuities to be bought or sold present, Reputed, or in Reversion: the first calculated by *R. C.* but corrected, and the latter diligently calculated by *Ro. Hartwell, Philomathemat.*

Scientia non habet inimicum nisi ignorantem.

Fide ——— sed ——— Vide.

LONDON,

Printed by *J. Flesher*, for *John Harrison*; and are to be sold by *Edw. Dod* and *Nath. Ekins* in Ivy-lane at the sign of the Gun. 1652.

That which my friend hath well begun
For very love to Common-weal,
Need not all whole to be new done,
But now epcreaſe I doe reveal.

Something herein I once redreſt,
And now again for thy behoof,
Of zeal I doe and at requeſt,
Both mend and adde, fit for all proof.

Of number infe, the endleſſe might,
No wit nor language can expreſſe:
Apply and try both day and night,
And then this truth thou wilt confeſſe.

J. Dee.

The Books Verdict.

TO pleaſe or diſpleaſe ſure I am,
But not of one ſort to every man:
To pleaſe the beſt ſort would I fain,
The froward diſpleaſe ſhall I certain;
Yet wiſh I will though not with hope,
All ears or mouths to pleaſe or ſtop.

To the moſt mightie Prince, *Edward* the Sixth, by the grace of God, King of *England*, *France*, and *Ireland*, &c.

THe Excellency of mans nature being ſuch, as it is by Gods divine favour (moſt mighty Prince) not only created in highneſſe of degree far above all other corporall things, but by perfection, reaſon, and ſearch of wit, much approaching toward the image of God, as not onely the holy Scriptures do teſtifie, but alſo thoſe naturall Philoſophers, which exactly did conſider the nature of man, and namely the far reach and infinite compaſſe of the words of the mind, were inforced to confeſſe, that man ſcarcely was able to know himſelf. And if he would duely ponder the nature of himſelf, he would finde it ſo ſtrange, that it might ſeem unto him a very miracle: And thereof ſprang that ſaying; *Magnum miraculum eſt homo, maximum miraculum ſapiens homo.* For undoubtedly, as man is one of the greateſt miracles that ever God wrought, ſo a wiſe man is plainly the greateſt.

And therefore was it that ſome did account the head of a man the greateſt miracle in the world, becauſe not onely of the ſtrange workmanſhip that is in it, but much more of the efficacy of reaſon, wit, memory, imagination, and ſuch other powers, and works of the mind, which can more eaſily conceive any thing in a manner then underſtand it ſelf. Amongſt all the creatures of God, it findeth none more difficult to be perceived then the ſame powers of it ſelf; whereby it doth conceive and judge: as it may bee well conjectured by the diverſity of opinions, that the wiſeſt Philoſophers did utter touching the ſpirit of man, and the ſubſtance of it: whereof I now intend to make no rehearſall; but who ſo liſteth to read thereof, may finde it largely

ſet forth, not onely in *Ariſtotle* his books *de Anima*, but alſo in *Galen* his book called *Hiſtoria Philoſophica*: and again in *Plutarch* his work, *De Philoſophorum placitis*, whoſe words are alſo repeated of *Euſebius* in the xv. book, Τῆς εὐαγγελικῆς προπαρασκευῆς, unto whom I remit them that have deſired to underſtand intricate difficulty of knowing our own ſelves, as touching our beſt part, and that part whereby we deſerve to bear the name of men.

This matter ſeemed ſo obſcure and difficult in knowledge; that *Galen*, who for his excellent wiſdom and judgement in naturall works, is called of many men a Miracle in Nature, yet in ſearching the nature and ſubſtance of the ſpirit of man, he not onely confeſſeth himſelf ignorant, but counteth it plain temerity to attempt to find it. So farre above the hope of mans knowledge is that part, whereby man doth know and judge of things. And although the ignorant ſort (which hate all things that they know not) doe little eſteem the profoundneſſe of mans ſpirit and reaſon, the chief power and faculty of it: yet as there is a kind of fear and obedience of unreaſonable beaſts unto man, by the working power of God, ſo is there in thoſe ſmall reaſoned perſons a certain kind of reverence toward wiſdom and reaſon, which they do ſhew oftentimes, and by power of perſwaſion, are inforced to obey reaſon, will they nill they. And hereby it came to paſſe, that the rudeneſſe of the firſt age of man was brought unto ſome more civill trade, as it is well declared by *Cicero*, in the beginning of his firſt book, *De Inventione Rhetorica*, where he ſaith thus. *Nam fuit quoddam tempus quum in agris homines paſſim beſtiarum more vagabantur, & ſibi victu fero vitam propagabant, nec ratione animi quicquam, ſed pleraq; viribus corporis adminiſtrabant. Nondum divinæ religionis, non humanitatis colebatur. Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos; non jus æquabile quid utilitatis haberet, acceperat: ita propter errorem atq; inſcitiam cæca ac temeraria dominatrix animi cupiditas, ad ſe explendam viribus corporis abutebatur pernicioſiſſimis ſatellitibus. Quo tempore quidam magnus videlicet vir & ſapiens, cognovit quæ materia eſſet, & quanta ad maximas res opportunitas in animis ineſſet hominum, ſi quis eam poſſit elicere, & præcipiendo meliores reddere*

reddere. Qui diſperſos homines in agris, & in tectis ſylveſtribus abditos, ratione quadam compulit in unum locum, & congregavit: & eos in unamquamq; rem inducens utilem atque honeſtam, primo propter inſolentiam reclamantes, deinde propter rationem atque orationem ſtudioſius audientes, ex feris & immanibus, mites reddidit, & manſuetos.

This long repetition of *Tullyes* words will ſeem tedious to them that love but little, and care much leſſe for the knowledge of reaſon, but unto your Majeſty (I dare ſay) it is a dilectable remembrance, and unto me it ſeemed ſo pleaſant, that I could ſcarce ſtay my pen from writing all that mine eyes did ſo greedily read.

This ſentence of *Cicero* am I loath to tranſlate into Engliſh partly for that unto your Majeſty it needeth no tranſlation, but eſpecially knowing how far the grace of *Tullyes* eloquence doth excell any Engliſh mans tongue, and much more exceedeth the baſeneſſe of my barbarous ſtyle: yet for the fruit of my ſentence, I had rather unto my meer Engliſh Countrymen utter the rudeneſſe of my tranſlation, then to defraud them the benefit of ſo good a leſſon, truſting they will ſo learn to love reaſon, that they will alſo gladly and greedily embrace all good Sciences, that may help to the juſt furniture of the ſame, when they conſider that informed reaſon was the onely inſtrument, or at leaſt the chiefeſt means to bring men into civill regiment, from barbarous manners, and beaſtly conditions. "For the time "was (ſaith *Tully*) that men wandred abroad in the fields "up and down like beaſts, and uſed no better order in "feeding then they: ſo that by reaſons rule they wrought "nothing, but moſt of their doings did they atchieve by "force of ſtrength. At this time there was no juſt regard "of religion towards God, nor of duty towards man. No "man had ſeen right uſe of marriage, neither did any man "know their own children from other; nor no man had "felt the commodity of juſt Laws: ſo that through error "and ignorance, wilfull luſt, like a blind and heady ru-"ler, abuſed bodily ſtrength as a moſt mortall miniſter for "the ſatisfying of his deſire. At that time was there one "which not onely in power, but alſo in wiſdome was "great, and he conſidered how that in the minds of men

" was both apt instruments, and great occasion to the due
" accomplishment of most weighty affaires, if a man
" could apply them to use, and by teaching of rules frame
" them to better trade. This man with perswasion of rea-
" son gathered into one place the people that were wan-
" dring about the fields and lay lurking in wild cottages,
" and woods, and bringing them into one common soci-
" ety, did trade them to all such things, as either were pro-
" fitable or honest, although not without repining at the
" first, by reason then they had not been so accustomed be-
" fore: Yet at length through reason and perswasion of
" words they obeyed him more diligently, and so of a wild
" and cruell people, he made them courteous and gentle.
Thus hath *Tully* set forth the efficacy of reason and perswasion, how it was able to convert wild people to a mildnes, and to change their furious cruelnes into gentle courtesie: were it not now a great reproach in this our time (when knowledge reigneth so large) that men should shew themselves lesse obsequious to reason? Unlesse it may be thought that now every man having sufficient knowledge of himself, needeth not to hearken to the perswasion of others.

Indeed he that thinketh himself wise, will not esteem the reason of any other, be he never so wise; so that of such a one it may well be said: He that thinketh himself wiser then he is, may justly be counted a double fool. Wherefore such men are not to be permitted in open audience to talk, but must be put to silence, and be made to give ear to reason; which reason consisteth not in a multitude of words heaped rashly together, and applied for one purpose, but reason is the expressing of a just matter with witty perswasions, furnished with learned knowledge: such knowledge had *Moses*, being expert in all learning of the *Egyptians*, as the Scriptures declare; and therefore was able to perswade the stubborn people of the Jews, although not without pain. Such knowledge, and such reasons did *Druys* shew which was the first Law-maker of all the West part of Europe. Like reason and wisdom did *Xamolxis* amongst the Goths. *Lycurgus* unto the Lacedemonians. *Zeleucus* to the Locrians. *Solon* to the *Athenienses*, and *Dunwallo Molmutius* two thousand yeers past amongst the old Britains of this Realm.

Druys was son to king *Samon*, and succeeded him in his kingdome.

Realm. And hereby came it to passe, that their Lawes continued long, till more perfect reason altered many of them, and wilfull power oppressed most of them.

As the beginning when these wise men perceived how hard it was to bring the rude people to understand reason, they judged the best means to attain this honest purpose, to depend of learning in every kinde: for by learning (as *Ovid* saith) *Pectora mollescunt, asperitasque fugit*; Stout stomackes doe waxe milde, and sharpe fiercenesse is exilde. Therefore as *Berosus* doth write, *Samon* that was the third King over all this West part of Europe, for to bring the people from beastly rage to manly reason: did erect Schools of liberall Arts, which tooke so good successe, that his name continued in that sort famous above two thousand years after: for *Diodorus Siculus* which was in the time of *Julius Cæsar*, maketh mention of the learned men of Gothes or Celtes, and nameth them *Samothides*; that is to say, *Samon his Scholars and followers*.

Among these Arts that then were taught some did informe the tongue, and make them able both to utter aptly their mind, and also to perswade; as Grammar, Logick, and Rhetorick, although not so curiously as in this time; some other did appertain to the just order of partition of lands, the true using of Weights, Measures and reckonings of all sorts of bargains, and for order of building and sundry other uses; those were Arithmetick and Geometry. Again, to incourage men to the honour of God, they taught Astronomy, whereby the wonderfull works of God were so manifestly set forth, that no mans tongue, nor pen can in like sort expresse his infinite power, his unspeakable wisdome, and his exceeding goodnesse toward man, whereby he doth bountifully provide for man all necessaries, not onely to live, but also to live pleasantly. And so was their confidence in Gods providence strongly stayed, knowing his goodnes to be such, that he would help man as he could, and his power to be so great, that he would doe nothing but that that was best. Besides these Sciences they taught also Musick, which most commonly they did apply partly to religious services, to draw men to delight therein, and partly to songs made of the manners of men, in praise of vertue and discommenda-

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This *Bardus Braydius* the 5. King of the Celtes, reigned 60. years, and died 1831 years before Christ.

tion of vice, whereby it came to passe, that no man would displease them, nor doe any thing evill that might come to their hearing: for their songs did make evill men more abhorred in that time then any excommunication doth in this time. The posterity of these Musicians continue yet both in Wales and Ireland, called *Bardes* unto this day, by the ancient name of *Bardus*, their first founder.

And as these Sciences did encrease, so did virtue encrease thereby. Again, as those Sciences did decay, so virtue lost her estimation, and consequently was little in use: whereof to make a full declaration were a thing meet for a Prince to hear, but it would require a peculiar Treatise. Wherefore at this present I count it sufficient lightly to have touched this matter in generall words, and to say no more of the particularity thereof, but onely touching one of those Sciences, that is Arithmetick, by which not onely just partition of lands was made, but also touching buying and selling, all Assises, Weights, and Measures were devised, and all reckonings and accounts driven; yea by proportion of it were the true orders of justice limited, as *Aristotle* in his Ethicks doth declare, and the degrees of estates in the common-wealth established; although that proportion be called Geometricall and not Arithmeticall, yet doth that proportion appertain to the art of Arithmetick, and in Arithmetick is taught the progression of such proportions, and all things thereto belonging. Wherefore I may well say, that seeing Arithmetick is so many waies needfull unto the first planting of a Common-wealth, it must needs be as much required to the preservation of it also: for by the same meanes is any common-wealth continued, by which it was erected and established. And if I shall in small matters in appearance, but indeed very weighty, put one example or two. What shal we say for the Statutes of this Realm, which be the onely stay of good order in manner now? As touching the measuring of ground by length and breadth, there is a good and an ancient Statute made by art of Arithmetick; and now it shal be to little use, if by the same art it be not practised and tried. For the assise of bread and drink, the two most common and most necessary things for sustentation of man, there was a goodly ordinance in the Law made, which by igno-

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rance hath ſo grown out of knowledge, and uſe, that few men doe underſtand it, and therefore the Statute bookes wonderfully corrupted, and the Commons cruelly oppreſſed: notwithſtanding ſome men have written that it is too doubtfull a matter to execute thoſe aſſiſes by thoſe Statutes, by reaſon they depend of the ſtandard of the coyn, which is much chang'd from the ſtate of that time, when thoſe Statutes were made. Thus ſhall every man read (that liſteth) in the Abridgement of the Statutes, in the title of Weights and Meaſures, in the ſeventh number of the Engliſh Booke, where he ſhould have tranſlated a good ordinance which is ſet forth in the French Book: but no marvell if the Abridgement doth omit it, ſeeing the great Book of Statutes doth omit the ſame Statute, as it hath done divers other very good laws. And this is the fruit of ignorance to reject and condemn all that it underſtandeth not, although they uſe ſome cloaks for it, but ſuch cloaks as being allowed, might ſerve to repell all good laws; which God forbid.

Again, there is an ancient order for aſſiſe of fire, wood and coals, which was renued not many years paſt; and now how avarice and ignorance doth canvaſe that Statute, it is too pitifull to talk of, and more miſerable to feel.

Furthermore, for the Statute of Coynage, and the ſtandard thereof, if the people underſtood rightly the Statute, they ſhould not nor would (as they often doe) gather an excuſe for their folly thereby: but as I ſaid, theſe Statutes by wiſdome and good knowledge of Arithmetick were made, and by the ſame muſt they be continued. And let ignorance no more meddle with the uſe of them, then it did with the making of them. Oh in how miſerable caſe is that Realm, where the miniſters and interpreters of the law are deſtitute of all good Sciences, which be the keyes of the lawes! How can they either make good lawes, or maintain them that lacke that true knowledge whereby to judge them? And happy may that Realme be accounted, where the Prince himſelf is ſtudious of learning, and deſireth to underſtand equity in all laws. Therefore moſt happy are we the loving ſubjects of your Majeſty, which may ſee in your Highneſſe not onely ſuch towardneſſe, but alſo ſuch knowledge of divers Arts, as ſeldome hath been

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ſeen in any Prince of ſuch yeats, whereby we are enforced to conceive this hope certainly, that he which in thoſe years ſeeketh knowledge when knowledge is leaſt eſteemed, & of ſuch an age can diſcern them to be enemies both to his royal Perſon, and to his Realms, which labour to withdraw him from knowledge to exceſſive paſtime, and from reaſonable ſtudy to idle or noyſome pleaſures, he muſt needs when he cometh to more mature years, be a moſt prudent Prince, a moſt juſt Governor, and a right Judge, not onely of his ſubjects commonly, but alſo of the miniſters of his Laws, yea, and of the Laws themſelves: and to be able to conceive the true equity and exact underſtanding of all his Laws and Statutes, to the comfort of his good Subjects, and the confuſion and reproach of them which labour to obſcure or pervert the equity of the ſame Laws and Statutes. How ſome of theſe Statutes may be applyed to uſe, as well in this our time, as in any other time, I have peculiarly declared in this Book, and ſome other I have omitted for juſt conſiderations, till I may offer them firſt unto your Majeſty to weigh them as to your Highneſſe ſhall ſeem good: for many things in them are not to be publiſhed without your Highneſſe knowledge and approbation; namely, becauſe in them is declared all the rates of alloyes for all ſtandards from one ounce upward, with other myſteries of Mint matters, and alſo moſt part of the varieties of coyns that have been currant in this your Majeſties Realm by the ſpace almoſt of ſix hundred years laſt paſt, and many of them that were currant in the time that the Romans ruled here.

All which, with the ancient deſcription of England and Ireland, and my ſimple cenſure of the ſame, I have almoſt compleated to be exhibited to your Highneſſe: In the mean ſeaſon moſt humbly beſeeching your Majeſty to accept this ſimple Treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your Majeſty is, that this amongſt other of my Books may paſſe under the protection of your Highneſſe, whom I beſeech God moſt earneſtly and daily, according to my duty, to advance in all honor, and Princely Regality, and to increaſe in all knowledge, juſtice, and godly policy, Amen.

Your Majeſties moſt obedient ſubject and ſervant,

ROBERT RECORD.

To the loving READERS, The Preface of M. *Robert Record.*

SOre oft times have I lamented with my self the unfortunate condition of England, seeing so many great Clerks to arise in sundry other parts of the world, and so few to appear in this our Nation: whereat for pregnancy of naturall wit (I thinke) few Nations do excell Englishmen: But I cannot impute the cause to any other thing, then to the contempt, or misregard of learning. For as Englishmen are inferior to no men in mother wit, so they passe all men in vain pleasures, to which they may attain with great pain and labour: and are slack to any never so great commodity, if there hang of it any painfull study, or travellsome labour.

Howbeit, yet all men are not of that sort, though the most part be, the more pity it is: but of them that are so glad, not onely with painfull study, and studious pain to attain learning, but also with as great study and pain to communicate their learning to other, and make all England (if it might be) partakers of the same; the most part are such, that unneath they can support their own necessary charges, so that they are not able to bear any charges, in doing of that good that else they desire to do.

But a greater cause of lamentation is this, that when learned men have taken paines to do things for the aid of the unlearned, scarce they shall be allowed for their well doing, but derided and scorned, and so utterly discouraged to take in hand any like enterprise again. So that if any be found (as here

there are ſome) that do favor learning and learned wits, and can be contented to further knowledge, yea only with their word; ſuch perſons though they be rare, yet ſhall they encourage learned men to enterpriſe ſomething at the leaſt that England may rejoice of. And I have good hope that England will (after ſhe hath taken ſome ſure taſte of learning) not only bring forth more favorers of it, but alſo ſuch learned men, that ſhe ſhall be able to compare with any Realm in the world. But in the mean ſeaſon, where ſo few regarders of learning are, how greatly they are to be eſteemed that do favour and further it, my pen will not ſuffice at full to declare.

Therefore, gentle Reader, whereas I do upon moſt juſt occaſion judge, yea and know aſſuredly, that there be ſome men in this Realm, which both love and alſo much deſire to further good learning, & yet am not well able to write their condign praiſe for the ſame, I think it better with ſilẽce to overpaſs it, then either ſay too little of it, or to provoke againſt them the malice of ſuch other, which do nothing theſelvs that is praiſ-worthy, & therefore cannot abide to hear the praiſe of any other mans good indeed.

And conſidering their great favor unto learning, though I my ſelf be not worthy to be reckoned in the number of great learn'd men, yet am I bold to put my ſelf in Preſs, with ſuch ability as God hath lent me, though not with ſo great cunning as many men, yet with as great affection as any man to help my country men, & wil not ceaſ daily, (as much as my ſmal ability will ſuffer me) to endite ſome ſuch thing, that ſhal be to the inſtruction, though not of learn'd men,

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yet at the leaſt of the vulgar ſort, whoſe argument alwaies ſhal be ſuch as it ſhal delight all learned wits though they do not learn any great thing out of it.

But to ſpeak of this preſent book of Arithmetick, I dare not, nor will not ſet it forth with any words, but remit it to the judgement of all gentle Readers, and namely ſuch as love good learning, beſeeching them ſo to eſteem it, as it doth ſeem worthy. And ſo either to accept the thing for it ſelf, either at the leaſt to allow my good endeavour. But I perceive I need not uſe any perſwaſions unto them, whoſe gentle nature and favourable mind is ready to receive thankfully, and interpret to the beſt all ſuch enterpriſes attempted for ſo good an end, though the thing do not always ſatisfie mens expectation. This conſidered, did bolden me to publiſh abroad this little Book of the Art of numbring, which if you ſhall receive favourably, you ſhall encourage me to gratifie you hereafter with ſome greater thing.

And as I judge ſome men of ſo loving a mind to their native Countrey, that they would much rejoice to ſee it proſper in good learning, and witty Arts: ſo I hope well of all the reſt of Engliſhmen, that they will not be unmindfull of his due praiſe, by whoſe means they are helped and furthered in any thing. Neither ought they to eſteem this thing of ſo little value, as many men of little diſcretion oftentimes do. For who ſo ſetteth ſmall price by the witty device and knowledge of numbring, he little conſidereth it to be the chief point, (in manner) whereby men differ from all brute beaſts: for as in all other things (almoſt) beaſts are partakers with us, ſo in nũbering

bering we differ cleane from them, and in manner peculiarly, ſith that in many things they excell us again.

The Fox in crafty wit exceedeth moſt men,
A Dog in ſmelling hath no man his peer.
To foreſight of weather if you look then,
Many beaſts excell men; this is cleer.
The wittineſſe of Elephants doth letters attain,
But what cunning doth there in the Bee remain?
The Emmet foreſeeing the hardneſſe of winter,
Provideth victuals in the time of ſummer.
The Nightingale, the Linet, the Thruſh, the Lark,
In Muſicall harmony paſſe many a Clark.
The Hedghog of Aſtronomy ſeemeth to know,
And ſtoppeth his cave where the wind wil blow.
The Spider in weaving ſuch Art doth ſhow,
No man can him mend, nor follow I trow.
When a houſe will fall, the Mice right quick
Flee thence before; can man doe the like?

Many things elſe of the wittineſs of Beaſts and Birds might I hear ſay, ſave that another time of them I intend to write, wherein they excel in manner all men, as it is daily ſeen: but in number was there never beaſt found ſo cunning, that could know or diſcern one thing from many. By daily experience you may wel conſider, when a Bitch hath many whelps, or a Hen many chickens: and likewiſe of other whatſoever they be, take from them all their young ſaving only one, & you ſhal perceive plainly that they miſs none, though they wil reſiſt you in taking them away, and will ſeek them again if they

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may know where they be, but else they will never misse them truly; but take away that one that is left, and then will they cry and complain; and restore to them that one, then are they pleased again. So that of number, this may I justly say, it is the onely thing almost that separateth man from beasts. He therefore that shall contemn number, declareth himself as brutish as a beast, and unworthy to bee counted in the Fellowship of men. But I trust there is no man so foul overseen, though many right smally do it regard.

Why the Author wrote in Dialogue wise.

Therefore will I now stay to write against such, and return again to this my Book, which I have written in the form of a Dialogue, because I judge that to be the easiest way of instruction, when the Scholar may aske every doubt orderly, and the Master may answer to his question plainly.

Howbeit I thinke not the contrary, but as it is easier to make another mans work then to make the like; so there will be some that will find fault, because I write in a Dialogue: but as I conjecture those shall be such as do not, cannot, or will not perceive the reason of right reaching, and therefore are unmeet to be answered unto, for such men with no reason will be satisfied.

And if any man object, that other bookes have been written of Arithmetick already so sufficiently that I needed not now to put pen to the book, except I will condemn other mens writings: To them I answer: That as I condemn no mans diligence, so I know that no one man can satisfie every man: and therefore like as many doe esteem greatly other

bookes, ſo I doubt not but ſome will like this my booke above any other Engliſh Arithmeticke *hitherto written; and namely, ſuch as ſhall lack inſtructors, for whoſe ſake I have ſo plainly ſet forth the Examples, as no booke that I have ſeen hath done hitherto: which thing ſhall be great eaſe to the rude Readers.*

Therefore (gentle Reader) though this Booke can be but ſmall aid to the learned ſort, yet unto the ſimple ignorant (which needeth moſt help) it may be a good furtherance and mean unto knowledge.

And though unto the King his Majeſty privately I doe it dedicate, yet I doubt not (ſuch is his clemency) but that he can be content, yea, and much deſirous, that all his loving Subjects ſhall take the uſe of it, and imploy the ſame to their moſt profit. Which thing if I perceive that they thankfully do, and receive with as good will as it was written, then will I ſhortly with no leſſe kindneſſe ſet forth ſuch introductions into Geometry, *and* Coſmography, *as I have at times promiſed, and as hitherto in* Engliſh *hath not beene enterpriſed, wherewith I dare ſay all honeſt hearts will be pleaſed, and all ſtudious wits greatly delighted.*

I will ſay no more, but let every man judge as he ſhall ſee cauſe. And thus for this time I will ſtay my Pen, committing you all to that true fountain of perfect number, which wrought the whole world by number and meaſure: he is Trinity in Vnity, and Glory, Amen.

Here

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Before

Before the Introduction of Arithmetick, it were very good to have ſome underſtanding and knowledge of theſe Figures and Notes.

i	1	one	xx	20	twenty
ii	2	two	xl	40	fourty
iii	3	three	l	50	fifty
iiii	4	foure	lx	60	ſixty
v	5	five	lxx	70	ſeventy
vi	6	ſix	xc	90	ninety
vii	7	ſeven	C	100	a hundred
viii	8	eight	CC	200	2 hundred
ix	9	nine	D	500	5 hundred
x	10	ten	DC	600	6 hundred
xi	11	eleven	M	1000	a thouſand
xii	12	twelve	MD	1500	a thou. 5 hund.
&c.	&c.	&c.	&c.	&c.	&c.

A Dialogue betweene the Maſter and the Scholar: teaching the *Art* and uſe of *Arithmetick with Pen.*

The Scholar ſpeaketh.

SIR, *ſuch is your authority in mine eſtimation, that I am content to conſent to your ſaying, and to receive it as truth, though I ſee none other reaſon that doth lead mee thereunto: whereas elſe in mine owne conceit it appeareth but vaine, to beſtow any time privately in learning of that thing that every Childe may, and doth learne at all times and houres, when he doth any thing himſelfe alone, and much more when he talketh or reaſoneth with others.*

Maſter. Lo, this is the faſhion and chance of all them that ſeek to defend their blinde ignorance, that when they thinke they have made ſtrong reaſon for themſelves, then have they pleaded quite contrary. For if numbring be ſo common (as you grant it to be) that no man can doe

any

any thing alone, and much lesse talk or bargain with other, but he shall still have to doe with number: this proveth not number to bee contemptible and vile, but rather right excellent and of high reputation, sith it is the ground of all mens affairs, in that without it no tale can be told, no communication without it can bee continued, no bargaining without it can duly be ended, or no businesse that man hath skilly completed. These commodities if there were none other, are sufficient to approve the worthinesse of number. But there are other innumerable, far passing all these, which declare number to exceed all praise. Wherefore in all great workes are Clerks so much desired? Wherefore are Auditors so richly fed? What causeth Geometricians so highly to be inhanced? Why are Astronomers so greatly advanced? Because that by number such things they finde, which else would far excell mans mind.

Scholar. Verily, Sir, if it be so, that these men by numbring, their cunning do attain, at whose great workes most men doe wonder, then I see well I was much deceived, and numbring is a more cunning thing then I tooke it to be.

Master. If number were so vile a thing as you did esteem it, then need it not to be used so much in mens communication. Exclude number, and answer to this question: How many yeers old are you?

Scholar,

Scholar. Mum.

Master. How many dayes in a weeke? How many weekes in a yeare? What lands hath your Father? How many men doth hee keep? How long is it since you came from him to me?

Scholar. Mum.

Master. So that if number want, you answer all by mummes: How many miles to London?

Scholar. A peak full of plums.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking it maketh men dumb, so that to most questions they must answer mum.

Scholar. This is the cause sir, that I judged it so vile, because it is so common in talking every while: For plenty is not dainty, as the common saying is.

Master. No, nor store is no sore, perceive you this? The more common that the thing is, being needfully required, the better is the thing, and the more to be desired. But in numbring as some of it is light and plain, so the most part is difficult and not easie to attain. The easier part serveth all men in common, and the other requireth some learning. Wherefore as without numbring a man can doe almost nothing, so with the helpe of it you may attaine to all things.

Scholar. Yea, sir, why then it were best to learn

learn the Art of numbring, first of all other learning, and then a man need learn no more if all other come with it.

Master. Nay, not so: but if it be first learned, then shall a man be able (I mean) to learn, perceive and attain to other Sciences; which without it he could never get.

Scholar. I perceive by your former words, that Astronomy and Geometry depend much on the help of numbring: but that other Sciences; as Musick, Physick, Law, Grammer, and such like, have any help of Arithmetick, I perceive not.

Master. I may perceive your great Clerkliness by the ordering of your Sciences: but I will let that passe now, because it toucheth not the matter that I intend, and I will shew you how Arithmetick doth profit in all these, somewhat grosly, according to your small understanding; omitting other reasons more substantiall.

Musick. First (as you reckon them) musicke hath not onely great help of Arithmeticke; but is made, and hath its perfectnesse of it: for all musicke standeth by number and proportion: And in Physicke, besides the calculation of criticall dayes, with other things which I omit, how can any man judge the pulse rightly, that is ignorant of the proportion of numbers.

Law. And as for the Law, it is plain, that the man

man that is ignorant of Arithmeticke, is neither meet to be a Judge, neither an Advocate, nor yet a Proctor. For how can hee well understand another mans cause, appertaining to distribution of goods, or other debts or of summes of money, if he be ignorant of Arithmetick? This oftentimes causeth right to bee hindred, when the Judge either delighteth not to heare of a matter that hee perceiveth not, or cannot judge for lacke of understanding: this commeth by ignorance of Arithmeticke.

Now, as for Grammer, mee thinketh you should not doubt in what it needeth number, sith you have learned that Nouns of all sorts, Pronouns, Verbs and Participles are distinct diversly by numbers: besides the variety of Nouns of Number, and Adverbs. And if you take away number from Grammer, then is all the quantitie of Syllables lost. And many other wayes doth number help Grammer. Whereby were all kindes of Meeters found and made? was it not by number? Grammer.

But how needfull Arithmeticke is to all parts of Philosophy, they may soon see, that doe reade either Aristotle, Plato, or any other Philosophers writings. For all their examples almost, and their probations depend of Arithmetick. It is the saying of Aristotle, that hee that is ignorant of Arithmetick, is meet for no Science. And Plato his Master wrote a little sentence Philosophy.

sentence ouer his Schoolhouse doore, Let none enter in hither (quoth he) that is ignorant of Geometry. Seeing hee would haue all his Scholars expert in Geometry, much rather hee would the same in Arithmetick, without which Geometry cannot stand.

Diuinity. And how needfull Arithmetick is to Diuinity, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much do write of it. And if I should go about to write all the commodities of Arithmeticke in ciuill acts, as in gouernance of Commonweales in time of peace, and in due prouision and order of Armies in time of war, for numbring of the Host, summing of their wages, prouision of Victuals, viewing of Artillerie, with other Armour; beside the cunningest point of all, for casting of ground, for encamping of men, with such other like: And how many waies also Arithmetick is conducible for all priuate Weales, of Lords and all Possessioners, of Merchants, and all other occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receivers, Stewards, Bailiffes, and such like, whose Offices without Arithmetick are nothing: If I should (I say) particularly repeat all such commodities of the noble Science of Arithmetick, it were enough to make a verie great booke.

Armies.

Scholar. No, no, sir, you shall not need: For I doubt not but this, that you haue said, were enough

enough to perſwade any man to think this Art to be right excellent and good: and ſo neceſſary for man, that (as I thinke now) ſo much as a man lacketh of it, ſo much he lacketh of his ſenſe and wit.

Maſter. What, are you ſo farre changed ſince, by hearing theſe few commodities in generall? by likelihood you would be farre changed if you knew all the particular Commodities.

Scholar. I beſeech you Sir, reſerve thoſe Commodities that reſt yet behinde unto their place more convenient: and if yee will be ſo good as to utter at this time this excellent treaſure, ſo that I may be ſomewhat inriched thereby, if ever I ſhall be able, I will requite your paine.

Maſter. I am very glad of your requeſt, and will doe it ſpeedily, ſith that to learn it you be ſo ready.

Scholar. And I to your authority my wit doe ſubdue, whatſoever you ſay, I take it for true. The duty of a Scholar.

Maſter. That is too much, and meet for no man to be believed in all things, without ſhewing of reaſon. Though I might of my ſcholar ſome credence require, yet except I ſhew reaſon, I doe it not deſire. But now ſith you are ſo earneſtly ſet this Art to attaine, beſt it is to omit no time, leſt ſome other paſſion coole this great heat, and then Perſeverance in ſtudy.

you leave off before you see the end.

Scholar. Though many there be so unconstant of mind, that flitter and turn with every winde, which often begin, and never come to the end, I am none of this sort, as I trust you partly know. For by my good will what I once begin, till I have it fully ended, I would never blin.

Master. So have I found you hitherto indeed, and I trust you will increase rather, then goe backe. For better it were never to assay, then to shrinke and fly in the mid way: But I trust you will not doe so; therefore tell me briefly: What call you the Science that you desire so greatly.

Scholar. Why sir, you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to have studied a Science, and yet cannot tell how it is named.

Scholar. Some call it Arsemetrick, and some Augrime.

Master. And what doe these names betoken?

Scholar. That if it please you, of you would I learn.

Master. Both names are corruptly written: Arsemetrick for Arithmetick, as the Greeks call it, and Augrime for Algorisme, as the Arabians found it: which doth betoken

Ἀριθμητική.

ken the Science of Numbring: for Arithmos in Greeke is called Number: and of it commeth Arithmetick, the Art of Numbering. So that Arithmeticke is a Science or Art teaching the manner and use of Numbering: This Art may bee wrought diverſly, with Penne, or with Counters. But I will firſt ſhew you the working with the pen, and then the other in order.

Scholar. This will I remember. But how many things are to bee learned to attaine this Art fully?

Maſter. There are reckoned commonly ſeven parts or workes of it.

Numeration, Addition, Subſtraction, Multiplication, Diviſion, Progreſſion, and Extraction of roots; to theſe ſome men adde Duplication, Triplation, and Mediation. But as for theſe three laſt they are contained under the other ſeven. For Duplation, and Triplation are contained under Multiplication, as it ſhall appear in their place: And Mediation is contained under Diviſion, as I will declare in his place alſo.

Scholar. Yet then there remain the firſt ſeven kinds of Numbring.

Maſter. So there doth: howbeit if I ſhall ſpeake exactly of the parts of Numbering, I muſt make but five of them: for Progreſſion is a compound Operation of Addition, Multiplication and Diviſion. And ſo is the Extraction

tractions of roots. But it is no harm to name them as kinds ſeverall, ſeeing they appear to have ſome ſeverall working. For it ſorteth not ſo much to contend for the number of them, as for the due knowledge and practiſing of them.

Scholar. Then you will that I ſhall name them: as ſeven kindes diſtinct. But now I deſire you to inſtruct mee in the uſe of each of them.

Maſter. So I will, but it muſt bee done in order: for you may not learn the laſt ſo ſoon as the firſt, but you muſt learn them in that order, as I did rehearſe them, if you will learn them ſpeedily and well.

Scholar. Even as you pleaſe. Then to begin: Numeration is the firſt in order, what ſhall I doe with it?

Maſter. Firſt, you muſt know what the thing is, and then after learne the uſe of the ſame.

Numeration.

NUmeration *is that* Arithmeticall *skill, whereby wee way duely value, expreſſe, and read any* Number or Summe *propounded: or elſe in apt* Figures *and* Places *ſet downe any* Number *knowne or named.*

Scholar.

Scholar. Why? then me thinketh you put a difference betweene the Value and the Figures.

Master. Yea so doe I. For the Value is one thing, and the Figures are another thing, and that cometh partly by the diversity of Figures, but chiefly in the places wherein they be set.

Scholar. Then must I know here 3 things, the Value, the Figure, and the Place.

Master. Even so. But yet adde Order to them as the fourth. And first mark, that there are but tenne Figures that are used in Arithmeticke; and of those tenne, one doth signifie nothing, which is made like an o, and is privately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying Figures, and be thus figured. A Cypher. Figures.

1. 2. 3. 4. 5. 6. 7. 8. 9.

And this is their value.

i, ii, iii, iiii, v, vi, vii, viii, ix.

But here you must mark, that every Figure hath two values: One alwayes certain, that it signifieth properly, which it hath of his form, and the other uncertain, which he taketh of his place.

A place is called the seate or roome that a Figure standeth in. And looke how many A place.

Figures are written in one ſumme, ſo many places hath that whole number. And that muſt be called the firſt place, that is next to the right hand, and ſo reckoning by order towards the left hand, ſo that that place is laſt that is next to the left hand. As for example. If there ſtood before you ſix men in a row, ſide by ſide, and you ſhould tell them as they ſtand in order, beginning with the man that were next to your right hand; then he that were next him ſhould be called the ſecond, and ſo forth to the furtheſt from your right hand, which is the firſt and the laſt.

Scholar. Sir, I perceive you well: ſo might I reckon Letters or any other thing. As if I ſhould write eight Letters after this order, a, b, c, d, e, f, g, h. then muſt I ſay, h, is the firſt, g, the ſecond, f, the third, e, the fourth, d, the fifth, c, the ſixth, b, the ſeventh, a, the eighth.

Maſter. That is well done. And after the ſame ſort uſe hereafter, that what I declare by one example, do you expreſſe by another: and ſo ſhall I perceive whether you underſtand it or no. And ſo paſſe over nothing, till you perceive it well, and be expert therein.

Schol. I pray you how many of theſe places be there in all?

Maſter. There is no certain number of them, but they are ſometimes more, and ſometimes fewer, according to the ſum that is expreſſed.

For

For ſo many as the figures are, ſo many are the places: and the laſt place is ſo called, not becauſe it is laſt of all other, but it is the laſt of that preſent ſumme, and it may bee the middle place in another ſumme.

Scholar. Me ſeemeth I perceive this very well, as touching the order of reckoning of the places: but as for the number of them, you ſay there is no certainty. Now there reſteth to declare the value of the figures by the diverſity of places, which you called the value uncertain. Value uncertain.

Maſter. But firſt let me hear whether you know perfectly the certain value. Value certain.

Scholar. Yes ſir, as you wrote them, ſo I marked them.

Maſter. How write you then five?

Scholar. By this figure 5.

Maſter. And how ſix?

Scholar. Thus, 6.

Maſter. Write theſe three numbers, each by it ſelf, as I ſpeak them, vii. iiii. iii.

Scholar. 7. 4. 3.

Maſter. How write you theſe foure other, ii. i. ir. viii.

Scholar. Thus (I trow) 2. 1. 6. 8.

Maſter. Nay there you miſſe: looke on mine example again.

Scholar. Sir, true it is, I was to blame, I take 6 for 9, but I will beware hereafter.

Maſter. Now then take heed, thoſe cer-

tain values every figure representeth when it is alone written without other figures joyned to him. And also when it is in the first place, though many other doe follow: as for example, This figure 9 is it, standing now alone.

Scholar. How is he alone, and standeth in the middle of so many letters?

Master. The letters are none of his fellows. For if you were in France in the middle of a thousand Frenchmen, if there were no Englishman with you, you would reckon your self to be alone.

Scholar. So it is. Then 9 without more figures of Arithmetick betokeneth it. whatsoever other letters be about it.

Master. Even so, and so doth it, if it be in the first place joyned with other, how many soever do follow, as in this example, 3769. You see 9 in the first place, and doth betoken nine as it were alone.

Scholar. I perceive that, and doth not 7 that standeth in the second place, between 7 and 6 in the third place, betoken 60. and so 3 in the fourth place betoken three?

Master. Their figures be as you have said, but their values are not so. For as in the first place every figure betokeneth his own value certain onely, so in the second place every figure betokeneth his own value certain tenne times: as in the example, 7 in the second place

is

is ſeven times ten, and is lxx. And in the third place, every Figure betokeneth his own value an hundred times, ſo the 6 in that place betokeneth vi. C. and in the fourth place every figure betokeneth his owne value a M. times, as in the aforeſaid number 3 in the fourth place ſtandeth for 3 M. and in the fifth place every figure ſtandeth for his own value x M. times, and in the ſixth place a CM. times, and in the ſeventh place a MM times, and in the eighth place xcMM, ſo that every place exceedeth the former ten times.

Scholar. As thus: if I make this number at all adventures, 91359684. here are eight places. In the firſt place is 4 and betokeneth but foure: in the ſecond place is 8 and betokeneth ten times 8 that is 80, in the third place is 6 and betokeneth ſix hundred: in the fourth place 9 is nine thouſand, and 5 in the fifth place is xM times 5, that is fifty M. So 3 in the ſixth place is a CM times 3 that is, CCCM. Then 1 in the ſeventh place, one MM and 9 in the eighth place, ten thouſand thouſand times 9, that is xcMM. But now I cannot eaſily nor quickly read it in order. A generall Rule.

Maſter. That ſhall you practiſe by this meanes. Firſt, put a pricke over the fourth figure, and ſo over the ſeventh. And (if you have ſo many) over the tenth, thirteenth, ſixteenth, and ſo forth, ſtill leaving two figures between each two prickes. And thoſe two

roomes

Ternaries. roomes between the pricks are called Ternaries.

Then begin at the laſt pricke, and ſee how many figures are between him and the end, which cannot paſſe three, reckoning himſelfe for one: then pronounce them as if they were written alone from the reſt, and at the end of their value, ſo many times thouſands as your numbers have pricks.

After that, come to the next three figures, and ſound them as if they were apart from the reſt, and adde to their value ſo many times thouſands, as there are pricks between them, and the firſt place of your whole number. And ſo do by every other three figures following, if you have more. As in example, 913509684 this was your number.

Put a prick over 9 in the fourth place, and over 1 in the ſeventh place, and then no more (for your places come not to ten) as thus: 91359684.

Now go to the laſt prick over 1, and take it and the figure 9 that followeth it, and value them alone.

Scholar. 91, that is xci.

Maſter. So it is, then adde for the number of your pricks twice m.

Scholar. That is, xci. thouſand thouſand.

Maſter. So it is. Then take the three other figures from one to the next prick, and value them.

Scholar.

Scholar. 359. that is, CCC. lix.

Master. Now adde for the one prick, that is between them and the first place, 9.

Scholar. CCC. lix. thousand.

Master. Then come to the other 3 figures that remain.

Schol. 684, that is, of C. lxxxiiii.

Master. Now have you valued all. And at the end of the last number you shall adde nothing, because there remaineth no pricke nor number after it, yet prove it in another number, as thus, 2308640891052̇40.

Schol. 2308640891052̇40. I have pricked them as you taught me, but I am in doubt whether I have done well or no, because of the Cyphers, for I remember you told me that they do signifie nothing, and therefore I doubt whether I should reckon them for a Figure in setting of the pricks; and againe, I know not wherefore they serve.

Master. That will I tell you now. Indeed they are of no value themselves, but they serve to make up the number of places, and so make the Figure following them to bee in a further place, and therefore to signifie the more value: as in this example, so the Cypher is of no value, but yet he occupieth the first place, and causeth 9 to be in the second place, and so to signifie ten times 9, that is 90. so that two Cyphers thrust the Figure following them into the third place, and so forth. The use of Cyphers.

Schol.

Scholar. Then I perceive in the example above, I have pricked well enough, for though that Cypher that is pricked signifie nothing, yet must he have the prick, because he came in the thirteenth place. Then will I prove to number that summe. First, there is 230, ℘, ℘, ℘, ℘ and then followeth 864, ℘, ℘, ℘. And what shall I now do? There is a Cypher in the third place, and no Figure after him, but they that I have reckoned.

Master. He did serve for them that you have already reckoned, to make them in a place further then they should bee, if he were away, and therefore now ye shall let him goe. And so doe alwayes when he occupieth that place next before any pricke, which is the last of that Ternary, and a Cypher in the last place doth nothing.

Scholar. Then shall I say but 89, ℘, ℘.

Master. So, but go forth.

Scholar. 105 thousand. Now are all my pricks spent, and yet remain 340, so that I must name them, CCC.xl. onely.

Master. Now can you reckon after this sort: and remember that every such room, so parted, is called a Ternary, or Trinity, for you have numbred or valued the summe most truly, and by the aid of the pricks each denomination is distinct most plainly.

Trinity.

Denomination. Scholar. What call you Denomination?

Master.

Maſter. It is the laſt value or name added to any ſumme. As when I ſay, an hundred two and twenty pounds: Pounds is the Denomination. And likewiſe in ſaying, 25 men: Men is the Denomination, and ſo of other: But in this place (that I ſpake of before) the laſt number of every Ternary is the Denomination of it. As for the firſt Ternary, the Denomination is Vnites, and of the ſecond Ternary, the Denomination is thouſands, and of the third Ternary, thouſand thouſands, or Millions; of the fourth, thouſand thouſand thouſands, or thouſand millions: and ſo forth.

Scholar. And what ſhall I call the value of the three figures that may bee pronounced before the Denomination, as in ſaying, 203000000. that is, two hundred three millions. I perceive by your words, that millions is the Denomination: but what ſhall I call CCIII. joyned before the millions?

Maſter. That is called the Numerator, or Valuer, and the whole ſumme that reſulteth of them both, is called the Summe, Value or Number.

Numerator.

Summe or value.

Scholar. Now is there any thing elſe to be learned in Numeration? or elſe have I learned it fully?

Maſter. I might ſhew you here who were the firſt Inventors of this Art, and the reaſon of

of all these things that I have taught you, but that I will reserve till ye have learned over all the practise of this Art, lest I should trouble you with over many things at the first.

Three kinds of numbers.

But yet this you must mark, that there are three kindes of Numbers, one called Digits, another Articles, and the third mixt numbers.

Digits.

A Digit is any number under ten, as these: 1.2.3.4.5.6.7.8.9.

Articles.

And 10 with all other that may be divided into ten parts just, and nothing remain, are called articles, such as are 10. 20. 30. 40. 50. &c. 100. 200. &c. 1000. &c.

Mixt.

And that number is called mixt, that containeth articles, or at the least one article, and a Digit, as 12. 16. 19. 21. 38. 107. 1005. and so forth: and for the more ease of understanding and remembrance, mark this. The Digit number is never written with more then one figure, but the article and the mixt number are ever written with more then one figure. And thus they differ, that the article hath evermore this Cypher 0 in the first place: and the mixt number hath ever there some Digit.

Scholar. By these last words I perceive, it much better then I did before, and now (I think) I will never misse to know those three asunder.

Master. If you remember now all that I have said, you have learned sufficiently this first kind of Arithmeticke called Numeration.

How

Howbeit I will exhort you now to remember both this that I have ſaid, and all that I ſhall ſay, and to exerciſe your ſelf in the practiſe of it; for rules without practiſe, are but a light knowledge, and practiſe it is that maketh men perfect and prompt in all things. Vſe maketh maſtery.

And as you have learned to gather and expreſſe the value of a ſumme propounded and ſet downe before you, ſo muſt you practiſe to marke, note, and write down with apt figures and in due places, any number onely named, or recited to you, or if your ſelf imagined; as for a proof. How note you, or write down this ſumme, five thouſand two hundred fifty and ſeven?

Scholar. This troubleth me now, whether I ſhould begin at the firſt, or at the laſt. For reaſon (me thinketh) ſhould cauſe me to begin at the firſt, and yet if I write it as you ſpeake it, I muſt begin at the laſt.

Maſter. When you know your places perfectly, you may begin where you liſt; but the more eaſe for your hand is to beginne with the laſt, that is to ſay, as I did ſpeak them, yet for the more ſurety, a while you may begin with the firſt, repeating my words backward thus: ſeven, fifty, two hundred, five thouſand: or elſe ſounding them all by their digit, or value, as thus: ſeven, five, two, five; for that way is eaſieſt: But then muſt you look well whether there be any cypher in your ſumme, that be

may

may be set in his place: as if the last value of your summe (as you speak it) be above 9, then is there a cypher in the first place. And if it be an hundred, or above, then is there two cyphers, one in the first place, and another in the second, and so forth.

But because this thing is such that cannot be set forth without many words; I think best here now at the end of Numeration, to adde a Table safe and ready for the first exercise of it.

Lo this is the Table.

The



The left hand or side. The right side or hand.

The determinators of the place or value uncertain.	Nine.	Eight.	Seven.	Six.	Five.	Foure.	Three.	Two.	One.	Cyph.	The order of places.
											Ones.
											Tens.
											Hundreds.
											Thousands.
											D. of Thousands.
											C. of Thousands.
											Millions.
											D. of Millions.
											C. of Millions.
											M. of Millions.
											D. M. of Millions.
											C. M. of Millions.

											Fifth.
											Fourth.
											Third.
											Second.
											First.

The names of Digits, values certain, or values.

This Table (as yee may see) hath eleven Places, and in each of them are set all the Digits, whose certain value is written on the right hand of the Table, and the value uncertaid on the left hand; so that by this Table you may learne both how to expresse any Number that you list, if that it exceed not eleven

eleven places) that is to say, ℔ C. thousand Millions, and so may you by help of it, value all sums proposed under the said number.

For example: take the summe that I proposed before which was five thousand, two hundred fifty and seven. And if you will ernestle it, take the first number (as I speak it) which is five ℣, whose valuer or certain value is 5, and his uncertain value, or Denomination is ℣. First, you shall seek at the right hand of the valuer 5. Then seek along under the title of Denomination toward the left hand till you finde thousands, and under it, right at the foote of the Table, is the number of the place that is in the fourth, wherein you must write your Digit, or valuer 5.

Afterward come to the second part of the number two hundred, whose valuer is 2, and his Denomination C. Seek two at the right hand of the Table, and go along under the Denomination toward the left hand, till you come under C. then look to the foot of the Table, and there you shall see the number of the place, that is to say, the third, wherein you must set your Digit 2.

Then do so by your other two numbers that remain, and you shall finde 5 in the second place for your fifty, and 7 in the first place for your seven. And thus you may do with other numbers.

Scholar. Master, I thank you heartily. I perceive you seek to instruct me most plainly and

and briefly, and not to hide your knowledge with fruitless words, as many do. For this rule is so plain, that I can declare it no plainer. And though it seem somewhat long, yet I perceive it to be a sure way.

Master. So it is, and though it be long, yet it is neither too long, neither too plaine for young learners that lack practise: for this Table is in stead of a Teacher to them that lack wit. But now I trust I have said enough of Numeration: which after you have well practised, then may you learn forth.

Scholar. Yet I pray you in one thing to tell me your judgement. Why do men reckon the order of the places backward, from the right hand to the left?

Why numbers are written backward.

Master. In that thing all men do agree, that the Chaldees, which first invented this Art, did set these Figures as they set all their Letters, for they write backward, as you terme it, and so do they read. And that may appear in all Hebrew, Chaldee and Arabick Books; for they be not onely written from the right hand to the left, and so must be read, but also the right end of the book is the beginning of it, whereas the Greeks, Latins, and all Nations of Europe, doe write and reade from the left hand toward the right; and all their Books beginne at the left side.

Scholar. That reason hath satisfied me.

Master. It neither satisfieth me, neither ought me well, because I see that the Chaldees

and Hebrewes doe not so vse their owne Numbers, as at another time I will declare. But this plain reason may best satisfie you presently, that seeing in pronouncing of Numbers wee keep the order of our own reading, from the left hand to the right: and again, wee doe euer name the greater numbers before the smaller: It were reason that the lesser places, containing the lesser numbers, should be set on the right hand, and the greater places containing the greater Numbers to proceed toward the left hand.

Scholar. This reason is to me so plain, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied: hoping that practise shall make me fully ready and expert in it. And in the mean season I desire to learn the other kinds of Arithmetick.

Master. That is well said: but what should you next learn? can you tell?

Scholar. I remember you said, that Addition was next.

Master. Even so, and what that is, must you first know.

Addition

Addition.

Addition is the gathering together and bringing of two numbers, or more into one ſumme. As if I haue 160 Bookes in the Latine tongue, and 136 in the Greeke tongue, and would know how many they bee in all, I muſt write theſe two numbers one ouer another, writing the greateſt number higheſt, ſo that the firſt figure of the one being vnder the firſt figure of the other, and the ſecond vnder the ſecond, and ſo forth in order.

When you haue ſo done, draw vnder them a right line, then will they ſtand thus:

Now begin at the firſt places toward the right hand alwayes, and put together the two firſt figures of theſe two numbers, and looke what commeth of them write vnder them, right vnder the line. As in ſaying 6 and 0 is 6, write 6 vnder 6, as thus:

160
136
6

And then go to the ſecond figures, and doe likewiſe: as ſaying 3 and 6 is 9, write 9 vnder 6 and 3, as here you ſee.

160
136
96

And likewiſe doe you with the figures that be in the third place, ſaying, 1 and 1 be 2, write 2 vnder them, and then will your whole ſumme appear thus:

160
136
296

So that now you see that 160, and 136, doe make in all 296.

Scholar. What? this is very easie to doe, me thinketh I can do it even since.

There came through Cheapside two droves of Cattell: in the first was 848 sheep, and in the second was 186 other Beasts.

Whose two summes I must write as you taught me, thus: Then if I put the two first Figures together, saying, 6 and 8, they make 14. What must I write under 6 and 8, thus:

848
186
14

Master. Not so: and here you are twice deceived. First in going about to adde together two summes of sundry things, which you ought not to doe except you seek onely the number of them, and care not for the things: For the summe that should result to that Addition, should be a summe neither of sheep, nor of other beasts, but a confused summe of both. Notwithstanding ye shall have summes of divers Denominations to be added, of which I will tell you anon: but first I will shew you where you were deceived in another point, and that was in writing 14, which came of 6 and 8, under 6, 8, which is impossible: for how can two figures of two places be written under one figure and one place?

Scholar. Truth it is, but yet I did somewhat understand you.

Master.

Master. I said indeed, that you should write that under them that did result of them both together: which saying is always true, if that summe doe not exceed a Digit. But if it be a mixt number, then must you write the Digit of it under your Figures, as you have said before: and if it be an Article, then write 0 under them, and in both sorts you shall keep the Article in your minde; and therefore when you have added your second Figures, which occupy the place of tens, you shall put that one thereto, which you kept in your minde for, though it were ten indeed, yet in that place it is but as one, because that every one of that place is tenne, for that it is the place of tens. And in like manner, if you have in the second place so great a number that it amounteth above 9, then write the Digit, and reserve the Article in your minde, ever adding it to the next place following, and so of all other places, how many soever you have. And if you have a mixt number when you have added your last Figures, then write the Digit under the last Figures, and the Article in the next place beyond them: so shall your number resulting of Addition, have one place more then the numbers which you shall adde together. A place.

Scholar. Now do I perceive you, and the reason of this is, (as I understand) because that no one place can contain above 9, which is the greatest Figure that is, and then all tens

or Articles must be put to the next place following: for every place (as I may see) exceedeth the other place next before him by 10.

Now, if it please you, I will return to my example of Cattell. But I remember you said I might not adde summes of sundry things together, and that I may see by reason.

Master. Truth it is, if you seeke the due summe of any thing, but if you onely seek a bare summe, and have no respect to the thing, then were it better to name the summe onely without any thing: as in saying 848, without naming sheep or any thing else. And likewise 186, naming nothing.

Now let me see how you can adde those two summes.

Scholar. I must first set them so that the two first figures stand one over another, and the other each one over his fellow of the same place; then shall I draw a line under them both. And so likewise of other figures, setting alwayes the greatest number highest, thus as followeth.

Then must I adde 6 to 8, which make 14, that is a mixt number, therefore must I take the digit which is 4, and write it under 6 and 8, keeping the Article 1 in my minde thus:

848
186
4

Next that, I doe come to the second figures, adding them together, saying, 8 and 4 make 12, to the which I put the one reserved in my minde, and that maketh 13, of which

which number I write the Digit 3 under 8 and 4, and keep the Article in my minde, thus:

```
  848
  186
 ----
   34
```

Then come I to the third Figures, saying, 1 and 8 make 9, and 1 in my mind maketh 10. Sir, shall I write the Cypher under 1 and 8?

Master. Yea.

Scholar. Then of 10 I write the Cypher under 1 and 8, and keep the Article in my mind.

Master. What needeth that, seeing there follow no more Figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last Figures, I must write the Cypher under them, and the Article in a further place after the Cypher thus:

```
  848
  186
 ----
 1034
```

Master. So now you see, that of 848, and 186 added together, there amounteth 1034.

Scholar. Now I thinke I am perfect in Addition.

Master. That will I prove by this example. There are two Armies of Soyldiers: in the one are 106800, and in the other 9400. How many are there in both Armies say you?

Scholar. First, I set them one over another, beginning with the first numbers on the right hand, thus:

```
 106800
   9400
```

But the nether number will not match the over number.

Master.

Master. That followeth not.

Scholar. Then do I adde 0 to 0, and there amounteth 0, that must I write under the first place thus:

```
106800
 9400
------
     0
```

Master. Well said.

Scholar. Then likewise in the second place I adde 0 to 0, and there ariseth 0, which I write under the second place thus:

```
106800
 9400
------
    00
```

Then I come to the third place, saying, 4 and 8 make 12, of which I write the Digit 2, and keep the Article 1 in my mind, thus:

```
106800
 9400
------
   200
```

Then I adde 9 to 6, which make 15, to that I adde the Article 1 that was in my minde, and it is 16, I write 6 under 6 and 9, and keep 1 in mind, thus:

```
106800
 9400
------
  6200
```

Master. Why doe you not write both Figures, seeing you are come to the last couple of numbers.

Scholar. Nay, reason sheweth me, that I must adde that Article that is in my mind unto the next Figure, of the ouer summe, though there be no more in the nether summe.

Master. That is well considered: then do so.

Scholar. Then say I, 0 in the ouer summe, and 1 in my minde maketh 1: that write I under

under 0. Then followeth there yet one more in the ouer ſumme, which hath none to be added to it, for there is none in the nether ſumme, nor yet in my mind: therefore I thinke I muſt write that euen as it is.

Maſter. Yes.

Scholar. Then doth my whole ſumme appeare thus:

	106800
	9400
	116200

Maſter. If you marke this you haue learned perfectly the common Addition of all ſummes which are of one Denomination: ſo that ye obſerue this alſo, that in Addition you muſt haue two numbers at the leaſt: or elſe how can you ſay that you doe adde? And euen let the greateſt number bee written higheſt, for that is the beſt way, though it be not neceſſary.

And forget not this, that (if you haue many numbers to adde together) you ſhall haue oftentimes an Article of a greater value then 10, ſometimes 20, ſometimes 30, ſometimes more, yea (peraduenture) 100. Therefore as you did with the Article 10, ſo doe with them, reſerving them in your minde, and adding to the number next following ſo many, as their valeur or value certain is: that is to ſay, 2 for 20, 3 for 33, 5 for 50, 10 for 100, 12 for 120, and ſo forth of other like. Now (if) the Article be 130, then muſt you ſet downe the 0 and keep 10 in minde, to be caried to the next row of Figures or place (for ſo may ſuch happen

pen to come. For your better understanding take this example for all.

I would adde these thirteen summes into one, which I set after this manner: then doe I beginne and gather the summe of the first row of Figures, which come to 107, (for I take 9 there tenne times, and that is 90) then 9 and 8 is 17, that is in all 107 of which summe I write the 7 under the first row of Figures, and then for that 100 is tenne tennes, I keepe tenne in minde, which tenne I must adde unto the next row of Figures, which are in the second place:

4889
4599
2290
3699
2299
4096
1099
3298
299
699
499
899
389

which second row of Figures (when they are added together with that tenne that I had in my minde) make in all 125, of which summe write the Digit 5 under the second row, and then (for that 120 containeth twelve tennes) I keepe twelve in minde to be added to the third place or row of Figures; which being added together, make in all 60, the Cypher o I set down under the row of Figures in the third place.

And the Figure 6 I keep in minde to be added to the row of Figures in the fourth place, which (when they are added together) make 29. The Figure or Digit 9 I set downe under the fourth place. And because it is my last number, I set downe the 2 also that I have in my

my minde to the 9 in the fifth place; 4889
so those summes doe make in all 4599
29057. 2290

¶ But (for your more ease in 3699
working) when you have an Addi- 2199
tion of so many summes to bee added 4099
together, you were best part that 1099
summe into two or three parts, and 3298
worke them severally, and so put 299
their Additions together, and this 699
were the best thing you could doe 499
when over many summes fall to bee 899
added. 389

Scholar. This seemeth somewhat 29057
hard, by the reason of so many numbers together.

Howbeit, I thinke (if I do often prove, even with the same example, either by working of it alone, or else by parting it as you said even now) that I shall be able to do so shortly with any other summe.

Master. So shall you. For it is often practice that maketh a man quick and ripe in all things: but because, as well in great summes as in small there may chance to be some errour, I will teach you how you shall prove whether you have done well or no.

Scholar. That were a great help and ease.

Master. Begin first with the highest number, and then to all the other orderly, and adde them together, not having regard to these places, but as though they were all Unites: and

The proof of Addition.

and ſtill (as your number ingreaſeth above 9) caſt away 9. Then go forth, ever caſting away 9 as often as it amounteth thereto: and ſo doe till you have gone over all the numbers that you intended firſt to adde; and whatſoever remaineth after ſuch Addition and caſting away of 9, write it in ſome void place by the end of a line, for the better remembrance: and thus is the firſt part of your worke proved. Then ſecondly, put together the Figures that reſult of the Addition under the line, ſtill caſting away 9 alſo. And then that that remaineth write at the other end of the line; and if thoſe two Figures be like, then have you well done, but if they be unlike, then have you miſſed. As for example, in this preſent ſumme. The firſt Figure of the firſt line is 9, let him goe, then 8 and 8 is 16, caſt away 9, there reſteth 7, and adde that 7 to 4 that followeth, and it maketh 11, ſecondly if you take 9, there reſteth 2. Then come to the next row, whoſe firſt and ſecond numbers are 9, therefore ouerpaſſe them both, & take the 5 to the 4 which did remain in the firſt row, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next unto that, goe to the third line, whoſe two firſt numbers you may let paſſe, becauſe they are nought: then take the two Figures of 2, which (with the other two that remained, in the ſecond row) make 6. Then goe to the fourth row, whoſe two firſt numbers let goe,

any

and take the 6 to the 6 that remained, and that maketh 12: take away 9, and there resteth 3, which with the 3 that is next, maketh 6. And so goe through all the other numbers, and you shall finde that there remaineth 5, after you have cast away 9, as often as you can finde it: therefore write 5, at the end of the line in a void place thus:

5 ———— 2

Then gather all the Figures of the Totall summe, which is under the lowest line, and cast away 9, as often as you can find it; as thus: 7 and 5 make 12, take away 9, there resteth 3. to that if you adde the 2 that is left, (for you may omit the 9) then doth it make 5, which 5 you must write at the other end of the line that you made in the void place, thus:

5 ———— 5

And then you see that these two Figures be like, whereby you may know that you have done well, and so you may prove in any other.

Scholar. (If it please you) I will prove in another summe.

Master. With a good will.

Schol. Then will I take one of your former examples, which was this.

First in the highest line 8 and 6 make 14, then 9 taken away, there remains 5. to which I adde the 1 that followeth, and that maketh 6: then come I to the second line, where I finde first 4, which with 6 maketh

106800	
9409	
116209	

both 10, from that I take 9, and there resteth one, the next figure is 9, and therefore I let him alone, so finde I 1 remaining, which I set at the end of a line, thus:

1———

Then I come to the Totall summe, and there I finde that all the figures put together make 10, from which I take 9, and there resteth 1 also, which I put at the other end of the line, thus: 1———1

And because they be like, I know that I have well added.

Maister. So you know now both how to adde two summes or more together, and also how to proove whether you have done well or no: and now I will teach you how to adde summes of divers Denominations together: which thing can never be but when the one Denomination is such that it containeth the other certaine times. And yet you shall adde them to the other, not after this sort (as you did them that were of one Denomination) but after such a sort as I will now shew you, that is to say:

Addition of numbers of divers Denominations.

If you have a summe of divers Denominations, then looke that you set every Denomination by himselfe, with some note or figure of his Denomination, as they are wont to be written. Then write your other summes so under that first, that every one be set under the other of the same Denominations: As for Example, if your Denominations be pounds,

pounds, shillings, and pence, write pounds under pounds, shilings under shillings, and pence under pence: and not shillings under pence, nor pence under pounds.

Scholar. Now that you have spoken it, me thinketh it needeth not to warne me of it, for it were against reason so to confound summes: but yet if you had not spoken of it, peradventure I should have been deceived in it.

Master. If you doe say it is plain, I will speak no more of it, but with an example make the matter to appear evidently.

First, one man oweth me 22 l. 6 s. 8 d. another oweth me 5 l. 16 s. 6 d. and another oweth me 4 l. 3 s. I would know what this is all together: Therefore must I first set down my great summe, and then the other, every one under his Denomination agreeing to the greatest summe, as here you see with a line under them.

l.	s.	d.
22	6	8
5	16	6
4	3	0

Then must I begin at the smallest numbers (which must alwayes be set next to the right) and adde them together: and if the summe will make 1 or 2 or 3 of the next Denomination) then must I keep it in my minde till I come to that place, and under that first place must I note the residue (if there remain any of the same Denomination) but if there remaine none, then need I to write under it nothing. And this is all that you must marke in this Addition; for all other things are like to the

manner of Addition before mentioned: *Therefore the chiefest point of this Addition is, to know the values of common Coines, and rated summes,* As how many shillings be in a pound, how many pence in a shilling, of which (and of other like things) I will instruct you hereafter in teaching of Reduction: But now I may not disturbe your wit from the thing that we are about.

Therefore let us returne to that former example which I proposed of the Debtors: which summes when I had set orderly, they stood thus with a line under them.

Then to adde them into one summe, I must begin at the right hand where the smallest Denomination is, and adde them together, first saying, 6 and 8 make 14. Now seeing these 14 are pence, which contain one shilling and 2 pences: the 2 pences I set down under the line of pence: and the one shilling I keep in my minde to carry to the next row being the place of shillings.

li.	s.	d.
22	6	8
5	16	6
4	3	0
		2

Then doe I adde the shillings together, saying, 1 in my minde and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the second place which standeth for 10, make 26, which is 1 pound 6 s. The 6 s. I set down under the place of shillings, as appeareth in the example. And the 1 pound I keep to carry to the pounds.

li.	s.	d.
22	6	8
5	16	6
4	3	0
	6	2

Then

Then come I to the pounds, adding them all together, saying, 1 that I keep and 4 make 5, and 5 make 10, and 2 make 12. The Figure or Digit 2 I set downe right under that place or row of pounds where I gather them, and the Article 1 I keep to carry to the next place, saying, 1 in minde and 2 is 3, which 3 I set downe directly under the 2. And then appeareth my whole summe thus:

li.	s.	d.
22	6	8
5	16	6
4	3	3
32	6	2

And thus must you doe with any such like summes whatsoever, whether they be money, weight or measure, which (if you practise divers summes) you shall be well acquainted with the feat of Addition.

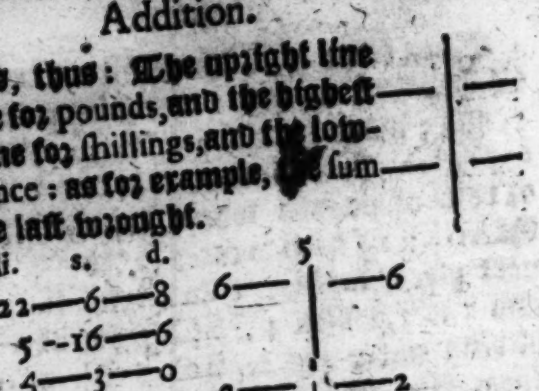
But now can you tell how to prove this Addition, or such other like of divers Denominations, and to try whether you have well done or no?

Scholar. I would I could.

Proof of Addition of divers denominations.

Master. That shall you doe by this meanes. You must make a Crosse which shall have as many lines as you have sundry Denominations in your Addition: As if you have but two Denominations then you may take it thus: that the ouer part and nether part may serue for one Denomination. And if you have three Denominations (as pounds, shillings, and pence) then must you make

three lines, thus: The upright line may serve for pounds, and the highest thwart line for shillings, and the lowest for pence: as for example, the sum which we last wrought.



li.	s.	d.
22	6	8
5	16	6
4	3	0

For the proof of which, because it containeth 3 Denominations, I must make a crosse of 3 lines, as in the example before. Then I reckon first at the right hand the pence, 6 and 8 make 14, from which I take 12 for the next Denomination, that is to say, a shilling, and there resteth 2, which I must write at one end of the nether thwart line.

After that I gather the summe of the shillings 3, 16, 6, which maketh 25, to whom I put 1 that I took of the pence, and that maketh 26, from those I take 20, the quantity of the next greater Denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds, 4 5, and 2, which make 11. to them I adde the one that came of shillings, and they make 12, from whence I cast 9, and there resteth 3, that 3 I joyne to the 2 in the next place, and they make 5, which 5 I set at the Crosse also

alſo. And thus is my firſt part of my worke proved.

That done I come to the totall ſumme under the line, and examine it, beginning at the pence, where I finde but 2, and cannot take 9 from him: therefore I ſet him at the other end of the netherthwart line: then I come to the ſhillings, where I finde onely 6, which (becauſe it is leſſe then nine) I ſet it at the other end of the line of ſhillings, that is, the overmoſt thwart line.

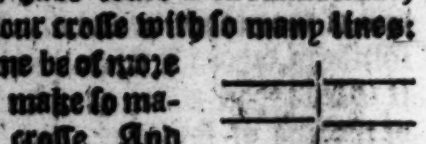
Laſt of all, of the 32 li. I take three times 9, which is 27, and there remaineth 5, which I write under the upright line: either elſe I may reckon them ſimply without any reſpect of their valuation or place: ſaying, 2 and 3 make 5, which becauſe it is leſſe then nine, I ſet under the upright line as before. Then I conſider every number, comparing it to the number that is againſt it: and becauſe I find them to be every one like his match, I know that I have well done.

Scholar. This Croſſe I perceive doth ſerve for theſe 3 Denominations, pounds, ſhillings, pence: but what if I had l. s. d. ob. and qd.

Maſter. Theſe lines as I have ſaid, doe ſerve for 3 Denominations, ſuch as they be, as here 3 doe ſerve for pounds, ſhillings, and pence: but if you have no pounds in your ſumme, then may they ſerve for ſhillings, pence, and half penies: yea, for d. ob. and q. or in weight for C. q. and l. or in meaſure for Elles,

Quarters; and Nailes, if you have no greater Denomination; so that you remember that the upright line serveth for the greatest Denomination, and the highest thwart line for the next, and the lowest for the least.

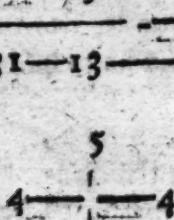
And so if you have foure Denominations, you must make your crosse with so many lines: And if your summe be of more Denominations, make so many lines in your crosse. And thus will I make an end of Addition, saving that here (for the better understanding of this Rule) I have set you downe certain examples both of money, weight, and measures with their workes and proofs.



Examples of Addition.

li.	s.	d.
23	10	4
45	6	8
37	2	9
25	13	6
131	13	3

li.	s.	d.
130	17	10
28		8
13	13	4
120	0	0
292	17	10



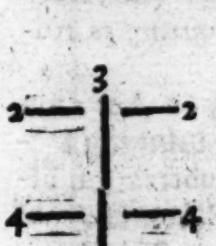
The proofs.



C.

C.	q.	li.
34	1	3
12	2	2
7	3	4
13	0	13
57	2	22

yards	q.	nayles.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3





Subtraction.

Scholar.

WHen have I learned the two first kindes of Arithmetick: *now as I remember*) *doth follow Subtraction, whose name* (*me thinketh*) *doth sound contrary to* Addition.

Subtraction.

Master. So it is indeed: for, as Addition increaseth one grosse summe, by bringing many into one: so contrariwise, Subtraction diminisheth a grosse summe by withdrawing of other from it. *So that* Subtraction *or Rebating is nothing else but an* Art *to withdraw and abate one summe from another, that the remainer may appear.*

Scholar. What doe you call the Remainer?

Master. That you may perceive by the name.

Scholar. So me thinketh: but yet it is good to aske the truth of all such things, lest in trusting to mine owne conjecture, I be deceived.

Master. So it is the surest way. And, as I see cause, I will still declare things unto you so plainly, that you shall not need to doubt. Howbeit, if I doe overpasse it sometimes, (as the manner of men is to forget the small knowledge of them to whom they speake) then doe you put mee in remembrance your selfe,

selfe, and that way is surest.

And as for this word that you last asked me, take you this description: The Remainer is a summe left after one Subtraction made, which declareth the excesse or difference of the two other numbers, as if I would abate or subtract 14 out of 18, there should remain 4, which is called the Remainer, and is the difference betweene those two numbers 14 and 18. Remainer.

Scholar. I perceive then what Subtraction is; now resteth to know the order how to worke it.

Master. That shall you doe by this means. First, you must consider, that if you should goe about to rebate, you must have two sundry summes proposed: The first, which is your grosse summe, (or summe totall) and it must be set highest: and then the rebatement (or summe to be withdrawn) which must be set under the first, (whether it be in one parcell or in many) and that in such sort, that the first figures be one just over another, and so the second and third, and all other following, as you did in Addition: then shall you draw under them a line, and so are your summes duely set to begin your working.

Then begin you at the right hand (as you did in Addition) and withdraw the nether number out of the higher, and if there remain any thing, write that right under them beneath the line: and if there remain nothing

(by

(by reason that the two Figures were equall) then writs under them a Cypher of nought: And so doe you with all the other Figures, evermore abating the lower out of the higher, and writs under them the Remainder still, till you come to the end: And so will there appear under the line what remaineth of your grosse summe after you have deducted the other summe from it, as in this example,

I received of your Father 48 s, of which I have laid out for you 36 s. now would I know what doth remaine. And therefore I set my number thus in order. First, I writs the greatest summe, and under him the lesser, so that the Figures at the right hoe be even one under another, and so the other, thus.

48
36

Then do I rebate 6 out of 8, and there resteth 2, which I writs under them right beneath the line thus.

48
36
2

Then I go to the second Figures, and do rebate 3 out of 4, where there remaineth 1, which I writs under them right, and then the whole summe and operation appeareth thus.

48
36
12

Whereby it appeareth, that if I withdraw 36 out of 48 there remaineth 12.

Scholar.

Scholar. Now will I prove in a greater summe, and I will subtract 2367924 out of 3468946, those summes I set in order thus:

3468946	
2367924	

Then do I begin at the right hand, and deduct 4 out of 6, and there resteth 2, which I write under them. Then go I to the second figures, and withdraw 2 out 4, and there remaineth 2, which I set under them also, then I take 9 out of 9 and there resteth 0, which I write under them (for you say, that if the Figures be equall, so that nothing doth remain, I must write the Cypher 0 under them.)

Master. It was well remembred: now go forth.

Scholar. Then I come to the fourth place, and draw 7 out of 8, and there remaineth 1, which I write under them also. Then in the fifth place, I take 6 out of 6, and there resteth 0, (for if I write under them the Cypher 0.) Then in the sixth place 3 rebated from 4 there remaineth 1, which I write under them, and likewise in the seventh and last place, 2 taken from 3 there is left 1. which I write under them: so have I done my whole working, and my summes doe appeare thus. Whereby I see, that (if I doe rebate 2367924 out of 3468946) there remaineth 1101022.

3468946	
2367924	
1101022	

Master. This is well done. And that you may be sure to perceive fully the Art of Subtraction,

traction, let mee see how you can subtract 52984732 out of 8250003456.

Scholar. First, I set downe the greatest summe, and after that I write under it the lesser number, beginning at the right side, and then my Figures will stand thus:

8250003456.
52984732

Note. Then take I 2 from 6, and the rest is 4. which I write under them. Then do I withdraw 3 from 5, and there remains 2, which I write under them. Then take I 7 out of 4, but that I cannot, what shall I now do?

Master. Marke well what I shall tell you now, how you shall doe in this case, and in all other the like: If any figure of the nether sum be greater then the figure of the summe that is over him (so that it cannot be taken out of the figure over him) then must you put 10 to the over figure, and then consider how much it is, and out of that whole summe withdraw the nether figure, and write the rest under them. Can you remember this?

Scholar. Yes, that I trust I shall. Now then in mine example where I should have taken 7 out of 4, and could not, I put 10 to that 4, which maketh 14, from it I take away 7 and there resteth 7 also, which I write under them.

Master. So have you done well: But now must you marke another thing also: that (when-

(whensoever you do so put ten to any Figure of the ober number) you must adde one still to the Figure or place that followeth next in the nether line: as in the example there followeth 4, to which you must put 1, and make him 5, and then go on as I have taught you.

8250003456	
5298473 2	
0187322	

Scholar. Then shall I say, 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3. but that cannot be; therefore I must put to it also 10, and then it will be 13, from which I take 5, and there resteth 8 to be written under them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the nether line, and that maketh 9, which I should take out of 0 and cannot; therefore I put thereto 10, and that maketh 10, from 10 I take 9, and there remains 1, which I write under them.

Thus do I adde 1 likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure above, but I cannot, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written under them.

Then come I to the next figure which is 2, and to him I doe adde 1, which maketh 3, that 3 I cannot take out of nought, therefore of that nought I make 10, and thence doe take 3. so there remaineth 7 to be written under them: likewise doe I put 1 to 5,

that

that 5, and make it 15, from which I rebate 6 and there remaineth 9, which I write under them. Now have I spent all the nether Figures, and what shall I do more?

8250003456
4298473 2
8197018724

Master. You should have added one to the next figure following (if there had been any) because you added 10 to the last figure before of the ober line: but being there is no figure following, you must adde that one to the place following, and then deduct that one from the number above.

Scholar. Then shall I say, because I borrowed 10 to the ober 5, I must put 1 in the next place beneath, that is under 2, then must I substract that 1 from 2, and there resteth 1 to bee written under that in the ninth place. Now I have no more to substract, for there is not any figure remaining beneath, neither yet any unite to bee added, because I borrowed not 10 to the Figure last before: and yet in there 8 remaining in the ober line, which I thinke (by reason) should be set at the end of the Figures in the lowest row, which is under the line, for because there was nothing taken from it.

Master. What is well considered, and reason toucheth so indeed.

Scholar. But Sir, I beseech you, shall I alwayes when any number so remaineth alone, as thus 8 did, write him under the line straight

straight against his own place?

Master. Yes, what else? whether they be one or many: and this well remembred, you have sufficiently learned Subtraction; Notwithstanding, because of certain things that might deceive you, if you did not take good heed to your working, I will propose to you another example of many numbers to be subtracted, as thus: I received of a friend of mine to keep 2869 Crowns, of which at one time I delivered him again 500. at another time 368, at another time 440, at another time 80, and another time 64, now would I know how many doe rest behinde. Therefore first I set down my grosse sum, and underneath it I set all the parcells thus, and under them a double line.

2869 *Crowns received.*

500
368
440 } *delivered.*
80
64

Then first I begin at the first place, and gather together the sum of all those lines (save the overmost) in their first Figures: and so I doe with all the figures of the second place, and so forth, as I did in Addition, save that I leave out the highest row of numbers (as the line warneth me) and that summe so gathered between the double line, is the summe delivered in all: which summe I doe afterwards subtract out of the highest row of numbers, and the remainder doe I set under the nethermost line:

Note.

as

as for example.

I set the summes as before: then do I gather the first figures of all the places delivered together: where I finde but 4 and 8, that maketh 12, (for three Cyphers increase no sum in Addition, as you learned before,) of the 12 therefore doe I write the Digit 2 between the double line and keep the Article in my minde, till I come to the second place, where I finde 6, 8, 4, 6, that maketh 24, to them I put the Article in my minde, and it is 25, of which I write under the second place, and keepe the Digit 2 in my minde for the third place, where I finde 4, 3, 5, that makes 12, to the which I adde the 2 in my mind, and it maketh 14, thereof I write the 4 under the third place, and because there remains no more Figures to be added, I write the Digit in the fourth place, as you see in the Example, and so it appeareth, I have delivered in all a thousand foure hundred fifty two Crowns.

2869	*Crowns received.*
500	*Delivered.*
368	
440	
80	
64	
1452	*Delivered in all*
1417	*Rest behinde.*

Then come I to the subtracting of this summe between the lines, for by Addition it is equall to the five parcells over it. Therefore I proceede to subtract it from the ovvermost summe, saying, 2 from 9 remains 7 to be written under them beneath the lowest line.

Then in the second place I take 5 from 6 and there resteth 1 to be written under them. Then in the third place, 4 from 8, resteth 4. Last of all in the fourth place, 1 from 2 remaineth 1. And thus I see that after those five sums are subtracted from 1869, the Remainer is 1417.

Scholar. This I perceive: but is there no shorter way and more speedy?

Maister. Yes, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest sum. But if in quantity those numbers added together, exceed the highest sum, or upper number, then shall you (as before hath been taught you) imagine to borrow 10, 20, or 30 more, as need shall require, and put them to the upper number, to help to further the abatement, reserving or restoring the Articles that you borrowed to the next place again: and so still goe forward till you have ended your worke: as for example. In the last summe proposed, I gather first in the first place 4 and 8 that maketh 12, which 12 I should deduct or take out of 9 in the upper number above the line, but I cannot; and therefore I adde unto 9 an Article of 10, and maketh the upper number 19, from whence I take 12, then there resteth 7, then for the Article 10 I adde to the next place of money delivered; saying, 1 that I bring and 6 make 7; and 8 make 15, and 4 make 19 and 6 make 25.

An abridgment of the former manner of Subtraction.

which 25 I should take out of 6 in the upper number, but I cannot. Therefore I adde 2 tens, or 20 unto 6 in the upper number, and that maketh 26, then 25 out of 26, resteth 1; then the tens which I borrowed, or have in minde, I adde to the next row or summe delivered; saying, 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14; then 14 out of 8 I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to bee added, the one that I borrowed, or have in minde, I rebate from 2 in the upper line, and there remaineth 1, which I set down in the remainer line: and so my summe appeareth (as before) to be 14 17 Crownes.

Lo thus have you now a shorter way.

Scholar. I like both wayes well: and I perceive both well: yet, as in the working seemeth somewhat long, so in the other it leaveth very much (mee seemeth) to remembrance, and therefore may cause errour quickly, except a man have a quick and an exercised remembrance. But yet for the sharpning of my wit by your patience (if you will give me leave) I will try what I can do in a like summe, to work it the shortest way: wherepon I would subtract out of 40301964, these three parcels.

Then

Therefore I set them first in due order: then I gather the parcells of the first place, which are 8. 2. 1. that is 11, which I should take or deduct out of 4, which is ober him, but I cannot; therefore I adde an Article, or one tenne to 4, which maketh 14, then 11 out of 14, there resteth 3 to bee written under the first place between the two lines.

49301964	*Charge.*
20603428	*Discharge.*
10002432	
10101461	
43	

Then come I to the second place, saying, 1 that I borrowed to have in my minde, and 6 make 7, and 3 make 10, and 2 make 12, which I cannot take from 6, therefore I adde 10 to 6, which maketh 16, and then 12 from 16 resteth 4, which I write under the second place between the two lines.

Then come I to the third place saying, 1 that I borrowed or have in mind and 4 make 5 and 4 is 9, and 4 make 13, which I should take out of 9 that is ober them, but I cannot: therefore I adde 10 to 9 which make 19 then 13 out of 19 resteth 6.

Then come I to the fourth place, saying, 1 in minde and 1 is 2, and 2 is 4 and 3 make 7. which because it cannot be taken from 1, I take it from 11, and there resteth 4.

After that I come to the fifth place, where are onely three Cyphers, which make nothing,

unto which I adde 1 in minde, then should I take that (that is to say) 1 from the Figure over them, which is also a Cypher: therefore I say thus, I cannot take 1 from 0, but 1 from 10 remaineth 9: so must I write 9 under them. Then in the fifth place I find but 1 & 1 in minde make 2, which I take out of 3 over him, and the remainer is 1: that must bee written betweene the two lines in the fifth place. So I goe to the seventh place, where I finde onely Cyphers, and in the grosse summe over them a Cypher also: therefore must I write the remainer (which is nothing) with a Cypher also. Then in the eight and last place, I gather 1, 1, 2. that maketh 4, which if I take out of that 4 that is over them, there will nothing remaine. And that must be noted with a Cypher between the two lines (as I have often said) and so have I ended my worke, and the figures stand as followeth.

Scholar. But Sir, I remember you taught me that Cyphers should not come in the last place, for because they serve onely to increase the value of other Figures which follow them and serve not those figures that goe before them: and now in my Example I have set two Cyphers in the two last places.

Master. I commend you for your remembrance. And truth it is, you should not have set them here, but onely because that I would make you plainly to perceive the Art of Subtraction,

traction. Therefore seeing that you doe now perceive it, whensoever you would write down a Cypher, looke whether any other figures be yet behind: and if not, then let go the o also, for it needeth not to write him in the latter places, where no other figure doth follow, except it be (as I did now suffer you) to teach the use of Subtraction the plainer.

Therefore your figures must stand thus when the worke is ended.

40301964	*Charge.*
20003428	*Dischar.*
10002432	
10101461	
194643	*Rest.*

Scholar. Sir, I do thinke with that that you taught me before, and by these two sums that you taught mee last also, that now I could Subtract any summe.

Master. So may you, if you have marked what I have taught you. But, because this thing (as all other) must be learned surely by often practise, I will propound here two Examples to you: wherein if you often exercise your selfe, you shall be ripe and perfect to subtract any other summe lightly, for in them is contained all the observances of whole numbers. And because you shall perceive somewhat both how to doe it, and also whether it be well done when you have proved to doe it; therefore have I written under them both the Remainers.

30606.	*Lent.*	308964.	*Debt.*
10354		103145	
10249	*Paid.*	102597	*Paid.*
163		101024	
20766	*Paid in all.*	02198	*Reſt.*
9840	*Reſt to pay.*		

Scholar. Sir, I thanke you: but I thinke I might the better doe it, if you did ſhew me the working of it.

Maſter. Yea, but you muſt prove your ſelfe to do ſome things without my aid, or elſe you ſhall not be able to doe any more then you are taught: And that were rather to learne by rote (as they call it) then by reaſon. And again, there is nothing in theſe examples, or any other of whole numbers, but I have taught you the rules of them already.

Scholar. Then I muſt by practiſe to attain the uſe of it. And is this all that I ſhall learne of Subtraction?

Maſter. Yea, ſaving that (as you have ſeen in Addition) there are numbers of divers Denominations, in which the working is not much unlike: yet (without ſome inſtructions be given of it) it might ſeem to a learner more difficult then indeed it is. Therefore I will briefly ſhew you the uſe of it onely by an example or two.

A certain man owed to mee: 14 l. 12 s. 8 d.
of which he paid mee at one time 4 l. 6 s. 8 d.
at another time 3 l. *at another* 2 l. 3 s. 4 d.
and laſt of all 6s. 8 d.

Now would I know what remaineth unpaid yet? therefore I ſet my ſums thus, every one in their due place: As pounds under pounds, ſhillings under ſhillings, pence under pence.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholar. Sir, I pray you why do you write 2 l. for the common ſpeech uſeth rather to ſay, 40 s.

Maſter. We muſt here uſe the Denomination that is greateſt in any ſumme, ſo that we may not write according as wee uſe to ſpeake, ſaying, 16 d. 18 d. or likewiſe 7 groats, 8 groats, 24 s. 40 s. 48 s. and ſuch other: but we muſt write every Denomination, that is in any ſumme, by it ſelf. Note how the penne differeth from the common order of Counters.

Namely, ſhillings and pounds. So muſt we write for the laſt ſummes now named, 16 d. 18 d. 1 s. 6 d. 2 s. 4 d. 24 s. 8 d. 1 l. 4 s. 2 l. 8 s. and ſo forth of other like.

Scholar. So that wee may not write in Arithmetick, pence, when the ſumme amounteth to ſhillings, nor ſhillings, when the ſumme maketh pounds. Now (if it pleaſe you) end your example.

Maſter. When my ſummes are ſo ſet as I ſhewed, then (according to the rules of Addition) I gather all the particular ſummes which bee paid mee into one totall ſumme, directly to be ſet under them between the two lines, not medling with the 14 l. 12 s. 8 d. as the line warneth mee: therefore muſt I beginne with the ſmalleſt Denomination, ſaying, 8, 4, 8, is 20, pence, which maketh one ſhilling and 8 pence, the 8 d. I ſet downe under the place of pence, and the one ſhilling I keep in minde to carry to the next Denomination of ſhillings. Then come I to the ſhillings, and ſay, one that I bring in minde, and 6 is 7, and 3 is 10 and 6 makes 16, which, becauſe it containeth not one pound, I ſet directly under the place of ſhillings. Then come I to the pounds, whoſe parcels are 2, 3, 4. that is in all 9, that 9 doe I ſet down directly under the pounds: And ſo the totall or whole Addition of all the particulars paid, amounteth to 91. 16 s. 8 d.

li.	s.	d.	
14	12	8	
4	6	8	}
3	0	0	}
2	3	4	}
	6	8	}
9	16	8	
4	16	0	*Reſt.*

Now for the proofe of Subtraction, I muſt rebate that totall ſumme of Addition out of the

the higheſt number, that is to ſay, from the 14 l. 12 s. 8 d.

Therefore to perform the worke I ſay, 8 d. out of 8 d. remaineth or reſteth nothing, therefore in the place of the reſt or remain, right under the Denomination, I ſet down o. Then comming to the ſhillings, where I finde 16. which ſhould be taken out of 12, but I cannot: therefore I imagine to borrow 1 of the next Denomination, that is, of the 14 l. and put that one pound ſo borrowed unto 12 s. that maketh 32 s.

Now 16 s. out of 32 s. reſteth 16 s. which 16 s. I ſet down directly under the place of the reſt.

Laſtly, comming to the pounds, ſaying, one pound in minde that I borrowed, and 9 make 10, then 10 out of 14, there reſteth 4.

So doth my whole reſt or remain, appear to be 4 l. 16 s. 0 d.

This I account the eaſieſt way for a young beginner to practiſe, though it be ſomething long.

Scholar. Is there any ſhorter way for this worke alſo?

Maſter. Yes, as in this laſt Example I will alſo ſhew you, for you may adde together the particular ſummes as

they

they are set in order, beginning with the pence, saying, 8, 4, 8, make 20 d. which 20 d. you should take out of the 8 d. above the line, but you cannot, therefore shall you borrow 1 of the next Denomination, that is to say, 1 of the shillings, and put it to the 8 d. that maketh 20 d. now 20, out of 20 d. resteth 0, which Cypher I set down directly under them.

li.	s.	d.
14	12	8
4	6	8
3	0	0
1	3	4
0	6	8
4	16	0

Then one shilling that I borrowed or had in mind, and 6 make 7, and 3 make 10, and 6 make 16, the 16 out of 12 I cannot take, therefore of the next Denomination I doe borrow one l. and put it to 12 s. which maketh 32 s. then 16 s. out of 32 s. resteth 16 s.

Lastly, I come to the pounds, saying, 1 l. in mind, as that I borrowed, and 2 make 3. and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remainer or rest appeare as before to be 4 l. 16 s. 0 d.

Scholar. Then doe I perceive very well, and if there be no other thing to bee learned in Substraction, then may I come to Multiplication, for that you reckoned to be next in order.

Master. Wee have done indeede with the Art of Subtraction, as touching the working.

But yet before we go to Multiplication, I will

will instruct you how to examine your works, whether it be well done or not. For the performance whereof, if you marke what I said in Addition, you may easily perceive what is to be done for the proofe of Subtraction, which is best made by the aide of Addition thus. Proofe of Subtraction.

Draw under the lowest number (which is your Remainer) a line, and then adde this Remainer and all the other that you did subtract before, together, and write that that ariseth under the lower line: and if the summe that cometh thereof, be equall to the highest of the Subtraction, then is the Subtraction well wrought, or else not: As you may see for example in the summes set down before, and first in summes of one Denomination, whereof, one were this.

Where the Number 529847 32 is subtracted	8250003456 529847 32
from 8250003456. and the Remainer is 8197018724	8197018724

Now to prove whether it be truly wrought or not, I adde the Remainer and the number subtracted together, beginning at the right hand; and first I say 4 and 2 is 6 which is set under the line: Example in a sum of one Denomination.

The number given	8250003456
The number to Subtract	529847 32
The Remainer	8197018724
The Proofe.	8250003456

Then again in the second place I say 2 and 3 is 5. which I write under, next that in the third

third place, 7 and 7 are 14, of which I write the Digit 4, and keepe the Article 1 in my minde. Then in the fourth place 8 and 4 is 12 and 1 in my minde maketh 13, whereof I write downe the Digit 3, and keep the Article 1 in my minde. Again in the fifth place, 1 and 8 is 9, and 1 in my minde is 10. Whereof I set downe 0 and keepe the 1 in my minde. And so going on to the rest (as it is taught in Addition) when I have made an end, I see that the lowest line of numbers and the highest be alike: wherefore I know that I have well done.

So likewise the Proofe is to bee made in numbers of divers Denominations: as for Example, in our summe of that kinde which in the first forme of working stood thus: (all the particular numbers to be subtracted, being drawne into one.)

Example in a sum of divers Denominations.

Where, in the title of pence, I find 8 and 0: the 8 I set downe directly under in that of pence.

Then in the place of shillings I finde 16 and 16 which make 32 shillings, wherein is contained 1 l. and 12 s. the 12 s. I set downe directly under them in the one place

li.	s.	d.
14	12	8
4	6	8
3	0	4
2	3	0
6		8
Paid in all.	9	16-8
Rest.	4	16-0
Proofe	14	12-8

of

of ſhillings, and one pound I keepe.

Then comming to the pounds, I ſay 1 that I keepe, and 4 is 5, and 9 is 14, which 14 in due order I ſet downe directly under them as this Figure ſheweth. And the whole ſumme is 14 l. 12 s. 8 d. agreeing with the upper number above. So I finde the worke is good, and the Subtraction well wrought.

The ſame thing is to be done for the latter forme of Subtraction (where the particular ſummes are not gathered together into one groſſe.) For the Remainer and all the particular ſummes ſubtracted, being added together, if the ſumme that commeth thereof bee equall to the higheſt number above, then is the Subtraction well wrought, or elſe not.

As for example alſo in the laſt ſums which ſtood thus,

First in the title of pence, I adde 8, 4, 8, that maketh 20 d. which containeth 1 ſhilling and 8 pence.

The 8 I ſet down under the loweſt line in the row of title of pence, and that 1 ſhilling I keep to carry the next Denomination or place of ſhillings.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

Example of a proof in the latter forme of Subtraction.

Then returning to the ſhillings, ſaying: one

one in minde, or that I keep, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 12 shillings, which amounteth to one pound, 22 s. the 12 s. I set down under the title of shillings: and 1 pound I keep or have in minde to carry to the next Denomination or place of pounds. Then come I to the pounds, saying, 1 that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14; then doe I write 14 under the pounds, and so have I ended the Addition: and I see that the lowest line is like unto the uppermost line in number, wherefore I know that I have well done.

And thus have I taught you the Art of Subtraction, and the means to prove whether it be well wrought or not. Therefore now will I make an end thereof, and will instruct you in Multiplication.

Multi-

Multiplication.

MUltiplication is an operation whereby two sums produce the third: which third sum so many times shall contain the first, as there are Vnites in the second. And it serveth in stead of many Additions. As for Example: When I would know how many are 30 times 48. if I should adde 48 thirty times, it would bee a long worke. Therefore was this worke of Multiplication devised, which shall doe that at once that Addition should do at many times. Multiplication what it is.

Scholar. I perceive the commodity of it partly, but I shall not see the full profit of it till I know the whole use of it. Therefore Sir, I beseech you, teach me the working of it.

Master. So I judge it best, but because that great summes cannot be multiplied but by the Multiplication of Digits, therefore I thinke it best to shew you the way of multiplying them: As when I say, 9 times 8, or 8 times 9, &c. And as for the small Digits, under 5 it were but folly to teach any rule, seeing they are so easie that every childe can doe it: but for the Multiplication of the greater Digits, thus shall you do. Multiplication of Digits.

First, set your Digits one right over the other,

other then from the uppermost downwards, and from the nethermost upward, draw straight lines, so that they make a crosse, commonly called Saint Andrews crosse, as you see here. Then looke how many each of them lacketh of 10, and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I must write those Digits thus.

The difference.

Digit difference.

8
X
7

Then doe I looke how much 8 doth differ from 10, and I finde it to be 2: that 2 doe I write at the right hand of 8, at the end of the line thus.

Digit difference.

8 2
X
7

After that I take the difference of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you see in this example.

Digit difference.

8 2
X
7 3

Then doe I draw a line under them, as in Addition, thus.

Last of all, I multiply the two differences, saying 2 times 3 make 6, that must I ever set under the differences, beneath the line: then must I take one of the differences (which I will, for all is like) from the other Digit (not from his owne) as the lines of the Crosse warne

learne it by rote, and that that is left, must I write under the Digits. As in this example, if I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write under the Digits, and then there appeareth the multiplication of 7 times 8 to be 56. And so likewise of any other Digits, if they be above 5, for if they be under 5, then will their difference be greater then themselves, so that they cannot be taken out of them. And again, such little summes every childe can multiply, as to say, 2 times 3, or 4 times 5, and such like.

Digit difference.

8 3

7 3

5 6

Scholar. Truth it is. And seeing me seemeth that I understand the multiplying of the greater Digits, I will prove by an example how I can do it. I would know how many are 9 times 6.

Master. It is all one in value to say 9 times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying, 6 times 9, and then all other summes.

Scholar. Then would I know how many are 6 times 9: therefore I put the Digits thus, and make the crosse, thus.

9

6

Then

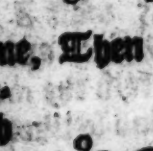
Then doe I set their differences from 10 at the right side, the difference of 9, which is 1, against it, and the difference of 6 which is 4, against it also, as in this example.



And under them draw a line, multiply the differences together, saying: 1 time 4 maketh 4, that 4 do I write under them thus.



Then take I one of the differences from the other Digit, as, 1 from 6, or else 4 from 9, and each wayes there resteth 5, which I doe write under the Digits thus. And so appeareth the multiplication of 6 times 9 to bee 54. Thus I see the feate of this manner of multiplication of Digits.



Then doe I

Master. Now might you go straight to the multiplication of great numbers, save that both for your ease and surety in working I will draw you here a Table, whereby shall appear the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seek your first or last Digit in the greater figures, and from it go right forth towards the right hand, till you come under the number of your second Digit, which is in the highest row, and then the number that is in the meeting of the rowes of little squares (which come directly from both your propounded Digits) is the Multiplication that amounteth of them. As if I would know by this Table the multiplication of 7 times 9, seek first 7 in the greater figures, and then go right forth toward the right hand, till you come under 9 of the highest row, in which place where you so come under the other Digit (as here for example you come under 9) is always contained the off-come or product, which you seeke,

and that place we terme to be in the common angle, in respect of the two numbers so taken on the outsides: as here in that common angle, where the rows of little squares directly proceeding from 7 and 9 do meet, you have 63, which 63 is the summe of the multiplication of 9 by 7.

To multiply greater summes.

Scholar. This is very good and ready. And so may I find the multiplication of any Digits: but now how shall I do in greater summes.

Multiplier. Times.

Master. *When you would multiply any* summe *by another, you shall mark that it is the meeteſt order to ſet the greateſt* number *higheſt, which is the place of the* number *that muſt be multiplied: and likewiſe the leſſer* number *under it, for that is the place of the* Multiplier *or* Multiplicator, *that is to ſay, the* number *by which the* Multiplication *is made, and is in* Engliſh *alwayes put before this word, Times: in ſuch ſpeaking when I ſay* 20 *times* 70. *And the* number *that followeth this word Times, is that which muſt be* Multiplied.

Therefore when I would multiply one number by another, I muſt write the greateſt highest, and the leſſer under it, as in Addition. And under them muſt I draw a line. As for example. If I would multiply 264 by 29, I muſt ſet them thus.

264	
29	

¶. Of which number thus ſet downe to be multiplied, may be formed a queſtion, as thus, There are 29 men, and each man hath

264

264 Lambes. The question is, how many Lambes they have in all.

To the performance whereof, I must multiply every figure of the high row, by every figure of the nether row: and that that amounteth I must set under the line, as thus.

First I doe multiply 4 by 9, saying: 9 times 4 (or 4 times 9 which is all one) and that maketh 36. as the Table before of Digits doth declare, of that 36 I must write the 6, that is the Digit, under the 9, and the Article 3 I keep in mind to carry to the next place.

264	
29	
	6

Then come I to the second figure of the higher row, which is 6, and say: 9 times 6 make 54, and with the 3 in my minde make 57. the 7 I set downe under the 2, and 5 I keep in mind.

264	
29	
	76

After that I come to the next figure, which is 2, and multiply it by 9, and that maketh 18, and with 5 that I have in minde, maketh 23: wherefore because it is the last figure of the Multiplier, I set it down in order as you see:

264	
29	
2376	

And so have I ended the first figure of the Multiplier. Wherefore I give it now a dash with my pen.

Then begin I with the next figure, and multiply it into all the higher figures as thus,

264
29
2376

First, 2 times 4 make 8, that 8 do I write under the second place: for evermore the Digit or first figure of the Multiplication that amounteth of the figure of the higher number, must be set under the Multiplyer of it, the other in their order toward the left hand.

Scholar. I understand you thus, that the Digit of the summe amounting of the Multiplication of the first figure of the higher row, by the first figure of the lower row, or Multiplier, must be set under the first place: and that that amounteth of the same first figure by the second Multiplier, must bee set under the second place, and so of the other, if there be more Multipliers.

Master. So mean I indeed: and if there amount but a Digit, then must it be set under the Multiplier.

And now to go forth: I multiply by the same 2, the second figure of the higher row, which is 6, saying, two times 6, make 12, wherefore I write the Digit 2 under the third place, and the Article 1 I keep in mind.

264
29
2376
28

Then doe I multiply the last figure of the higher summe by that same 2, saying, two times 2 is 4, and with the 1 that I have in minde maketh 5, which I write under the fourth place. And so

ſo have I ended the whole Multiplication: wherefore I alſo give the 2 a daſh with my pen, thus: and ſo I doe ever as ſoone as I have diſpatched any Digit by which I multiply: and the ſummes ſtand thus.

Then muſt I drawe a line under all thoſe ſummes that mount of the multiplication, and muſt adde all them into one ſumme, as in the Example, you may ſee.

264
29
2376
528
264
29
2376
528
7656

Where in the firſt place I finde but 6, and therefore write I it under the line. Then in the ſecond place 8 and 7 make 15, whereof I write 5, and keepe one in my minde, and ſo forth as you learned in Addition. And ſo appeareth the whole ſumme to be 7656, which amounteth of the Multiplication of 264, by 29 and that is the juſt number of the Lambes that 29 men had.

Scholar. If there be no more to be obſerved in it, then can I doe it, I ſuppoſe, as by this example I ſhall prove.

¶ There is a peece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards ſquare there is in all this peece of ground: which numbers I ſet downe with the greater above, and the leſſer under, as you ſee.

1365
236

Then doe I multiply 5 by 6, saying, 6 times 5 make 30, of which I write the Cypher in the first place, and the Article 3 I doe keep in minde to carry to the next place.

Then doe I by the same 6 multiply the second figure of the higher summe, which is 6, saying, 6 times 6 make 36, and 3 in my minde make 39, of which I write the 9 under the second place and the Article 3 I keep in minde.

Then do I multiply the third figure, which is 3 by the same 6, and that maketh 18, and 3 in my minde make 21. The 1 I set below, and keep 2 in minde.

Then come I to the last figure of the higher summe, and multiply it by 6, saying, 6 times 3 make 6, and 2 in my minde make 8 that 8 do I write under the fourth place. And so have I ended the first Multiplier, and dish him sleightly with my Pen.

1365	
236	
0	
1365	
236	
90	
1365	
236	
190	
1365	
236	
8190	

Then begin I with the second Multiplier, and say, first 3 times 5 that maketh 15, of which I set the 5 under the second place, because that the Multiplier is there, and the Article 1 I keep in mind.

Then

Then comes I to the second Figure that is 6, and multiply it by 3, which maketh 18, and with one in mind, maketh 19, the 9 I set down under the third place, and 1 I keep in mind.

Then comes I to the third Figure, which is 3, and multiply it by 3, saying, 3 times 3 make 9, and with 1 in minde, make 10, the Cypher I set under the fourth place, and the Article 1 I keepe in mind.

And then comming to the last figure 1, I multiply it by 3: and it maketh 3, and with the one in mind, it maketh 4, which 4 I set in the fifth place, and then have I ended two of the Multipliers, and the summes stand as you may see in the latter end of the page going before, and then I give 3 his oath.

1365
236
8190
4095

1365
236
8190
4094
0

Then came I to the third Multiplier, and multiply it into every figure of the Cypher sum, and first I say, 5 times 5 make 10, of which I set the Cypher under the Multiplier in the third place, and the Article 1 I keep in minde.

And by multiplying the second figure 6 by

that

that same 2, there amounteth 12, and 1 in my minde maketh 13, whereof I write the Digit 3 under the fourth place, and the Article 1 I keep in minde.

Then do I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and the one in minde make 7, which 7 I set down under the fifth place, as appeareth by the example.

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the sixth place, and then doth the summe stand thus.

And so have I ended the whole Multiplication.

But now (as you taught me) to know what this whole summe is, I must adde all those parcels together, and then under the line will appear, as you may see, the grosse or totall summe is, 322140. Whereby I know there is so many yards square in that piece of ground.

	1365
	236
	8190
	4095
	30
	1365
	236
1	8190
	4095
	730

	1365
	236
	8190
	4095
	2730
	322140

Master. This is well done.

Scholar. Then me thinketh I could call it well done, when I know, whether I had well done or no.

Master.

Master: It is to be proved by 9 as Addition was, but the surest proof is by Division, and therefore I will reserve that proof by Division, till you have learned the Art of Division. And anone I will shew you how it is commonly proved.

¶ *But first, for your further instruction in this exercise of* Multiplication, *I will with one example try your cunning, and so make an end: And the* question *is this. I would know how many dayes it is since the Nativity of our Lord and Saviour Jesus Christ, unto this yeare* 1630. *Which to performe, you must* multiply *this present yeare* 1630. *by the dayes in one whole yeare, which are* 365.

Scholar. Now for that you have given me so much light into the question, you shall see I will handsomely finish the work, for according to your former instructions, I set them down with a line under them, thus.

1630
365

Then say I, 5 times 0 is 0, which I set downe under the first place, as here appeareth. Then say I, 5 times 3 make 15, the Digit 5 I set down in the second place under 3, and the Article 1 I keep in minde to bee added to the next Multiplication. Then saying five times 6 make 30, and 1 in minde 31, the 1 I set downe in the third place, and 3 I keepe in minde. Then comming to the last Figure, I say once 5 is 5, and 3 in minde make 8, that I doe I set down under

the

the fourth place: and thus have I ended my first Multiplier, and therefore I give it a dash with my Pen.

Then come I to the second Multiplier, which is 6, and do likewise multiply it into the upper number, saying; 6 times 0 is 0, which I set down in the second place, right under his Multiplier: then say I, 6 times 5 make 18, the 8 I set down under the third place, and 1 I keep in mind. Then say I 6 times 6 make 36, & 1 I keep in mind make 37. the Digit 7 I set down in the fourth place, and 3 I keep in mind: Then say I 6 times 1 is 6, & once 6 is 6, and 3 in mind make 9, which I set down next, and so have I ended two Multipliers: wherefore I dash the 6 with my pen.

Then I begin to multiply the third Multiplier into the over number, saying, 3 times 0 is 0; the 0 I set down in the third place right under his Multiplier. Then say I 3 times 3 make 9, which I set down in order next: then say I, 3 times 6 is 18, the 8 I set down, and 1 I keep. Lastly, I say, once 3 is 3 and 1 I keep is 4, which I set down in order next: And so have I ended the Multiplication, and my figures stand thus.

		or thus
1590	1630	1630
365	365	365
7950	8150	815
9540	9780	978
	4890	489
	594950	594950

Master

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one summe it will be 594250; which is the grosse or totall summe of that Multiplication, and declareth the number of dayes since our Lord and Saviour his incarnation, unto the end of 1630 *yeares, besides* 407 *dayes, and twelve houres for leape yeares.*

Scholar. This is marvellous, me think, that such great matters may so easily be atchieved by this Art, which heretofore I ever thought hadbeen impossible, as infinite sorts of people are of that minde.

Master. Truth it is, that knowledge hath no greater enemy then ignorance, for this is one of the least of ten thousand things that may be done by this Art, as hereafter you shall be able to justifie.

Scholar. The manner of Multiplication I perceive, if there be no more in it.

Master. *Yes, there are other formes and helps for ease and shorter labour of the worke of* Multiplication, *but I will remit them till you have some little tasted* Division, *where also the like helps into* Division *may be used: and so therefore under one example for both, will I shew you both ease in* Multiplication, *and also in* Division.

But sith the other formes and workings be nothing differ from these workes in effect, but onely in setting of the numbers, I will over-passe them till a more meete place and time.

time. And now will I instruct you in Division, so that you think your self sufficiently to perceive what I have taught you.

Scholar. Yes Sir, I thank you, but I dos not perceive how to examine my work, to try whether I have well done or no: therefore as you promised me ere-while, I pray you first shew me how I shall prove it.

Master. *That is commonly used by the proofe of 9, as you learned before in* Addition, *saving that it differeth from that forme in divers respects: As for example.*

First, must you make a crosse after this manner. X

Proofe of Multiplication.

Then must you examine your summe that should be multiplied, and look what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoever remaineth in it after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, and looke what amounteth thereof, if it be under 9, write at the higher part of the crosse: but if it be above 9, then take thence 9 as often as ye can, and write the rest at the head of the crosse: As for example, we will prove the example you put forth of the piece of ground that contained 1365 yards in length, and 236 yards in breadth.

Therefore first I cast away all the nines from the summe to be multiplyed, saying, 5

and

and 6 make 11, cast away 9 rest 2: then 3 and 2 makes 5, and 1 is 6, that 6 I write at one side of the crosse thus.



Then do I examine the Multiplyer which is 236, wherein when the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the crosse.



Then doe I multiply 6 by 2, and it maketh 12, from which 12 I withdraw 9, then resteth 3, which 3 doe I set at the head of the crosse. Then do I examine the grosse summe, amounting of the Multiplication, which is 322140, where I finde 9 once, and 3 remaining; that 3 I set at the foot of the crosse, and then I see it to agree with the other 3 at the toppe of the crosse, and so know I that I have done well: for if they two did differ, then were my work vain, and the Multiplication false.



This is the common proof: but the most certain proof is by Division, of which I will anon instruct you.

Scholar. Sir, what is the chief use of Multiplication? A sure proofe of Multiplication.

Master. The use of it is greater then you can yet understand: howbeit, these plain commodities it hath, that if you would resolve any great and whole value into many small and lesse portions, as if you would change pounds

pounds in shillings and pence, or any other greater or smaller parcels by Multiplication, ye shall doe it speedily and easily. Also if you should need to adde one summe to it selfe, or to any other often times, you shall doe it by Multiplication much more speedily, readily, easily, and surely, then by often and tedious Additions. Take you these commodities greatly shewed for an answer at this time, and hereafter I will more abundantly make you to perceive the use of it.

Division.

Scholar,



Well Sir, then in Division *I pray you to instruct me. But me thinketh by the name of it, that it should be all one with* Multiplication: *for I call that* Division, *when any thing is parted into divers and many parts.*

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceive that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but contrariwayes, it bringeth many parcells into few, but yet so, that these few taken together, are equall in value to the other many: for by Division, pence are turned into shillings, and shillings into pounds: As for example, of 120 shillings, it maketh 6 pounds, so are 120 turned into 6, which is a smaller number: but then if you consider the Denominators, you shall see that they are such, that one of the latter is equall to 20 of the first, and so in value the summes are one, though in number they do differ, and the latter summe is the lesser, and so it is alwayes in Division, howbeit, yet in the

woꝛking the ſumme is parted by another, and thereof doth it take the name.

Scholar. I thinke I ſhall better underſtand the reaſon of the name when I know the uſe of the woꝛk, therefoꝛe now would I gladly learn that.

Diuiſion what it is.

Maſter. *Diuiſion is a diſtributing of a greater ſumme by the unites of a leſſer*: Or, *Diuiſion is an Arithmeticall producing of a third number, in reſpect of two propounded numbers; which third number ſhall ſo often contain an unite, as the greater of the two propounded numbers can contain the leſſer.* So that as Multiplication did ſeem to ſerue in ſtead of many Additions, ſo Diuiſion may ſeem to be in place of many Subtractions: Becauſe that third number briefly expreſſeth how many times the leſſer of your two propounded numbers may bee ſubtracted from the greater: as in practiſe will more largely appeare. Therefoꝛe (as you may perceiue) unto Diuiſion are required three numbers: The firſt, which ſhould be diuided, and that muſt (generally) bee the greater: and the ſecond, by which the other muſt bee diuided, and that is (generally) the leſſer, and is called the Diuiſion: And the third, which anſwereth to the queſtion (How many times?) and therefoꝛe is called the Quotient.

A generall rule for placing the figure.

The firſt muſt bee firſt written, and the ſecond ſo ſet under it, that the laſt figure of the lower number bee right under the laſt of the

the higher, contrariwiſe to the manner of other kindes of Arithmetick: for in them the two firſt figures were ſet ever meet one under the other; but in Diviſion, the laſt figures muſt be ſet meet, except it chance ſo that the laſt figure of the Diviſor, be greater then the laſt of the higher number, for then you ſhall ſet the laſt of the Diviſor under the laſt ſave one of the higher number, as for example: An exception.

If you ſhould divide 365 (which are the ſumme of the dayes of a year) by 28, which are the dayes of a common Moneth, then ſhould you ſet them thus.

365	
28	

But if you ſhould divide thoſe 365 daies by 52, which is the number of weekes in one yeare, then ſhould you ſet them thus.

365	
52	

Likewiſe, if I would divide the ſame 365 by 4, which is the ſumme of the quarters of years, then muſt I ſet them thus.

365	
4	

Scholar. Sir, this doe I underſtand, but now how ſhould I doe to divide the one by the other?

Maſter. You muſt beginne with the laſt Figure next the left hand, and ſee how many times the laſt Figure of the Diviſor may bee taken out of the laſt Figure of the other Number, and that ſhall you note within a crooked line toward your right hand: As for example, I would divide 365 by 28,

then set I those two summes thus.

365 (
28

And I looke how many times I may find 2 (which is the last figure of the Divisor) in 3, (which is the last of the number to be divided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. When because that when 2 is taken out of 3, there remaineth 1, I must write that 1 over 3, and deface or cancell the 3 and the 2, then will the figures stand thus.

Quotient numbers.

1
365 (1
28

Then come I to the next figure of the Divisor, and take it likewise so many times out of the figures that be over it, and looke what doth remain, that I must write over them, and cancell them, as in this example.

Therefore now doe I take once 8 out of 16, and there remaineth 8, which I must set over the 6, and cancell or crosse out the 16, and the 8 of the Divisor: and then will the figures stand thus. And so I have once wrought.

18
365 (1
28

Scholar. So I perceive that you take the nether figure, not onely out of the other that is right over him, but out of that with the other also that

remaineth

remaineth before, and are written toward the left hand.

Maſter. So muſt you doe: for you muſt ſo take the Diuiſor out of the ouer number, that there remaine not ouer it ſo great a ſumme as it ſelfe is, for then were your worke in vaine.

But yet againe here muſt you marke, that when you ſeeke how many times the laſt figure of the Diuiſor may be found in the number ouer him, that you looke alſo whether you may as often finde all the figures following in thoſe that are aboue them (conſidering all the remainers, if there be any) if not, take your Quotient leſſe by one, and then proue againe, and ſo ſtill you finde a meet Quotient: and by that meet Quotient muſt you alwayes multiply your Diuiſor, and ſet the product vnder your Diuiſor, ſo that the firſt figure ſtand vnder the firſt figure of your Diuiſor, and the ſecond vnder the ſecond, and ſo forth: and then ſubtract that product from the number to be diuided that ſtandeth directly ouer it, as you haue ſeen me do.

When you haue thus wrought once, then muſt you begin again, and write your Diuiſor anew, neerer toward the right hand by one place, as in this example, you ſhall ſet 2 vnder 8, and 8 vnder 5, thus.

18	
365	(1
288	
2	

When (as before) ſeeke how many times you may

take your Diuisor out of the number ouer him now.

Scholar. That may I do here 4 times.

Master. Truth it is, that you may finde 2 foure times in 8: but then marke whether you can finde the figure following so many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholar. No, neither yet once.

Master. Therefore take 2 out of 8 once lesse.

Scholar. That is three times.

Mark how to consider this kinde of Remainer.

Master. Well then, 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following make 25, in which sum I finde 8 three times also: and therefore I take 3 as a true Quotient, and write it behind the crooked line of the Quotient before the 1, thus:

2 | 8 | 365 (13
2 | 88

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancell the 8, and write ouer it the 2 that doth remain, thus:

Then do I take 8 as many times out of 25, saying, 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1, so then I cancell 25 and 8, and ouer the 5 set 1 thus:

Or you might (after you found 3 to be a fit Quotient) straight way have multiplyed the whole Diviſor 28, by that at once: which giveth 84, which being ſet under 28, and duely ſubtracted from 85, of the number divided, giveth 1, the remainer of the whole Diviſion, as before you had. Work which way you liſt, here you may ſee alſo the ſame.

	2
	281
	365 (13
	288
	77

And now have I done with the dividing, for I cannot find my Diviſor 28 no more in the over ſumme.

Scholar. No, except you would part the 1 that remaineth into 28 parts.

Maſter. That is well ſaid, and ſo muſt we doe in ſuch caſes, when there remaineth any thing: but I will let that paſſe now, and will make you perfect in Diviſion of whole numbers, and will hereafter teach you particularly of broken numbers called Fractions. Now if you do perceive the order of Diviſion, then do you divide this ſumme 136280 by 452.

Scholar. Firſt, I ſet downe the number that ſhould be divided; then doe I ſet the Diviſor under the laſt figure of the over number. Then will it be thus.

136280	
452	

Maſter. Can you take the laſt of your diviſor (which is 4) out of one, which is the laſt of the ouer number?

Scholar. I had forgotten, becauſe the laſt of the diviſor cannot bee taken out of the laſt of the ouer number, in ſo much as it is the greater, therefore muſt I ſet the diviſor one place more forward toward the right hand, thus.

136280
452

And then muſt I looke how often I may finde the laſt figure of the diviſor (that is 4) in 13, which I may doe 3 times, therefore doe I ſay, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doe I make at the right hand of my ſummes a crooked line, and write before it my Quotient 3, and I cancell 13 and 4, and ouer the 3 I ſet the 1 that remaineth, and then the figures ſtands thus.

1
136280 (3
432

Then I multiply the ſame Quotient into euery figure of the Diviſor, and withdraw the ſumme that amounteth out of the numbers ouer them, as firſt I ſay, 3 times 5 make 15, which I take from 16, and there reſteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remaineth, thus.

11
136280 (3
452

Then doe I ſay likewiſe, 3 times 2 make 6, which I take out of 12 and there reſteth 6, there-

therefore I cancell the 12 and the 2 over, and then I write the 6 that remaineth thus.

```
 216
2136280 (3
 452
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Then ſhould I ſet forward the Diviſor into the next place toward the right hand thus.

```
 216
2136280 (3
 4522
  45
```

Maſter. But you may ſee that over the 4 is no figure, therefore I muſt ſet the diviſor yet forwarder by another place.

And marke, whenſoever it chanceth ſo, that you ſhould ſet forward the diviſor, and that it cannot ſtand there, becauſe there is no number over the laſt place, or if there bee any, it is leſſer then the laſt figure of the diviſor, then muſt you remove the diviſor, yet once again: and becauſe that his firſt place of removing ſerved not to ſubtract him ſo much as once, therefore you ſhall write in the Quotient a Cypher, and if you ſhould by chance need to do ſo oft times, for every time write a Cypher in the Quotient. The reaſon of this will I ſhew you hereafter.

Scholar. Then muſt I ſet my ſummes thus.

```
 216
2136280 (30
 4522
  45
```

And becauſe I removed the diviſor, ſo that I overskipped one place, I muſt write a Cypher in the Quotient: and then muſt I ſeeke a new Quotient, as

in

in this example I muſt ſay; How many times 4 is there in 6? (and ſith it can be but once) therefore doe I write 1 in the Quotient: and then ſay I, 1 time 4 taken out of 6, remaineth 2. I cancell the 6 and the 4, and write 2 over them, thus:

Then ſay I againe, once 5 out of 28, remaineth 23: I let the 2 ſtand as it did, and over that 8 I ſet 3; cancelling the 8 and the 5 under it, thus:

```
      2
   1263
1362280 (301
  43222
   455
     4
```

```
      2
   1263
1362280 (301
  43222
   455
```

Maſter. You might as well have ſaid, once 5 out of 8, and ſo remaineth 3, but now go forward.

Scholar. When once 1 out of 0 cannot bee: What ſhall I now do?

Maſter. Borrow of the next number that is behinde (for there is 230) and do as you learned in Subtraction in a like caſe.

Scholar. When muſt I borrow 1 out of the 3 comming behinde next, and make that 0

ſo

to bee 10, and then take 3 2
out of 10, and there resteth 8: and because I borrowed one of the 3, I must cancell the 3, and write 2 over it, then only the figure stand thus.

22
21658
136280 (301
45222
4

Master. Now have you done, and yet remaineth 228, and your Quotient notwithstanding. that if you divide 136280 by 452, you shall finde your Divisor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example (where I divided 365 by 28, the Quotient was 13, and 1 remaining: whereby I knew that in a yeare (which containeth 365 dayes) there are 13 moneths, reckoning 28 dayes (or 4 weeks) unto a moneth, and 1 day more.

Scholar. Why then do we call a yeare but 12 moneths?

Master. Of that at a more convenient time will I fully instruct you: but now it is not convenient to intangle your minde with other things then do directly pertain to your matter. Therefore if you remember what you have heard, you have learned a short manner of Division, which I would have you often to practise, so that you may be perfect in it, and hereafter I will shew you certaine other proper points touching it.

Scholar. When I pray you tell mee how

I shall examine and try my worke, whether I have done well or no, that though no man be by me to tell me, yet I may perceive it my selfe.

Proof of Division.

Master. Some men (yea and commonly most) doe try it by the Rule of 9, as in all the other kindes, save that their order is; First, they cast away 9 as often as they can out of the Divisor, and that that remaineth they set at one side of a Crosse, as in our first example the Divisor was 28, from which you may take 9 three times, and 1 remaineth: which they set by a Crosse, thus.

1
X

Then they likewise examine the Quotient, (which in our example is 13) and from thence they cast away 9 as often as they can, and the remainder they set at the other side of the Crosse, and then they multiply together those two remainers: and to it that amounteth they adde the remainer of the Division, if there were any from that whole summe they withdraw 9 as often as they can, and the rest they set at the head of the Crosse, as in our example, the Quotient is 13. from which take 9, and there remaineth onely 4, and therefore must you set 4 at the other side of the Crosse, thus:

4
1 X

Then multiply 4 by 1, and it yeelds but 4, thereto adde the remainer of the Division (which was 1) and it will be 5, which summe doth

doth not amount to 9, and therefore must be set wholly at the head of the Crosse, as you see here.

5
4 X 1

And this number on the head of the Crosse, is the first proof, to which if you find another like in the number that was divided, then you have done well.

Therefore now shall you likewise examine the whole summe that was divided, and take away 9 as often as you can, and that that remaineth, set at the foot of the Crosse: and if it be equall to that in the head of the Crosse, then have you done well, else not.

As in our example the whole sum was 365, which maketh 14, from that take 9, and there resteth 5, which set at the foot of the Crosse, thus.

4 X 1
5

And you shall see that they agree: therefore have you well done.

Now will I likewise examine our second example, where the Divisor was 452, which maketh 11; from thence I take 9, and the 2 that remaineth I set at the right side of the Crosse, thus.

X 2

Then examine I the Quotient, which was 301, where I finde but onely 4: that I set at the other side of the Crosse, thus:

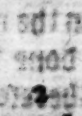
4 X 2
1

Then I multiply 4 by 2, and

2, and it maketh 8: to that I adde the remainer of the Division (which was 228, and it maketh 12) and they two make 20, wherein I find twice 9. and 2 remaining: that 2 muſt I ſet at the head of the Croſſe, thus:



Then I examine the whole number to be divided, which was 136280, where I finde twice 9. and 2 remaining, which I ſet at the foot of the Croſſe, thus:



And becauſe it doth agree with the figure at the head of the Croſſe, I know that the Diviſion was well wrought.

The Proof of Diviſion more certain by Multiplication.

Maſter. This is the common proof: Howbeit, the more certain working is by the contrary kinde: as to prove Diviſion by Multiplication, thus:

Multiply the quotient by the Diviſor, and if the ſumme that amounteth, be equall to the ſumme that ſhould be divided, then have you well divided: elſe not.

Howbeit, this muſt you marke, that if there remained any thing after the Diviſion, that muſt you adde to the ſumme that amounteth of the Multiplication. As in our firſt Example our Quotient was 13, and the Diviſor was 28: Now multiply the one by the other, and the ſumme will be 364: to that if you adde the 1 that remained after the Diviſion;

then

then will it be 365, which was the ſumme that ſhould be divided: and therefore I know that I have well done.

Scholar. Now will I proue the ſame in the ſecond example, whoſe Diviſor was 452, and the Quotient 301: theſe doe I multiply together, and there amounteth 136052: to which if I adde the 228 that remained, then will it be 136280, which was the whole ſumme to be divided: and therefore I perceiue that I haue well done.

Maſter. This is the ſureſt way to examine Diviſion by Multiplication: and contrariwiſe, the ſureſt proofe of Multiplication is by Diſion.

And therefore (according to my promiſe) now will I ſhew you how you may proue Multiplication by Diviſion.

Proof of Multiplication by Diviſion.

When you haue ended Multiplication, and would know whether you haue well done or not, ſet the groſſe ſumme that amounteth of the Multiplication ouermoſt, and diuide it by the Multiplier: and if the Quotient be the ſame number that ſhould be multiplied, then haue you well wrought, elſe not, as in that example where we multiplied 264 by 29, the groſſe ſumme was 7656.

Now if you will know whether that Multiplication be true, you ſhall diuide that 7656 by the multiplier 29, and you ſhall perceiue that the Quotient will be 264, and that is a token that you haue well wrought.

Scholar.

Scholar. By your patience I will prove that, and firſt ſet downe the groſſe ſumme and the multiplier, not after the rule of Multiplication, but after the rule of Diviſion, for now that number is become the Diviſor, that was before the Multiplier, I ſhould ſet them therefore thus.

	7656
	29

Then ſhall I ſeek how many times 2 in 7, that may be three times and one remaineth: but then may not 9 be found ſo often in 16? therefore muſt I take a leſſer Quotient, that is to ſay 2: then ſay I, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then do I cancell 7 and 2, and over 7 I write 3, and in the Quotient I ſet 2: ſo the figures ſtand thus.

3	
7656	(2
29	

Then ſay I further, two times 9 make 18, which I abate out of 36, and there reſteth 18: then cancell I 3, and over him ſet 1, and likewiſe I cancell 6 and 9, and over them I ſet 8: ſo that thus ſtand the Figures.

1	
28	
7656	(2
29	

Then I ſet forward the Diviſor by one place, and ſeeke a new Quotient, that is to ſay, how many times 2 are in 18, which I finde to be 9 times: but then can I not finde 9 ſo many times in 5: but therefore I take a leſſer Quotient, as to ſay 8: but yet that is too great: for if I take 8 times 2 out of 18, there remaineth

remaineth but 2, and I cannot finde 8 times 9 in 25: therefore yet I take a lesse Quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seek a lesser quotient, as to say 6, then say I, 6 times 2 make 12, that I take out of 18, and there remaineth 6, so I cancell 18, and the 2, and write 6 over 8, thus:

Then say I forth, 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus:

Then must I set forth the Divisor again, and seek a new quotient, which will be 4: for though I may find 2 in 11, 5 times, and 1 remain, yet I cannot find 9 so often in 6, therefore I set the figures thus:

And the 4 in the quotient I multiply into the figures of the Divisor, saying: four times 2 makes 8, which I take out of 11, and there resteth 3, therefore I cancell the 11, and the 2, and set 3 over the first place of 11 thus:

And then doe I say forth, 4 times 9 maketh 36, which I take from 36, and there remaineth nothing,

so that the Quotient of this Division also (where 7656 is divided by 29) is 264: Which doth declare, that if 264 be multiplied by 29, the summe will be 7656. And thus I perceive now how both Multiplication is proved by Division, and Division also by Multiplication.

Master. Now have I ended the five common kindes of Arithmetick: For (as touching Mediation, Duplation, Triplation, and such other) they are no severall kindes of Arithmetick, but are contained under the other. For Mediation is contained under Division, and is nothing else but dividing by 2: and so are Duplation and Triplation contained under Multiplication: for Duplation is nothing else but multiplying by 2, and Triplation, is multiplying by 3, of which I will onely propose an example, for the Rules you have heard already.

An example of Mediation.

If you would mediate, or divide into 2, this summe 4531010, you shall set 2 for the Divisor, and worke as you learned before, as thus:

```
4531010 ( 2
2
```

Then I finde 2 in 4 two times, therefore my Quotient must be 2: so I cancell 4 and 2, and remove the Divisor forward thus, as the worke requireth, and as before in Division hath been declared.

```
4531010 (2265505
22222222
```

Which mediation or division by 2 being finished,

finished, you shall have for your Quotient 2165 $\frac{165}{1010}$, which is the parts of 453 $\frac{1010}{1010}$, as you may try by Duplation; for double your Quotient, or multiply it by 2, and the same number will amount. Duplation.

I will no longer tarry about these, seeing they are but members of the other kinds. But here now (according to my promise) I will teach you certain easie formes both of Multiplication and of Division. And first of Multiplication.

If you would therefore multiply any summe by 10, you shall need to do no more but adde a Cypher before his first place; as for example, 36 multiplied by 10, makes 360. Easie formes of Multiplication.

Likewise if you would multiply any summe by 100, put two Cyphers at his beginning. So if you would multiply any summe by 1000, adde three Cyphers to the beginning of it.

Scholar. This doe I well perceive, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmetick also: and as for these matters now look, as I have told you, that you have remember it, and also often practise it.

And now you have learned how to multiply easily by 10, 100, 1000: and of like manner may you do with any other of like sort.

But now if you will multiply by 20, 30, 40, and so forth; or by 200, 3000, and such like, where there is one Cypher in the first place,

or many orderly in the first places, you shall take away those Cyphers, and multiply the summe onely by the other figure or figures (if they be many) and then at the beginning of the summe that amounteth, you shall set so many Cyphers as you tooke away.

Example of 2873, which I would multiply by 300. First, I omit the 2 Cyphers from the Multiplier, and I multiply the summe by those onely that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted or tooke away, and then is it 861900. And that is the summe that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures before the Cyphers, I must onely take away the Cyphers, and multiply by the other figures, as I learned before: As if I would multiply 93648 by 25000, I should take away the three Cyphers, and multiply the same by 25, and then at the beginning of that totall summe should I adde the 3 Cyphers again.

Master. Even so: but if it chance the number that should bee multiplied, or both the summes, as well the number that should be multiplied as the multiplier, to have Cyphers in their first places, evermore omit the Cyphers and worke by the rest. But remember to restore as many Cyphers to the amounting summe as you tooke before. As in this example: 30200 shall bee multiplied by 206, I shall onely take away two Cyphers from the greater number

number, and then multiply 302 by 206, and afterward adde the two Cyphers again. But if I would multiply the ſame 30200, by 2060 I ſhall not onely take away the two Cyphers from the number that ſhould bee multiplied, but alſo I may take away the one Cypher from the Multiplier, and then muſt I adde three Cyphers to the ſumme that amounteth: but take heed that you take away no Cypher that commeth after any ſignifying figure, as in the laſt example, you muſt not take away that in the fourth place of the higher number, neither that in the third place of the multiplier: howbeit, yet thus you may doe: If one Cypher or more come in the laſt of your ſummes, you may multiply the other figures, and overskip them: but ſo, that you give every figure his one place: as thus, I will multiply 3026 by 2004, therefore I ſet them thus.

3026
2004
12104
52

And thus I doe multiply them. Firſt 4 times 6 make 24, I ſet 4 under the firſt place, and keepe the two ſtill in my mind. Then ſay I againe: 4 times 2 make 8, and the 2 that is in my minde maketh 10, I ſet downe the Cypher 0, and keepe the Article 1 in my minde: Then four times 0 is 0, and the 1 in my mind maketh 1, I ſet downe the figure 1. and ſay again, 4 times 3 is 12, I ſet downe 2, and keeping the 1 ſtill in my mind (having no more places of the upper number to multi-

tiply it withall) I put it downe next o in the fifth place.

But now when I come to the next place (being a Cypher o) I let it go, because it multiplieth nothing, and likewise the second Cypher.

But then when I come to the 2. and multiple it into the 6 of the vper number, you must take heed (according as I taught you in Multiplication) that the first number of morning of the multiplication be set right vnder the multiplier, and the vther ordering towards the left hand, according as you may see in this example, which being finished, with the addition thereof gathered together, will stand as this Example sheweth.

Which is indeed wrought so much the sooner and shorter by ouerskipping of the two Cyphers: which otherwise, (if the same example were wrought at length) it would haue had two workings more, as by the same example here also set down doth appear.

	3026
	2904
	88104
	6052
	6064104
	3026
	2904
	12104
	0000
	0000
	6052
	6064104

Scholar. Sir I thanke you, for I see great ease in this way of Multiplication: and (if you can shew me such like in Division) you shall greatly further me.

Master.

Maſter. Yes, I will teach you ſome eaſie wayes in Diviſion alſo, and firſt this: If you would divide any ſumme by 10, you ſhall onely with your Pen make a ſquare line between the firſt figure of your ſumme and the ſecond, and then have you done: for the whole number that followeth the line, ſtandeth for the Quotient, and the figure that is before the line, is the remainer: As for example, 3648 divided by 10. Where 364 is the Quotient, and betokeneth that ſo many times are 10, in 3648, and the 8 after the line is the remainer, which cannot be divided into 10, but by breaking it into Fractions, wherewith I will not meddle yet. Eaſie forms of diviſion.

364 |8

And ſo likewiſe if you would divide any ſumme by 100, with your Pen you ſhall cut away the two firſt figures, and if you would divide by 1000, you muſt cut away the three firſt figures, and ſo of any other diviſor, whoſe laſt figure is 1 and the other Cyphers, look how many Cyphers the diviſor hath, and ſo many figures at the beginning ſhall you cut away with the ſquare line, and they ſtand alwayes for the remainer, becauſe they are leſſe then the diviſor, and cannot be divided by it, and the other figures that are behinde the line ſtand for the Quotient.

But now if your diviſor have any other figure in his laſt place then 1, and in all his other places have Cyphers, looke how many

cyphers they be, cut away so many of the first figures of the number that should be divided, and divide the rest that followeth the line by that figure that is in the last place, as if it were the whole divisor.

Example of 64284. which I would divide by 300, here must I cut away the two first figures, (for so many cyphers my divisor hath) and must divide the rest by 3, which is the figure in the last place of the divisor. First, therefore I part away the two first figures, and the summe standeth thus:

64|284 (2
3| 00

Then do I divide 642 by 3, and the Quotient is 214: for in 6 I finde twice 3, and in 4 once, and 1 remaining, which 1 with the 2 next before, doth make 12, wherein I finde 3 foure times: And this is a ready way to turn shillings into pounds: for sith one pound doth contain 20 shillings, I must divide the whole number of shillings by 20. Therefore easily do it, I see that my divisor hath one cypher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then I do mediate or divide by the 2 other figures or summe that followeth.

Scholar. I will put an Example.

If you would divide 64287 shillings, by 20: that is to say, If I would turn so many shillings into pounds, I must cut away the first figure, that is 7, and divide the rest, that is 6428 by 2, so shall the Quotient bee 3214, whereby

whereby I know that 64287 ſhillings make 3214 pounds, and 7 ſhillings remaining.

Maſter. Now prove by Multiplication whether you have well done or no.

Scholar. The Quotient is 3214, which I do multiply by the diviſor 2, and it doth amount to 6428.

Maſter. Hereby you may perceive not onely that you have well done, but alſo how by Diviſion you may turn ſhillings eaſily into pounds: and contrariwiſe by Multiplication you may turn pounds into ſhillings.

But here ſhall you ſee amongſt divers men divers forms of ſuch diviſion: but if you marke what I have told you, you ſhall perceive eaſily all the wayes. For ſome men doe not cut away ſo many of the firſt figures of the ſumme that they would divide, as there are Cyphers in the firſt places of the Diviſor: but they ſet all their Cyphers orderly under the firſt places of the Number that they would divide: and then with the other figure or figures, (if there bee many) they divide the reſt of their ſumme. Another manner of the Abridgement.

Example. If they would divide 725931 by 3400, they doe ſet their ſummes thus:

725931	
34	00

And then do they divide orderly till they come to the Cyphers: for there they ſtay and end their worke, as in this example.

They ſeek how often 3 may bee found in 7, which is two times, and 1 remaining: there-

therefore they set 2 in the quotient, and cancell 8, and 7, and over 7 they set the 1 that remaineth, thus:

Then doe I goe forth, saying, two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus:

Then remove they the divisor forward, and seeke how often 3 may bee found in 4, which is but once, and 1 remaineth, then set they 1 in the Quotient, and cancell 3 and 4, and over them they set that 1, thus:

Then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth 1: so they cancell the 4 and 5, and set 1 over them, thus:

Then set they forth the divisor again, and seek how many times 3 are in 11, which they finde three times, and 2 remaining: so they set 3 in the Quotient, and cancell 11 and 3, and over them set 2, thus:

Then doe they multi-

ply

ply 4 by 3, which maketh 12, that withdraw they out of 29 and there resteth 17, of which the 7 must bee set over the 9, and the 1 ober the 2, thus:

And now are the two Cyphers next ensuing, so that the diuisor can no more be set forward, and therefore is the diuision ended, and the remainer is 1731.

Now the Quotient which is 213, doth declare, that if you diuide 725931 by 3400, you shall finde it therein 213 times, and there remaineth 1731: so shall you finde it, if you work as I taught you, by cutting away the two first figures, because of the two Cyphers.

But this must you mark (as you may perceiue by this last example) that if there be left any other Remainer in the summe that was behinde the square line, that the Remainer must be set to the latter end of the first remainer, which was cut away with the square line: as if you would diuide 725931 by 3400, after the forme that I taught you, then would your summes appear, thus: Note.

So that 17 which remaineth after the line,

line, must bee set to the 31 (that was cut away with the line) in higher places, as you see here: where that 17 with the 31, do make 1731.

Scholar. Sir, is there no other form of Division in practise but this?

Master. Yes verily, there are other forms in practise, but because I love brevity, I will declare onely one, which I first learned of, and is practised by that worthy Mathematician, my ancient and especiall loving friend, Master Henry Bridges, wherein not any one figure is defaced or cancelled. As if I should divide 72 by 6, first place them thus. 6)72

Then if you please you may write the divisor in a loose paper that it may more easily without cancelling or defacing of the worke be applied to, and removed from the dividend at pleasure; then apply your Divisor 6 to 7, the first figure of the dividend, and inquire how oft it may be had in 7, and seeing 6 is but once in 7, set 1 in the Quotient line, thus:

Write the Divisor in a loose paper, to remove at pleasure.

Then multiply the divisor 6, by the quotient 1, and set the Product 6 under 7, thus: 6)72(1

Then draw a line under 6, and subduct 6)72(1 6

6 out

6 out of 7, setting the remainer 1 under 6, thus:

Then bring down the next figure of the Dividend, and set it with the Remainer 1 under the line, thus:

And bring the moveable divisor 6 under the 2, and as before enquire how oft 6 is in 12, and finding it to bee twice in 12, set 2 in the quotient, thus:

```
6) 72 (1
   6
   -
   1

6) 72 (1
   6
   --
   12

6) 72 (12
   6
   --
   12
   12
   --
   0
```

And multiply 6 by that new Quotient 2, setting the Product 12 under the other 12, and subducting it out of the upper number, there resteth nothing. And since the unites of this Product do stand under the unites of the dividend, the division is ended: otherwise you should proceed as before, bringing down the next figure; removing the divisor, dividing, multiplying, subducting, &c.

Scholar. This is very easie, but if there be greater numbers propounded, is the operation the same?

Master. If the numbers be never so great, the work is the same without any difference, as shall appear by this example.

Divide

Divide 7890 by 33.

First set them thus: then bring the divisor under 78, and see how oft it is there found, which is twice, and therefore set 2 in the Quotient, by which multiply the divisor 33, and set the Product 66 under 78, and subduct it out of it thus.

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33)7890(2
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Then bring the next figure 9 down, and set it with the Remainer 12, it maketh 129, and removing the divisor 33 thereto, enquire how often 33 is contained in 129, and I finde it but thrice, (though at the first it made shew to be more) wherefore set 3 in the Quotient, and multiplying 33 by 3, the product gives 129, subtracting that product out of the number above, and placed as before.

```
33)7890(239 3/33
 66
 129
  99
  300
  297
    3
```

Then shall you finde the Divisor 9 times in the Remainer. Wherefore setting 9 in the Quotient, multiply, and subduct as before, and at the last you shall have onely 3 remaining, which must be set above a line after the Quotient, and the divisor under, as above appeareth.

Scholar. Is there no more difficulty in the whole Rule?

Master.

master. Not any, although your Number bee never so great, as before I have said.

¶ And here will I make an end of Division; (saving that I doe request you to exercise your selfe well herein by many examples, till you have attained some experience therein.)

For the reasons and conclusions thereof are so many, and so available for all sorts of men whatsoever; that if I should speak of the infinite uses thereof, I should rather lack words, than matter. And therefore recommending it to your judgement hereafter, upon your further travell into the Art, I will here end this Treatise, representing unto you one example, or simple question of Division and Multiplication, in stead of many, which is this:

A question of shooting in Ordnance

There are foure brasse Peeces: The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed against the battery of a Hold, and there is allowed by the Master Gunner, 700 pounds of powder to be spent by these foure Peeces, in this assault. The question is two-fold. The first how many shot each Peece shall justly make above with this 700 pounds of powder? And lastly, how many pounds of powder ought justly to be allowed to each Peece for his proportion?

Scholar.

Scholar. Why Sir, you make me smile, to bring mee in hand, that these two demands may be simply resolved by Multiplication and Division.

Master. Truly that they may, and that you may by and by worke your selfe with a little labour: First adde together their quantities of powder, that is 9 pounds, 5 pounds, 4 pounds and 2 pounds, all which make 20: Divide the 700 pounds of powder by that 20, and your Quotient giveth 35, as here appeareth, which sheweth for most certainty that they shall make just 35 shootes about.

9	
5	
4	
2	
700	
200	(35
2	

Scholar. Sir, all this have I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To try the truth of the same, multiply the first peece that spends 9 pounds by 35, and you shall see his allowance, which is 315 pounds of powder. Multiply also the second peece that spends 5 pounds by 35, and you shall find 175 pounds his allowance: then 4 by 35, and you shall finde 140 pounds his allowance. Lastly, multiply 2 by 35 and you shall find 70 pounds his allowance. All which four particular summes you shall adde together by Addition, as here appeareth, and it maketh just 700 pounds, and so is the question truly absolved.

315
175
140
70
700

Scholar.

Scholar. Truly Sir, these excellent conclusions do wonderfully more and more make me in love with the Art.

Master. It is an Art, that the further you travell, the more you thirst to goe on forward. Such a Fountaine, that the more you draw, the more it springs: and to speake absolutely, in a word, (excepting the study of Divinity, which is the salvation of our Soules) there is no studie in the world comparable to this, for delight in wonderfull and godly exercise, for the skill hereof is well known immediately to have flowed from the wisdome of God into the heart of man, whom he hath created the chiefe image and instrument of his praise and glorie.

Scholar. The desire of knowledge doth greatly incourage me to be studious herein: and therefore I pray you cease not to instruct me further in the use hereof.

Master. With a good will. And now therefore for the further use of these two latter, that is, Multiplication and Division, I will briefly shew you the feat of Reduction.

Reduction.

Reduction what it is.

Reduction is, by which all ſummes of groſſe Denomination *may be turned into ſummes of more ſubtile* Denomination. And contrariwiſe, all ſummes of ſubtile Denomination may be brought to ſummes of groſſer Denomination.

Groſſe denomination.

Scholar. What call you groſſe Denomination, and ſubtile Denomination?

Maſter. That I call a groſſe Denomination, which doth contain under it many other ſubtiler or ſmaller: as a pound (in reſpect to ſhillings) is a groſſe Denomination: for it is greater then ſhillings, and containeth many of them. And ſhillings (in compariſon to pounds) are a ſubtile Denomination, for becauſe they are leſſer then pounds, and many of them are contained in one of the other: and ſo likewiſe of other things: whatſoever thing is compared to other, if it be greater and containeth many of them, it is a groſſe Denomination, but if it be leſſer (ſo that many of them are in the other) then are they called the ſubtile Denominations: whereby you may perceive that one Denomination may be called a groſſe Denomination, and alſo a ſubtile (that is to ſay, a great and ſmall) in divers compariſons. For ſhillings compared to pounds, are a ſubtile or ſmall Denomination: but compared to pence, they

Subtile denomination.

they are a groſſe, or great Denomination.

Scholar. Now I underſtand the name; I pray you teach me the uſe.

Maſter. The uſe is eaſily learned, if you remember what you have learned before. For if you will reduce any ſumme of a groſſe Denomination into a ſumme of a ſmaller or ſubtiler Denomination, you muſt conſider how many of that ſubtiler Denomination, doe make one of the groſſer Denomination, and by that number or numerator doe you multiply the ſumme. As if you would reduce 30 pounds into ſhillings, you muſt conſider that in a pound are included 20 ſhillings, therefore multiply the one 20 by the other 20, and there will amount 400 whereby you may know that in 20 pounds are contained 400 ſhillings. Likewiſe, if you would reduce 30 ſhillings into pence, conſidering that in a ſhilling are 12 pence, you muſt multiply 30 by 12, and it will be 360, whereby you may finde that in 30 ſhillings are contained 360 pence. And thus may you reduce any groſſe Denomination into a more ſubtiler, by Multiplication, if you know how many of the leſſer doe make the greater: of which thing I will anon give you a briefe Table for the moſt accuſtomed kinds of Money, Weights, Meaſures, and Time, and ſuch like: whereby you may know how often each ſubtile Denomination is contained in the groſſer, when you ſhall need it for the foreſaid kinde of Reduction. And alſo the ſame ſhall ſerve you, if you would

To reduce groſſe denomination to ſubtile.

reduce any summe of a subtiler Denomination, into a summe of a greater Denomination. For in such Reduction you must consider (as in the other form) how many of the smaller do make the greater: and by that number you must divide the other summe, and the Quotient will declare how many of the greater Denomination are comprehended in that summe: as for example; If you would know how many shillings are contained in 3240 d. consider that 12 pence doe make 1 s. you must divide that 3240 by 12, and your Quotient will be 270, whereby you may know that so many shillings are in 3240 d. But if you would know further, how many pounds are in these 270 shillings, seeing that every pound containeth 20 shillings: divide that 270 by 20, and it will be 13 and 10 remaining, whereby you may know, that in 3240 d. (or 270 shillings) are 13 pounds and 10 shillings. For evermore the Remainer must be named by the name, or Denomination of that summe that was divided, which in this place were shillings. And thus may you do with any other kindes of Denominations.

¶ Wherefore, to the intent you may have certain light or knowledge in most common Coynes, Weights and Measures, (which is the chief and principallest thing in traffickes to be known) I have in each Reduction, as they come in order, set down certain instructions incident thereunto. And first I have hereunto added

added this Table, wherein is comprehended, not onely our current and common coynes, but also the moſt part of the uſuall coynes of Chriſtendome, with their juſt weights and value currant in this Realme of England, intending at the latter end of my Addition to this Book, to write of the ordinary Money uſed in divers places, and their common values currant for traffick, with the manner of their exchanges from place to place, &c.

A Table of the names, and now valuation of the most usuall Gold-coyns throughout Christendom, with their several weight of Pence and Grains: and what they are worth of currant English money, this present year, 1630.

The names & titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	Pence.
Great Sovrain,	10	0	33	0
Double Sover K. H.	8	1	22	0
Double Sov. of K. E.	7	7	22	0
Royall.	4	23	16	6
Half Royall.	2	1 d.	8	3
Old Noble.	4	6	14	8
Half Noble.	2	3	7	4
Angell.	3	8	11	0
Half Angell.	1	16	5	6
Salute &	2	5	6	11 ob.
parts of Salute.	1	11	4	7
George Noble.	3	0	9	9 ob.
Half George Noble.	1	11	4	11 q.
First Crown K. H.	2	9	6	11 ob.
Base Crown K. H.	2	0	5	6
Sover. K. H. best.	2	14	11	8 ob. q.
Soverain K. H.	4	0	11	0
Edward Sover.	3	15 d.	11	0
Elizabeth Sover.	3	15 d.	11	0
Elizabeth Crown.	1	9	5	6
Half Crown.	0	19	2	9
Unite.	0	12	22	0

Double

Double Crown.	3	6	11	0
Britain Crown.	1	1	5	6
Thistle Crown.	1	7	4	4 ob. q.
Half Crown.	0	19 d.	2	9
Crosse Dagger.	3	6 d.	11	0
Half Crosse Dagger.	1	15	5	6
Rose Royall.	0	21	33	0
Spur Royall.	4	10 d.	16	0
The Angell.	2	23 d.	11	0
Half Angell.	1	11 d.	5	6

All the severall pieces of gold heretofore mentioned, are set down according to their valuation by the Kings Majesties Proclamation for gold, dated the 23 of November, 1611.

A Table of forain Gold-coyn, according to their ancient valuation and severall weight in Pence and Graines.

The names & titles of the Gold.	The weight in Pence.	Grains	The value in Shil.	Pence.
Vnicorn of Scot.	2	10	6	0
Scottish Crown.	2	5	6	0
French Noble.	4	16	13	4
Al sorts of French Crowns.	2	5	6	0
Flanders Riders.	2	6	6	6
Gelders Riders.	2	2	3	6
Philips Royall.	2	10	10	0
Philips Crown.	2	5	5	0
Collen Gilden.	2	2	4	8
New And. Gild.	2	2	5	0

The names & titles of the Gold.	The weight in Pence, Grains.		The value in Shil. Pence.	
Flanders Noble.	4	10	11	0
Half Flan. Noble.	2	6	5	0
Flan. Angel. beſt.	3	6	3	0
Flan. Royall oike.	3	10	10	0
Carolus Gilden.	0	12	3	6
Flanders Royall.	2	6	5	0
Saron Gilden.	2	2	4	8
Flanders Crowne.	2	5	6	0
Philips Gilden.	2	3	4	2
Half Phil. Gild.	1	1	2	1
Golden Lion.	2	16	7	8
3 parts of golden Lio	0	21	2	5
1/3 parts of golden Lion	1	19	4	11
Davids Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old rnare Gilden.	2	3	4	10
Cruſa. long Creſſe.	2	6	6	0
Cruſa. ſhort Creſſe.	2	6	6	2
Milreyes.	4	20	1	4
Half Milreyes.	2	10	6	8
Portague 1 ounce.	2	16	68	0
Golden Caſtile.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungary Ducket.	2	7	6	4
Dowble Piſtolet.	4	9	11	8
Single Piſtolet.	2	4 d.	5	10
Ducket of Floren.	2	5		4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
Double duc. of Rome	4	13	12	8

It is to be underſtood (gentle Reader) that whereas in theſe Tables, the weight is called by the name of a penny, it is not meant a penny of ſilver money, but a penny of Gold-ſmiths weight, which containeth 24 Barley-corns. Concerning which ſee Troy weight in folio 133.

So if a man have not the weight wherewith to weigh any peece of gold, he may do it with barley-corns, being dry, and as it is ſaid, folio 133.

The prices of Gold which the bringers in of forrain Gold ſhall receive at the Mint, according to the Kings Majeſties Proclamation, Dated the 14 of *May, Anno* 1612.

For an ounce of *French* crowns being 22 Karacts fine.	3 li. 6 s.
For every ounce of *Spaniſh* Piſtolets, being 21 Karacts, 3 grains and a half fine.	3 li. 6 s.
For Duckets of *Spaine*, being 23 Karacts, 1 grain fine at leaſt the ounce.	3 li. 8 s. 8 d.
For Milreas Cruſado long croſſe. Cruſado ſhort croſſe, the ounce.	3 li. 6 s. 2 d.
For *Hungary* Duckets being 23 Karacts, 1 grain fine at leaſt the ounce.	3 li. 9 s. 2 d.
For the Checkeen of *Venice*, being 23 Karacts, 1 graine fine at leaſt the ounce.	3 li. 10 s.

For

For *Barbery* Gold being 23 Karects and di. grain fine, at the leaſt the ounce. —— 3 li. 9 s.

¶ *And if the ſaid Barbary Gold be of leſſe fineneſs, abatement to be made according to the rate.*

For Sultaines being 23 Karects, 1 Graine fine at leaſt the ounce. —— 3 li. 8 s. 8 d.

For all other Gold, being 22 Karects fine the ounce. —— 3 li. 6 s.

¶ *And being finer, a greater price according to that rate, and being courſer, a leſs, ſo that the bringer in ſupply the leſſe fine, with the more fine, in ſuch ſort, that in the totall it makes good the ſame rate of 22 Karects fine.*

The price of Silver, which the bringers in of forain Silver ſhall receive at the Mint, according to the Kings Majeſties aforeſaid *Proclamation.*

For the ounce of *Spaniſh* Silver money of *Civill* —— 5 s.

For the ounce of *Mexico* money. —— 4 s. 10 d.

For

For Ingots of Silver, being 11 ounces, 2 d. weight fine according to the Standard of *England*, the ounce. } 5. s.

¶ *And for other Silver of more fineneſſe, a better price according to that rate, and for courſer a leſſe: ſo that the bringer in ſupply the leſſe fine with the more fine, in ſuch ſort, that in the totall it makes good the ſaid rate of* 11 *ounces,* 2 *pence weight fine, according to the Standard of England.*

Of Silver Coynes currant in this Realme.

The *Edward* crown of 5 s.
The *Edward* half-crown of 2 s. 6 d.
The *Edward* ſhilling, half ſhilling, and the three pence.
Philip and *Maries* ſhilling, and half ſhilling.
The *Mary* groat, and *Mary* two pence.
Queen *Elizabeths* ſhilling, 9 d. 6 d. 4 d. 3 d. 2 d. 1 d. three farthings, and half penny.

HEre would I now expreſſe the values of ſundry other Coynes of divers Countries, but for three cauſes I now refraine. **The firſt and chiefeſt is, becauſe they are not currant by the Statutes of this** Realme. **Another cauſe is, by reaſon they are ſo uncertain, that they bee never long at one** rate

rate. And againe, they are ſo different in ſo many places, that it were matter enough for a great Booke to ſpeak ſufficiently of them all. Howbeit, becauſe you ſhall not be altogether ignorant of them, I will ſhew you the values of ſome that are moſt in uſe, and firſt of France.

French Coynes. The moſt common Money are Deniers, Soulx, and Frankes: 12 Deniers make 1 ſhilling, 20 Soulx make 1 Frank: ſo that as you ſee theſe three kindes are like in the rate to pence, ſhillings and pounds with us; but that this is the difference, that their Denier is but the ninth part of our penny, and ſo their Soulx (commonly called Souſes) goe 9 to our ſhilling, and 9 of their Franks to an Engliſh pound of money. So that three of their Franks make a Noble. And by theſe three you may practiſe how to reduce French money into Engliſh money, according as I have ſet forth here following.

2160 } 3240 } 8352 } Deniers make { 240 d. or 20 s. / 360 d. or 30 s. / 928 d. or 3 li. 17 s. 4 d.

2160 Soulx make 240 ſhillings. And ſo of other in like rate. As for the reſt of their Coynes I omit them till hereafter, that you have ſome underſtanding in broken numbers.

Flanders Coynes. But now as for the Coynes of Flanders, they bee ſo changeable, that you muſt know them from time to time: elſe you cannot reduce them into our money certainly: but yet that

that you have an example of their money to exercise you withall, you shall take those that be most common: as Stivers both single and double, Groats Flemish, Carolus and Gyldens. A Flemish Groat is a little above 3 farthings English. A single Stiver is 1 d. ob. q. half farthing. The double Stiver Carolus is 4 d. ob. half farthing. Then there is also the Carolus Gylden, which is worth 10 Stivers. And the Flemish Noble is worth 3 Carolus Gyldens, and 12 Stivers.

So that if you would convert Flemish money or any other kind of money whatsoever it be, justly into Sterling, you must reduce it first into the smallest part of English money that is in that Coyne. As for example: If I would reduce 368 double Stivers into English money (considering that a double Stiver containeth 3 d. farthing) you shall first look how many farthings be in the double Stiver, and you shall find them 13, therefore multiply the summe of the Stivers by 13, and then have you their value in farthings, which is 4784. Now if you divide that by 4. then there will appear the number of pence: but better it were to divide it by 48 (for so many farthings are in one shilling) and then will the Quotient declare the summe of the shillings.

Likewise if you would reduce any summe of single Stivers into English money, you must multiply the summe first by 13, and then have you reduced them into a certain summe, that is

is to wit, half-farthings, which ſumme if you divide by 8, then will amount the ſumme of pence: or if you divide it by 96, the ſumme of ſhillings will appear.

But mark this in all Diviſion: when yee do reduce to bring one Denomination into another, if there be any Remainer after the Diviſion, that muſt be named by the Denomination of the groſſe ſumme that was divided: As for example, I would bring 254 farthings into pence, therefore I divide that 254 by 4 (for ſo many farthings make a penny) and the Quotient is 63, which is the ſumme of the pence, and then remaineth yet 2; which are farthings ſtill, as one may prove by dividing. And this muſt be marked in all Diviſion, namely, when it is done for Reduction.

Note wel.

Danks Money.

¶ Touching Danks Money, they have their Soulz, whereof 20 is a Liver: which is 2 ſhillings ſterling. They have alſo their Graſh whereof 80 make a Gilder, which is 4 ſhillings ſterling. They have alſo Dollors, and their common or old Dollar is 35 Graſh. New Dollors they have which be divers, ſome valued at 24 Graſh, ſome at 26, and ſome at 30. And thus much I thought good to adde to the Author, touching Danks Money.

Spaniſh Money.

Conterning Spaniſh Money, whereof the moſt common are Cornadoes, Marveides, Marveide. 4 Marveides make a Ryall, and

11 Ryalls

11 Rials make one Ducket, ſo the Ducket containeth 374 Marveides, which is about 5 ſhillings 10 pence ſterling. Therefore if you would convert 124 pounds 5 ſhillings ſterling into Duckets, conſider that pence is the leaſt value or Denomination named in this queſtion: therefore reduce 124 pounds 5 ſhillings into pence, and it maketh 29820 pence: which if you divide by pence that a Ducket is worth, (which is 70) you ſhall have for your Quotient 426 Duckets, your deſire.

In Venice they have Bettes, Souldyes, Lieures. 5 Bettes make an Engliſh penny, 60 Bettes a ſhilling, which is 2 Souldyes, and 20 Souldyes a Lieure of Venice which is a pound ſterling. Venice money.

Thus much have I ſaid of Money: Now will I ſhew you in like ſort the diſtinction of weights. Weights.

After a Statute made anno 11. *H.* 7. *there ought to be but one ſort of weight: as* 24 *Barley-corns dry, and taken out of the middeſt of the Ear, do make a penny weight,* 20 *of thoſe penny weights make an Ounce: and* 12 *Ounces a pound of Troy weight, by which is weighed Bread, Gold, Silver, Pearl, Silke, and ſuch like.* But commonly there is uſed another weight called Haberdupoiſe; in which 16 Ounces make a pound. Therefore when you would reduce Ounces into pounds, you muſt conſider whether your weight be Troy weight or Haberdupoiſe: and if it bee Troy weight you muſt divide

Trow weight. A penny weight. An ounce. A pound Troy.

Haberdupoiſe weight.

divide your ounces by 12, to bring them to pounds, but if it be Haberdupoise, you must divide them by 16. Now again, there be greater weights, which are called a hundred, halfe a hundred, and a quartern, and also a half quartern, &c.

A hundred weight.

Scholar. Why? so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they have not common weights made to their rate as the other have. And again, these that I did name, are not just in number as they seem by their name: for an hundred is not just 100, but is 112 pound. And so the half hundred is 56, the quarter 28, and the half quarter 14. And these be the common weights used in most things that are sold by weight.

Howbeit there are in some things other names, as in Woolt 28 pound is not called a quartern: but a Todde: and 14 pound is not named half a quartern, but a Stone, and the 7 pound half a Stone. Other names because they differ in many places, and agree in few, I let them passe.

Woolt weights. Todde. Stone.

But a Sack of Woolt by the Statutes is limited to be 26 stone.

Sack of Woolt.

¶ Now in Cheese, though it be sold by the hundred, and by the stone in some places, yet the very weights of it are Cloves, and Weyes. So that a Clove containeth 8 pound, and a

Cheese weights.

Wey

a wey 32 Cloves, which is 256 pound, that is 12 ſcore and 16 pound, and ſo much weigheth the wey of Suffolk cheeſe, and the like is or ſhould be the Barrel of Suffolk butter.

The Wey of Eſſex cheeſe containeth ſix ſcore and ſixteen pound: and ſo much is alſo the barrell of Eſſex butter.

Moreover this weight is uſed by the Apothecaries in their Phyſicall compoſition, and mixture of medicine, wherein the leaſt is a grain,

And	20 *Graines*	make	*A Scruple*	thus	℈
	3 *Scruples*		*A Drachme or Dragme*	cha-	ʒ
	8 *Dragmes*		*An Ounce*	racte	℥
	16 *Ounces*		*A Pound.*	red.	lb.

The Apothecaries weights.

Now of weights are made other meaſures both for grain and liquor. For a pound in Troy weight, maketh a pint in meaſure, ſo that 8 pound or 8 pints do make a gallon: half a gallon is named a pottle, and half a pottle is called a quart, which containeth two pints: Now above a gallon the next meaſure is a Firkin: then the Tertian, a kilderkin, or half a barrel, and a barrel: And by theſe meaſures are ſold commonly Ale, Beere, Wine, and Oyle, Butter and Soap, Salmon, Herrings, and Eeeles.

Meaſures for liquor.

A Pint.
Gallon.
Pottle.
Quart.
Ferkin.
Tertian.
Kilderkin.
Barrell.

But as theſe be unlike things, ſo the meaſures of their veſſels do differ, for the meaſures of them all are as followeth.

Of Ale	*The firkin*	contai-	8	*Gallons.*
	The kilderkin	neth	16	
	The barrell		32	

Ale meaſure.

Beer measures.

Of Beer { the Firkin, the kilderk. } containeth { 9, 18 } Gallons.
{ the barrell } { 36 }

Sope measures.

Sope measures, both Firkin, Kilderkin, and Barrell should be equall to Ale measure.

Moreover the Statutes do limit the weight of every of those three vessels being empty.

{ *A barrell* } { *to weigh empty* } { 26 }
{ *Half a barrell* } { 13 } *pounds.*
{ *A firkin* } { $6\frac{1}{2}$ }

Herrings.

Herrings also sold by the same measures that Ale and Soape be sold by.

Herrings are sold by the tale, 120 to the hundred, ten thousand to the Last.

Salmon & Eeles.

Salmon and Eeles have a greater measure.

Salmon & Eeles { the butte, the barrell, half barrell, the firkin } holdeth { 84, 42, 21, $10\frac{1}{2}$ } gallons.

Howbeit, some Statutes did limite Eele vessels equall with Herring vessels.

Wine measures.

Now as for wine vessels they are seldome smaller then Hogsheads, which are of 63 Gallons: Every Hogshead, is two Barrels: yet there are many other wine vessels, but of them all see this Table; and marke the measures one by another.

Of

Of wine and oyle	the runlet	holdeth	18½	Gallons.
	the barrell		31½	
	the hogſhead		63	
	the tertian		84	
	the pipe		126	
	the tunne		252	

But you ſhall marke that there bee other kindes of Tertians: for there bee Tertians, (that is to ſay) Thirds of pipes, of hogſheads, and of barrels, as well of other things as of wine. Tertians.

Alſo Malmſeyes, and Sacke, &c. the halfe Tun is not called a Pipe, but rather a Butte. A Butte.

And thus much have I thought meet to tell you at this time.

Scholar. And is that alwayes true?

Maſter. I have told you how it ſhould be, but how it is, I may not ſay: how they doe differ daily from their juſt meaſure, that Gaugiers can tell you better then I. But I will let this paſſe now, and ſpeak briefly of the other meaſure.

And as of weights there did ſpring the liquid meaſures (whereof I ſpake laſt) ſo of the ſame ſpringeth dry meaſures, as Pecks, Buſhels, Quarters, and ſuch like, whereby are meaſured corne and like grains, alſo Salt, Lime, Coals, and other like. And this is the order and quantity of them. Drie meaſures.

A pecke is the meaſure of two Gallons.

A Buſhell. A Quarter. A Wey.

A buſhell, A quarter, A wey } containeth { *foure pecks. eight buſhels. ſix quarters.*

Theſe are the common names and meaſures, but in divers places there be divers ſorts.

Strike. Cornock.

The Buſhell in many places is two buſhels, but then is that buſhell there called a ſtrike: and in ſome places half a quarter is called a Cornock. But theſe diverſities are too many to tell you briefly them all: and again, ſith they are againſt the Law and Statutes, I count them unmeet to be uſed.

Meaſure to mete length, bredth, and thickneſſe.

But now remaineth yet another kinde of meaſures, whereby men mete length, breadth, and thickneſſe, and theſe are, an Inch, a Foot, and ſuch other: whoſe names and quantities this Table ſheweth.

An Inch. Foot. Yard. Elle. Pearch.

3 Grains of barley in length make an inch.
12 Inches, 3 foot } *make* { *a foot. a yard.*
3 Foot and 9 Inches } *make* { *an ell.*
5 Yards and a halfe } *make* { *a pearch.*

1 Pearch in breadth, and 40 in length, doe make a rodde of land, which ſome call a rood, ſome a yard land, ſome a farthendele.

Acre.

2 Farthendels, 4 Farthendels } *make* { *half an acre of ground. an acre.*

¶ *More, 40 rods in length do make a furlong, 8 furlongs make an Engliſh mile, which containeth 320 Perches.*

So that an Engliſh mile, grounded upon the Statute, is in length 1760 yards, 5280 foot,

foote, and 63360 Inches.

Somewhat greater then the Italian mile of 1000 paces, and 5 foot to a pace.

Here might I tell you many things elſe touching meaſures, and alſo how to reduce ſtrange meaſures to our meaſures, but becauſe it cannot be well done without the knowledge of Fractions, which as yet you have not learned, I will let them paſſe till another time, that I have taught you the knowledge of broken numbers.

Scholar. But yet ſir of the parts of time, I pray you tell me ſomewhat. The parts of time.

Maſter. You know that a naturall day hath 24 houres, and every houre hath 60 minutes. It needeth not to tell you, that 7 dayes make a weeke, and 4 weekes make a common moneth, and 13 moneths make a yeare, lacking one day, and certain houres and minutes: but of that I ſhall inſtruct you hereafter.

A day.
An houre,
Weeke,
Moneth,
Yeare.

Here will I make an end of Reduction *for this time, which though it bee counted no kinde ſeverall of* Arithmetick, *you ſee it is no leſſe needfull to be known, or eaſier to be done, then any of the other.*

Scholar. Marry ſir, it ſeemeth unto me much harder then any other ſort, for it requireth the knowledge of ſo many things: but now Sir when you ſee time, I am ready to learne forth, as much of Reduction as you have taught mee, I remember; but and if I doe at anie time forget, I ſhall have re-

course to the Tables which you set forth for me.

Master. So doe you: for it will not be remembred without exercise. But in as much as you understand so much as wee have intreated of, I will now instruct you in Progression.

Progres-

Progreſsion.

ALthough untill this day the moſt part of Writers have deſined Progreſſion as a compendious kinde of Addition, yet truely it is not ſo: for Progreſſion (as the very nature of the word doth inform any man) is a going forward and proceeding in numbers, and that regularly and orderly, whoſe place is aptly choſen to bee very neare, or rather next after the expoſition of the foure principall parts of Arithmeticke: for in it after a moſt eaſie manner, are all the foure former parts exerciſed and practiſed: and not onely Addition, as cuſtomably is done. Which cuſtome hath been the cauſe, why it hath ſo ſpecially been named a kind of Addition, and defined to be a quick and brief Addition of divers ſummes, proceeding by ſome certain and reaſonable order. What Progreſſion is.

You ſhall alſo underſtand there are infinite kinds of Progreſſions, but for you (as yet) two are ſufficient to be exerciſed in, of which the one I call Arithmeticall, and the other Geometricall.

Arithmeticall Progreſſion is a rehearſing or placing down of many numbers, number after number, in ſuch ſort, that between every two next numbers rehearſed or placed down, the difference, diverſity, or exceſſe, be equall and alike. Arithmetical Progreſſion.

Scholar. Sir I thank you for that you have both opened unto me what progreſſion is truly, and alſo why it is here placed.

But I pray you with an example make plain your definition.

Maſter. Examples cannot want, ſeeing all reaſonable creatures naturally uſe the order of one kind of Arithmeticall progreſſion (which therefore is alſo named naturall) whenſoever they diſtinctly do count or number any multitude by one, ſaying, 1, 2, 3, 4, 5, 6, whereby the proceeding from number to number, and every one ſurmounting and exceeding his fellow next before by a like quantity (which here is 1) declareth the ſame to be Arithmeticall progreſſion. And for the more plainneſſe, I ſet it down in this manner.

The common exceſſe. 1 1 1 1 1

The Progreſſion. 1 2 3 4 5 6

Scholar. This is moſt evident. And I think that I am able to tell you now of any progreſſion Arithmeticall propounded, what is that common exceſſe or difference whereby it proceedeth, if this order be kept in it.

Maſter. What ſay you of 3, 6, 9, 12, 15?

Scholar. They exceed each other by 3: And that may I ſet down in ſuch evident order, as you did your example of naturall progreſſion, in this wiſe.

The

The common exceſſe. 3 3 3 3



The Progreſſion. 3 6 9 12 15

Maſter. And doe you not alſo now perceive, that the whole Table of Multiplication may be made by the order of progreſſion Arithmeticall? either if you will begin at the firſt number of any of them on the left hand, & ſo proceed right overthwart: or at any of the firſt numbers of the upper row, & go directly downward.

Scholar. I pray you let me conſider the thing a little, and I will anſwer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceive it now very well, for the common exceſſe or difference between any two next, is continuallie as much as the firſt number of everie row, either from the left hand overthwart taken, or from ante

of

of the uppermoſt overthwart rowes downeward.

To know the totall ſumme of an Arithmeticall progreſſion.

Maſter. Now then, if of any ſuch Progreſſion, you would ſpeedily know the totall ſumme much quicklier then by common Rules of Addition: firſt tell how many numbers there are (which numbers here we call places or parcels) and if they be odde, write their ſumme downe by it ſelf: as in this Example, 2, 4, 6, 8, 10. 12, 14. where the numbers are 7. as you may ſee: therefore ſet down 7 in a place alone, then adde together the firſt number and the laſt, as in this Example: Adde 2 to 14. and that maketh 16, take half of it, and multiply by the 7 which you noted for the number of the the ſame places, and the ſumme that amounteth, is the ſumme of all thoſe figures added together: as in this example 8 multiplied by 7, maketh 56. and that is the ſumme of all thoſe figures.

Scholar. That will I worke by another example. I would know how much this ſumme is, 5, 8, 11, 14, 17, 20, 23, 26, 29, I tell the places and there are 9, that I note. Then I put the firſt number 5, and the laſt 29 together, and they make 34. I take the half of it, that is 17, and multiply by 9, and it maketh 153. That you ſay is the ſumme of all the numbers.

Maſter. So ſhall you find it if you try it.

Scholar. How ſhall I try it?

Maſter. By your common Addition, for if

if you adde all the parcels together, you ſhall ſee the ſame ſumme amount, if you did worke well. And that manner of Addition trieth all kinds of ſumming any Progreſſion.

Scholar. Then can I ſumme any Progreſſion, if the number of the parcels be odde. But what if they bee even, as in this example, 1,2,3,4,5,6,7,8 ?

Maſter. When the number of the parcels is even, then note that alſo as you did before, and likewiſe adde the firſt ſumme to the laſt, and by the half of the number of the places, doe you multiply it: as in our example, the parcels are 4, that I note: then adding the firſt ſumme to the laſt, there amounteth 9, that doe I multiply by half of the parcels, that is, by 4, and it maketh 36, which is the ſumme of the parcels.

But if you will take one Rule for theſe both, doe thus: Multiply the halfe of the one by the other whole, and the ſumme will amount all one. For ſometime it chanceth that the number of the parcels be odde, ſo their half cannot be taken: and that ſometime it chanceth the Addition of the firſt number and the laſt, doe bring forth an odde number, ſo that halfe of it cannot be taken: but they will never be both odde.

A generall rule.

Scholar. Then I perceive this, if there be no more belonging to it.

Maſter. As acuſtomably it hath beene taught, this hath beene the chiefe and onely exer-

exerciſe in Progreſſion uſed. But that you may perceive how divers waies, and to how great profit ſo ſimple a thing (as this Arithmeticall Progreſſion is) may be conſidered and uſed, I will here propound you ſixe Propoſitions: of which foure of them were invented by a friend of mine, and never before thus publiſhed: and the two firſt were never to my knowledge written of but by three Men.

Scholar. This doth greatly encourage me to be attentive unto your words, ſeeing I ſhall not onely be inſtructed at your hands in the common known Rules of this excellent Art, but beſides that ſo abundantly in other new Rules enformed, as my very entrance ſhall ſeeme to paſſe a great many mens further ſtudie, and longer continuance. Therefore Sir, I beſeech you, let mee know your ſixe Propoſitions.

Maſter. Theſe they are.

1 *To know the laſt* number *without proceeding by continuall* Addition, *till you come unto it, ſo that the common exceſſe, the firſt* number, *and the* number *of the places be known.*

2 *The firſt* number *of the* Progreſſion *and the laſt being known, with the common exceſſe to finde the* number *of the places.*

3 *The exceſſe being given, and the* firſt *or* laſt, *to know the quantity of any middle* number, *whoſe place is given from the* firſt *or* laſt.

4 *The* totall ſumme *being given, and the* firſt *and* laſt, *to finde out the* number *of the places.*

The

The totall ſumme *of any* Arithmeticall Progreſſion *being given, and the firſt and laſt, to find out the common exceſſe.* 5

The totall ſumme *being given, and the mutuall exceſſe with the* number *of the places, to give the firſt or laſt* number *of the ſame* Progreſſion. 6.

Many moe conſiderations could I propound you in theſe *Arithmeticall Progreſſions*, but theſe are ſufficient, to give you occaſion to think, that Rules of Knowledge and Arts are infinitely capable of enlargement.

Scholar. Happy were I, if I did but well underſtand that which is already invented and written: And yet in my ſimple fantaſie, theſe things offer themſelves (in manner) to be ſtudied for about Progreſſion, therefore I pray you to proceed to the Rules anſwering to theſe propoſitions.

Maſter. I will orderly for every of theſe 6 propoſitions give you Rules, and with every one an example, unleſſe the plainneſſe and eaſineſſe need no farther exemplifying.

For the Solution of the firſt multiply the exceſſe by a number *leſſe by* 1 *then the* number *of the places, and the off-come adde to the firſt* number, *ſo you ſhall have the laſt* number, *which is ſought for.* 1 Propoſition.

As for example. If there were ſeven places in a Progreſſion Arithmeticall, whoſe continuall encreaſe or mutuall exceſſe were 4, and the firſt number were 5, and I would know what the laſt and ſeventh number is: I multiply 6 which is one leſſe then 7, (the number

number of the places) by 4, thereof commeth 24, which I adde to 5, that maketh 29 and that is the laſt number which I deſire to know. And this you may ſtraightway prove by continuall proceeding from 5, till the ſeventh place, encreaſing every one by 4, as thus.

5 9 13 17 21 25 29.

Loe here, the laſt, being alſo the ſeventh, is 29.

Scholar. I perceive already one good property in this Rule, which in all workes is to be deſired: that is, it will eaſe one from great labour, if a Progreſſion were propounded of a hundred or two hundred places, or moe: And alſo it is very eaſie to worke, and moſt neceſſary for the totall ſumme finding, in a very long Progreſſion.

Maſter. It is true, and therefore now let me ſee if you can anſwer me this queſtion by this propoſition.

A Merchant buyeth 50 pounds of Spices, and agreeth to pay for the firſt pound 4 pence, for the ſecond 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The queſtion is, how much he muſt pay for the laſt pound, and then how much the 50 pound commeth to?

Scholar. According to the propoſition, I multiply 49 (which is leſſe by one then the number of the places) by the exceſſe, which is 3;

to

to the product 147, I adde the firſt number which is 4, it maketh 151 pence, the price of the laſt pound. Now I adde 4, the price of the firſt pound, to 151 the price of the laſt pound, it maketh 155, which I multiply by halfe the number of the places, which is 25 the Product, 3875 pence is the totall ſumme or price of the 50 pounds of Spices, as appeareth.

49 Places 1 leſſe.	151 laſt
3 exceſſe	4 firſt
147	155
4 firſt	25 halfe places
151 the laſt.	775
	310
	3875 totall ſumme

which amounteth to

li	s	d.
16	2	11

Maſter. It is truly wrought.

Scholar. Then I intreat you to proceed to your ſecond propoſition.

Maſter. *The ſecond* Rule *is this. From the laſt ſubtract the firſt, the* remainder *divide by the common exceſſe, to the* Quotient *adde* 1, *and you have the* number *of the places, which you would know: as in this* Progreſſion. 2 Propoſition.

6 11 16 21 26 31

If I know onely 6 and 31 and that they encreaſe

encreaſe by 5, then according to the Rule, from 31 I ſubtract 6, there remaineth 25: which 25 I divide by 5 (the common exceſſe) the quotient commeth forth 5, to which I adde 1 that maketh 6: and ſo many are the places, as you ſee.

Scholar. This Rule is ſo eaſie, that I were much to blame, if I could not remember it.

3 Propoſition.

Maſter. *The third* Propoſition *may alwayes thus be ſolved. Multiply the* exceſſe *by a* number *leſſe by one, then the diſtance of the place is from the* firſt, *or the* laſt number given: *the offcome adde to the* firſt, *if the* diſtance *be reckoned from the* firſt, *and the* firſt *alſo known, or ſubtract from the* laſt, *if the diſtance be from the* laſt counted, *and the* laſt given *alſo, and that which commeth forth, either in that* Addition to the firſt, *or* Subtraction from the laſt *is the* number ſought. As for Example, *I propound you this* Progreſſion.

8 15 22 29 36 43 50 57

And for the apt conſidering the manner of this queſtion, I will note over every place his diſtance from the firſt, and under every place his diſtance inclinſively from the laſt, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the exceſſe whereby this Progreſſi-

on

ſion ſtandeth, be knowen to be 7, and the firſt numbers given, being 8, I would know what number ſtandeth under 4, that is to ſay, in the fourth place. I multiply 7 by 3 (which is leſſe by 1 then the number of the place proponed) that yeldeth 21, to which I adde 8, the firſt number, ſo commeth 29: which I ſay to belong to the fourth place, as you ſee in the example it alſo both: or if in the third place from the laſt, you would know what number in this example ſhould ſtand, the laſt number being knowen to be 57, and the common exceſſe 7, then by 2 which is leſſe by 1 then the place propounded, I multiply 7, that giveth 14, which appertaineth to the third place inclufively reckoned from the laſt, and ſo my example giveth you.

Scholar. I perceive right good uſe of this rule: for if I had forgotten what the firſt number were, and remember ſtill but the laſt the common exceſſe, and the number of the places, then might I come by the knowledge of my firſt number again.

And me thinketh, that it differeth not much from the firſt Propoſition, ſaving that which you make here a middle number, there was made the laſt: and alſo in this point it differeth, that in it the laſt was only ſought, and no conſideration had in numbring the places from the laſt, as here I marke in your numbers noted under your Progreſſion.

Maſter. And thinke you not, the middle

numbers of Progression standing of a hundred or three hundred places or more, may as much cumber a man to come to the knowledge of them by continuall encreasing from the first by the common excesse, or abating from the last continually the common excesse as the very finall Numbers in a shorter Progression would doe?

Scholar. Yes sir, that I think right well, and therefore I am glad of this new framed proposition, and the manner of the working of it.

4 Proposition.

Master. *The rule of the fourth is this. Adde the first and the last together, and by the off-come divide the totall* summe. *Double the* Quotient, *and that will be the* number *of the places.*

Scholar. Then if in a Progression, whose summe were 207, and the first number 12, and the last 57, if I adde 57 and 12 together, that maketh 69, and by it I divide 207, the Quotient will be 3, which I double; and so I have 6, and so many must be the number of the places that this Progression standeth on.

Master. Whether it be so or no, how will you trie?

Scholar. Half 6, which is 3, being multiplied by 69, must make 207, the totall summe; if 6 be the number of the places. For so the whole worke of your Rule in summing any Arithmeticall Progression did enforme me. I will then multiply 69 by 3, thus.

69
3
207

It commeth forth justly.

Master.

Master. I must much herein commend your promptnesse both in memory and in well applying your Rule: although in manifest words it did contain no such matter.

Scholar. Sir, I pray you beare me frame one example or more.

Master. I am well pleased so that ye be short, for you make me more longer here then willingly I would have been: but I cannot perceive how I could have omitted any thing as yet, without your great lack thereof.

Scholar. If I had received 85 pounds of certaine men, but of how many I have forgotten, yet I remember that the first gave me 7 pound, and the last 27 pound, and every payment after other did rise by a like summe. And the man for whom I received this money, conditioned with mee, that of every payment I should have twelve pence for my labour: now unlesse I can by Art finde the truth of this case, I am like to lose the most part of my reward. Aquestion of money.

Master. I perceive you can handsomely frame an example, which should concern your owne gaine: I pray you let mee see how you would do justice in this point.

Scholar. I adde the first and the last together, that maketh 34: by which I divide 85, thus:

	27	17
85 (		
34	34	

Why how now Sir, here is a remnant of 17, in which 34 cannot bee had

had: ſo that now I am in the briers for doubling of my Quotient, and farewell then both my Juſtice, and a good lump of my gains.

Maſter. Ye are never the farther from the matter, though it fall into a Fraction. For you ſhall underſtand that the Fraction which of any ſuch work proceedeth, is ever halfe of one ſuch, as the unites of the Quotient before are. And that you may try, if you double that which ſo remaineth, for then it will be equall to your Diviſor, as if ye double 17, (the remnant) it maketh 34, and your Diviſor alſo was 34, this noteth the remainder to be halfe of one.

Scholar. Now I am glad of this hard Example: For with it I have a generall rule for the Fraction that may hap in this work. So that the Quotient being two and a halfe, I double that, it maketh 5, therefore ſhould my gain be 5 ſhillings. And to be ſure (by your leave) I will trie it, for I will multiply halfe of 34: (which is the firſt and laſt number joyned together) by 5, thus.

	17
	5
	85

It is moſt true (I ſee) that I ſhould leeſe nothing by the former working.

5 Propoſition.

Maſter. *The fifth propoſition hath this rule appertaining unto it: By the fourth rule finde the* number *of the places, that being done, from the laſt ſubtract the firſt, and the reſidue divide by a* number *leſſe by* 1, *then the* number *of the places, and the* quotient *will ſhew the exceſs, which is ſought for.*

An

An example hereof ſhall be this: If ye had disburſed 685 pounds to a certain number of men, you neither can tell how many they were, or how much the ones money exceeded his next before, but you are ſure that the exceſſe was equall between every two next: and alſo you remember that the firſt had 19, and the laſt 180 Pounds, how would you find the number of the name and the exceſſe continually obſerved in the ſucceſſion of their payments? Example.

Scholar. Your Rule doth plainly bid, firſt to finde the number of the places, which I will do according to the fourth rule: I adde 19, and 118 together, thus.

By this 137, divide 685 thus.

Seeing there is no Fraction but a whole number, being 5, I double that, and then muſt the number of the places bee 10. Now from the laſt I ſubtract the firſt, as 19 from 118, thus: And ſo remaineth 99.

This 99 I divide by a number leſſe by one then the number of the places, and ſeeing the places were 10, I divide 99 by 9, thus:

The quotient is 1, and ſo was the exceſſe, if I have followed your rule right.

```
118
 19
---
137

137
685 (5
685

118
 19
---
 99

 99
 99 (11
 99
```

Maſter. You have wrought every part of this queſtion both well in order, and truly in the practiſe of your Rules.

Scholar. I will then ſet it down alſo formably, ſo that the number of the places, the exceſſe, and the totall ſumme may ſtraight appear, as your firſt example ſtood.

The common exceſſe. 11 11 11 11 11 11 11 11 11 11
19 30 41 52 63 74 85 96 107 118

The progreſſion. That the places be 10, and that from the firſt to the laſt, the common exceſſe is 11, I perceive moſt evidently: but whether the totall ſumme be 685, I have not yet proved, which I will now doe: I adde 19 and 118 together, that maketh 137; I multiply that by halfe the number of the places, thus.

137
5
685

All things agree moſt exactly, ſo that I am perfect enough in theſe Rules, if I forget them not again.

Maſter. Uſe maketh all things perfect.

6 Propoſition. *Your ſixth rule is this. By the number of the places divide the totall ſumme, double the quotient, and that will be the firſt and laſt joyned in one ſumme. Then by a number leſſe by* 1, *then the number of the places, multiply the exceſſe, that off-come ſubtract from the firſt doubled quotient, and the halfe of the reſidue is the firſt number. The laſt number you may diverſly find*

ont

out, as by the first of our 6 Rules, or by subtracting this first number from the summe which here contained both the first and last joyntly, (or thirdly) by continuall adding the excesse.

Scholar. I pray you make this somewhat more plain with an example.

Master. If every moneth in the year (counting them now as 13) you gained clearly 40 shillings more then you did the moneth next going before, and at the years end you find the whole gain 5720 shillings, but ye remember not how much either the gaine of the first moneth or the last was, by this Rule it may be tried out. Example of gaine.

Scholar. So that here ye seeme to apply the 13 moneths to thirteen places, the 40 shillings every one more then the other next before it to be the common excesse, and 5720 shillings to the totall summe.

Master. It is true: by 13 then I divide 5720 in this manner.

I double this Quotient, so have I 880 for the first and the last summe joyned together, by 12 which is lesse by one then the number

1	
25	
1720	(440
1333	
11	

ber of the places, I multiply 40 (the common exceſſe) ſo commeth 480.

This 480 I ſubtract from 880, ſo remaineth 400: half whereof is the firſt number which we deſired to know: that is 200.

40	
12	
480	
880	
4	
400	

And as for the laſt number, I can give you it three wayes. As by the firſt of my ſix rules I multiply the exceſſe, by a number leſſe by 1 then the number of the places, as 40 by 12 that giveth 480, which I adde to the firſt, being 200 ſo ſhall the laſt be 680.

The ſame ſumme commeth forth if ye ſubtract 200 from 880.

And thirdly, If I begin at 200, and ſo proceed, encreaſſing by 40, I ſhall at the thirteenth place have 680, as thus:

200	240	280	320	360	400	440
480	520	560	600	640	680	

Scholar. I thanke you moſt heartily for theſe ſix Rules. Now if it be your pleaſure I would hear and learn ſomewhat of Progeſſion Geometricall.

Maſter. There are yet very many Rules and Propoſitions which fall into this Arithmeticall Progreſſion.

And for the uſe and practiſe of them, I will propone unto you certain pleaſant and neceſſary Queſtions of Arithmeticall Progreſſion, and to the performance of their workings, ſuch

ſuch neceſſary rules and documents, as are requiſite for the better underſtanding of them, or any ſuch like.

A queſtion of Velvet.

A certain Mercer *ſold* 20 *yards of Velvet to be paid in* 12 *weekes, by* Arithmeticall Proportion; *that is to mit, to receive the firſt weeke* 6 ſhillings, *the ſecond week* 12 ſhillings, *the third week* 18, *and ſo forth, increaſing the* number *of weeks by* 6 ſhillings, *till the twelfth and laſt week were expired.* The queſtion is how many pounds he had for 20 yards of Velvet.

To the performance of this queſtion, and ſuch other the like, I ſet forth the 12 payments in ſuch ſort, as for example, here appeareth.

Then touching the adding together of theſe ſummes, without the aid of Addition, according to the rules I taught you in Progreſſion Arithmeticall, I note the number of the places, which are 12, then adding the laſt number of the Progreſsion, which is 72, and the firſt number together make 78; and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 13 pounds 8 ſhillings. And ſo much hath the Mercer for his 20 yards of Velvet, which is nigh about 23 ſhillings 5 pence, a yard.

6
12
18
24
30
36
42
48
54
60
66
72

Scholar, I underſtand this worth very well, but is there any proofe for the juſtifying hereof,

hereof, as you have of other workes?

Maſter. The work of it ſelf (being ſo perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantity, is all that is demanded for juſtifying of the ſame: yet notwithſtanding, becauſe your requeſt is reaſonable, I will propone an example for the proofe hereof.

The proof of the laſt queſtion. *A certain man is bound to pay for* 20 *yards of Velvet, the* ſumme *of* 23 pound 8 ſhillings, *and it is to be paid weekly, in* 12 *weeks or termes by* Arithmeticall Progreſſion. The queſtion is therefore to know with what number the ſame Progreſſion is to be begun and continued in ſuch equall proportion Arithmeticall, that in 12 weekes the ſame may be juſtly accompliſhed.

For the reſolution whereof, and of all ſuch other like, reduce 23 pound 8 ſhillings all into ſhillings, which maketh 468 ſhillings.

A generall rule. Then adde 1 unto 12, the number of the termes, it maketh 13, which 13 you ſhall multiply by half the number of the termes, which is 6, it maketh 78; then divide 468 by 78, and you ſhall finde 6 in the Quotient, which is the true number that ſhall begin and continue the ſaid Progreſſion. That is to ſay, the firſt week 6 ſhillings, the ſecond 12 ſhillings, and the third weeke 6 ſhillings more, which is 18 ſhillings, and ſo every weeke as they riſe, 6 ſhillings

ſhillings more then the week before, as is manifeſt in the queſtion aforeſaid.

A queſtion of a Farm.

A Farm is to be ſold to be paid by the weekes in a year; the firſt weeke to pay 4 ſhillings, the ſecond weeke 8 ſhillings, the third weeke 12 ſhillings, and ſo forth, increaſing each number by 4, till the number of 52 (which are the number of weeks in a year expired.) The queſtion is, what the price of the Farm commeth to?

Scholar. I doubt not, but by that you have already taught me, to end this queſtion very well; wherefore I ſet forth the Progreſſion with his exceſſe 52 times.

Maſter. Nay ſtay a while: And here for your further eaſe, (to abridge you of great labour that appeareth to fall out in this queſtion, and ſo may doe in any other the like) If a queſtion were propoſed of 100 or 200 places, or more, and that this queſtion, or any other the like can be ended, unleſſe you may know abſolutely what the laſt number of the Progreſſion at the 52 place is, (or ought to be) I will give you a generall rule how to know the laſt number of any Progreſſion Arithmetical, as well as if you had ordinarily proceeded by continuall Addition, till you had come to the laſt worke, which is this.

A generall rule.

Multiply the exceſſe by a number leſſe by one then the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall have your deſire.

Scholar.

Scholar. This Rule is well worth the noting: for if I underſtand you aright, I conſider that my exceſſe is 4. which I multiply by 51, which is one leſſe then the number of the places, and it maketh 204, whereunto I adde the firſt number of the Progreſsion, which is 4, and then it is 208, which you ſay is, or ſhould bee the laſt Number of the Progreſsion.

Maſter. This is a moſt approved truth, if there were never ſo many places.

Scholar. This Rule is ſo eaſie, that I were much to blame, if I do not remember it. For by the benefit hereof, I have ſuch an eaſe and light into this excellent Art, that my firſt entrance doth ſeem to paſſe a great many mens further ſtudy, and longer continuance.

Maſter. Many moe Conſiderations could I propound you in theſe *Arithmeticall Progreſſions*; but theſe are ſufficient for a taſte, to give you occaſion to think that *Rules* of Knowledge and *Arts*, are infinite capable of enlargement.

Scholar. Happy were I, if I did but well underſtand that which is already invented and written. But theſe things in my ſimple fantaſie, offer themſelves to be greatly beneficiall unto the aide of Progreſsion, Therefore now I will goe forward with your Queſtion.

Now conſidering that the 52 and laſt place is 208, I adde thereunto the firſt number

numbers of the Progreſsion, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, and it amounteth to 5512 ſhillings. And ſo much is the totall ſumme or addition of this Progreſsion: which maketh 275 pounds 12 ſhillings, as appeareth here by my Tables.

Maſter. I like well your labour, and commend you for your diligence: I will here propone one example more, and therewithall for this time will end Progreſsion Arithmeticall.

A certain man bought 20 Ells *of* Holland, *to be paid in* 17 *weekes, or termes, by* Progreſsion Arithmeticall. *And the firſt week to pay* 1 ſhilling 8 pence, *the ſecond week* 3 ſhillings 4 pence, *the third weeke* 5 ſhillings, *the fourth weeke* 6 ſhillings 8 pence, *and ſo forth, each weeke ſucceeding* 20 pence *more then the weeke before.* The queſtion is, what the ſumme of his 20 Ells cometh to. A queſtion of Holland

Scholar. Becauſe here is mention made both of ſhillings and pence, I feare there is ſome harder matter contained herein, then in the other before: therefore I pray you worke it your ſelfe, and I will diligently mark your labour.

Maſter. There is no more to be done in this, then in the other before; but becauſe your requeſt is ſo reaſonable, be attentive unto me.

Firſt, by the generall Rules, I ſeek to finde out the laſt Number of the 17 place, what this

this Progreſſion ought to be. Therefore here in my Tables multiplying the exceſſe 20 by 16, which is one leſſe then the number of the terms or places, and it commeth to be 320; and thereunto adding the firſt number of the Progreſſion, which is 20 pence, all is 340 pence, or 28 ſhillings 4 pence: for ſo much ought the laſt number of the payments to be.

Then finally, to know what the whole 17 places amount unto, I adde the firſt number of the Progreſſion and the laſt together, which make 360. Now becauſe 17 is an odde number, whoſe halfe cannot be taken, I take the halfe of 360, which is 180, and multiplying 180 by 17, commeth to 3060 pence, which maketh as you ſee by Diviſion 12 pound 15 ſhillings. And ſo much is the buyer to pay for his 20 Elles of Holland. Which 3060 pence if you divide by 20, the number of Elles that was bought, you ſhall find 12 ſhillings 9 pence, and ſo much payed hee for an Elle one with another.

The Proof.

Aqueſtion of debt.

A certaine man doth owe 12 pound 15 ſhillings, *to be paid in* 27 weeks *or* terms *by* Arithmeticall Progreſſion. *The queſtion is, to know with what* number *he ſhall begin and continue the* Progreſſion, *in ſuch equall proportion, as the ſame may be truly paid and ſatisfied in* 17 weeks.

The

The Anſwer.

Firſt I reduce 12 pounds 15 ſhillings, all into pence, which as you ſee here in many Tables, make 3060 pence, that I let ſtand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I ſhould multiply by halfe the number of the weekes or termes, which is $8\frac{1}{2}$ which $8\frac{1}{2}$ multiplied by 18 cannot well be done, unleſſe you were acquainted with Fractions or broken Numbers, therefore you ſhall let that paſſe and multiply 17 by the half of 18, which is 9, (for that is all one with the multiplication of $8\frac{1}{2}$) and the multiplication of 9 into 17 maketh as you ſee 153, with which number you ſhall divide the 3060 pence beforeſaid, and the Quotient bringeth forth 20 pence, which is the firſt number or payment to beginne the Progreſſion withall: and ſo each week ſucceeding to riſe 20 pence more then the weeke before, and thereby in 17 weeks ſhall 12 pound 15 ſhillings be payed: as before was ſufficiently declared. Thus much for Progreſſion Arithmeticall.

Scholar. Certainly Sir, I know not how to render you condigne thanks for theſe benefits ſhewed me, which me thinketh are ſo eaſie, delightfull, and pleaſant, that I count my ſelfe happy to be in your company.

Maſter.

Maſter. I am glad you delight ſo well herein, which is an Art of wonderfull dexterity to all ſorts of men, of what degree or profeſſion ſoever they be. And now will I proceed to Progreſsion Geometricall, wherein I will be more briefe, both becauſe I have been ſo long in this part of Arithmeticall Progreſsion, and alſo for that it would require the knowledge of Roots and ſurd numbers, (whereof ye have learned nothing) if I ſhould frame the like propoſitions in them as I have done in theſe. Therefore I will onely teach you two practices about it, and ſo end the conſiderations and workes of theſe Progreſsions.

Progreſſion Geometricall.

PRogreſſion Geometricall, *is when the numbers increaſe by a like proportion, that is, if the ſecond number contain the firſt, 2, 3, or 4 times and ſo forth, then the third containeth the ſecond ſo many times alſo: and*

ſo the fourth the third,	3	6	12	24	48
and the fifth the fourth,	1	3	9	27	81
wherefore I ſet theſe	3	2	10	50	250

examples.

Here in the firſt Example you ſee, that every number containeth the other (that goeth next before him) two times; and in the ſecond example three times, and in the third example five times. Now if you will know how to finde eaſily the ſumme of any ſuch number, doe thus: Conſider by what numbers they be multiplied;

multiplied, whether by 2, 3, 4, 5, or any other, and by the ſame number multiply the laſt ſum in the Progreſſion.

To finde the totall ſumme in any Geometricall Progreſſion.

Scholar. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, which I have framed by proceeding from 2, and continuall multiply by 4.

Maſter. Then muſt I multiply the laſt ſumme (which is 2048) by 4 alſo, and it will be 8192. Now muſt I abate from this ſumme the firſt number of the Progreſſion, which here is 2, then reſteth 8190. which ſumme I muſt divide by 1 leſſe then was the Number that I multiplyed by. Seeing then I multiplyed by 4. I muſt divide by 3, ſo dividing 8190 by 3, the Quotient will bee 2730, which is the ſumme of all the Progreſſion. And now to prove whether you can doe the ſame, I give you theſe Numbers to adde by this Rule 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tell by what number this Progreſsion doth increaſe.

Maſter. In any ſuch doubt doe thus: Divide the ſecond number by the firſt, and the Quotient will ſhew you the number that engendreth the Progreſsion.

Scholar. Then is that number in this example 5, for ſo many times is 3 in 15.

Maſter. So is it. Now worke as I taught.

Scholar. The laſt number is 46875, which I multiply by 5, and it yeeldeth 323475, from

which I abate the laſt number of the Progreſſion, that is 3, and there reſteth 234372, which I divide by 4, for that is one leſſe then 5, and the Quotient is 58593, which is the whole ſumme of the Progreſſion.

Maſter. If you remember well this, you have learned the Art of Progreſſion both Arithmeticall, and alſo Geometricall, which you may prove either by ſubtracting of each number alone from the ſumme, and ſo will there nothing remain: or elſe by adding together of all the parcels, for ſo will the ſame ſumme amount.

Aqueſtion of Satten.

A Mercer hath 12 yards of Satten, which he valueth at 16 ſhillings the yard, and ſelleth the ſame 12 yards to another man to be paid as followeth: That is to wit, for the firſt yard to have one ſhilling, for the ſecond yard two ſhillings, for the third yard foure ſhillings, for the fourth yard eight ſhillings, &c. doubling each number following, till the twelfth and laſt yard. The queſtion is, who hath made the better bargain of the buyer or ſeller.

Firſt you may ſet down 12, the number of the yards, as you ſee here in this Example. And againſt each number the number of ſhillings due to be paid as the order of Duplation or Multiplication by two teacheth.

Then reſorting to the adding up or ſumming of this Progreſſion, where I conſider that the increaſe of this ſumme proceeded by the Multiplication of 2, and therefore after I have made

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4095	

a line under the 12, I worke and multiply the laſt ſumme by 2 alſo and it yeeldeth 4096: from whence I abate the firſt number of the Progreſſion, which is 1, and then reſteth 4095: which I ſhould divide by one leſſe then I did multiply by, but ſeeing it is 1, I neede not to divide it: for 1, (as I have ſaid before), doth neither multiply nor divide, therefore I take that ſumme 4095 for the whole ſumme of the ſhillings, which by Reduction amounteth to 304 pounds 15 ſhillings, and ſo much hath the Mercer for his twelve yards of Satten: which is 17 pound, 1 ſhilling, 3 pence a yard. But I think you will buy none ſo deare.

Scholar. No Sir, by the grace of God this yeare.

A queſtion of an horſe

Maſter. *Then what ſay you to this* queſtion? *If I ſold unto you an horſe having 4 ſhoes, and in every ſhoe 6 nailes, with this condition, that you ſhall pay for the firſt nayl one* ob. *for the ſecond nayl two* ob. *for the third nayl foure* ob. *and ſo forth, doubling until the end of all the nayls. Now I aske you, how much would the price of the horſe come unto?*

Scholar. First, to know the number of the nayls, I muſt multiply 6 by 4, and it maketh 24. Then will I doe thus: I will write the number of the nayls every one in order from 1 to 24, and againſt each number of the nayls the ſumme of halfe pence duly, as the order of Duplation or Multiplication by 2 teacheth, and as in the next figure following appeareth.

Then doe I reſort to the Rule of ſumming up the Progreſſion, where I conſider that the increaſe of this ſumme proceedeth by the Multiplication of 2, as the laſt Example did. And therefore multiplying the laſt ſumme by 2 alſo, and it yeeldeth 16777216, from which I abate the firſt Number which is 1, and then reſteth 16777215, which I ſhould divide by one leſſe then I

did

1	1 did multiply: but ſeeing
2	2 that it is 1, I need not to
4	3 divide it, for 1 (as you
8	4 have before ſaid) doth nei-
16	5 ther multiply nor divide,
32	6 therefore I doe take the
64	7 number 16777215 for
128	8 the whole ſumme of the
256	9 half pence, which by Redu-
512	10 ction I find to bee 699050
1024	11 ſhillings and 7 pence halfe
2058	12 penie: that is 34952
4096	13 pounds, 10 ſhillings, 7
8192	14 pence, ob.
16384	15 Maſter. That is well done,
32768	16 but I thinke you will buy
65536	17 no horſe of the price.
131072	18 Schol. No ſir, if I be wiſe.
262144	19 Maſter. Well then, an-
524288	20 ſwer mee to this Queſti-
1048576	21 on.
2097152	22 *A Lord delivered to a* A queſtion
4194304	23 *Bricklayer a certain number* of Bricks.
8388608	24 *of loads of Bricks, whereof he*
16777216	*willed him to make twelve walles, of ſuch ſort, that the firſt wall ſhould receive two thirdells of the whole number, and the ſecond two thirdells of that which was left; and ſo every other two thirdells of that that remained: and ſo did the Bricklayer: and when the 12 walls were made, there remained one load of Brick.*

Now I aske you, how many load went to each wall, and how many load was in the whole?

Scholar. Why Sir, it is impoſſible for mee to tell.

Maſter. Nay, it is very eaſie, if you marke it well. Marke well that I ſaid, that every wall ſhould receive two thirdels of the ſumme that was left. Now take away two thirdels from any ſumme, and you muſt needes grant that that which remaineth, is one thirdell of the ſumme laſt before, Example of 9, from which if you take two thirdels, there will remain three, which is one thirdle of 9. Likewiſe from three bate two thirdels, and there will remain 1.

Scholar. This is true, and now I perceive the leaſt wall had but two load of brick.

Maſter. And by the ſame reaſon may you know how many load every wall had, according as this figure following doth ſhew, and likewiſe what the whole ſumme of brickes was, for if you make 12 ſummes, multiplying by 3, Still from the laſt remainer, as you may ſee here on the left ſide of the Table, there will appeare all the remainers of the whole wall: and if you multiply the laſt of thoſe 12 ſummes by 2 alſo, then will that be the ſumme of the loads which was delivered to the Bricklayer,

The

	1	12	2
The remainer after every wall.	3	11	6 *Loads due to each wall.*
	9	10	18
	27	9	54
	81	8	162
	243	7	486
	729	6	1458
	2187	5	4374
	6561	4	13122
	19683	3	39366
	59049	2	118098
	177147	1	354294

Summe of the loads delivered. 531440

Again if you double every Remainer, as you may ſee at the right ſide of this Table, thoſe numbers will ſhew the ſumme of loads that went to each wall, whereby you may perceive that each wall was three times ſo great as the next leſſe.

Scholar. Lo now it appeareth eaſie enough. Now ſurely I ſee that Arithmetick is a right excellent Art.

Maſter. You will ſay ſo when you know more of the uſe of it: For this is nothing in compariſon to other points that may bee wrought by it.

Scholar. Then I beſeech you ceaſe not to inſtruct mee further in this wonderfull cunning.

The Golden Rule or Rule of Proportion direct, called *the Rule of Three.*

Maſter:

The Rule of Proportion.

BY order of the Science (*as Men have taught it*) *there ſhould follow next* the Extraction of Roots of Number, *which becauſe it is ſomewhat hard for you yet, I will let it paſſe for a while, and will teach you the feat* of the Rule of Proportion, *which for his excellency is* called the Golden Rule. Whoſe uſe is, by three numbers known to find out any other unknown; which you deſire to know, as thus.

The Golden Rule.

Queſtion of boardings.

If you pay for your board for three moneths ſixteen ſhillings, *how much ſhall you pay for* eight moneths?

To know this and all ſuch like queſtions, you ſhall conſider which two of your numbers be of one Denomination, and ſet thoſe two the one over the other, ſo that the undermoſt be it that the queſtion is of: as in my queſtion 3 and 8, be both of one Denomination, for they both be months; and becauſe 8 is the number that the queſtion is asked of, I ſet the one over the other, and 8 undermoſt thus, with ſuch a crooked draught of lines. When you I ſet

3
8

the

the other number which is 16, against 3 at the right ſide of the line, thus.

3 8 Z 16

And now to know my queſtion, this muſt I do: I muſt multiply the lowermoſt on the left ſide, by that on the right ſide, & the ſumme that amounteth, I muſt divide by the higheſt on the left ſide: or in plainer words, thus, I ſhall multiply the number of which the queſtion is asked (which is called the third number) by the number of another Denomination (which is called the ſecond) and the ſumme that amounteth, muſt I divide by the ſumme of like Denomination (which is called the firſt) Then for the knowledge of this queſtion, I multiply 1 into 16, and there amounteth 128, which I divide by 3, & it yeeldeth 42 ſhillings, and 2 ſhillings remaineth, which I turne into pence, and they bee 24 pence, of which third part is 8 pence, ſo the third part of 128 ſhillings, is 42 ſhillings, 8 pence, which ſumme I write at the right hand of the figure againſt 8 thus,

Note.

The third number. The ſecond number. The firſt number.

3 Z 16 ſhillings
8 42 ſhil. 8 pence.

Hereby I know that if three moneths boarding, doe come to 16 ſhillings, that 8 moneths boarding, will come to 42 ſhillings 8 pence, and likewiſe of any other like queſtion.

But here muſt you marke, that the firſt number and the third be of one Denomination, and alſo the ſecond and the fourth, for which you

you ſeek: or elſe be of ſuch Denominations, that you in working may bring them into one; As if a man ſhould aske me this queſtion.

Queſtion of expence

Twelve weeks journeying coſt me 14 *French Crowns at* 6 *ſhillings the peece, how many pounds is that in one yeare?* Here you ſee no two numbers of one Denomination, but yet in working you may turn them into like Denomination: as thus, turn the one yeare into 52 weekes, and the fourth ſumme will be French Crowns, by the order of the working. Then to know this queſtion multiply the third ſumme 52, by the ſecond 14. and the ſumme will bee 728: that divide by your firſt number 12, and the Quotient will be 60. Crownes, and 8 Crownes remaining: which if you turn into ſhillings, they will be 48 ſhillings, which if you divide by your firſt number 12, the Quotient will be 4, which ſignifieth 4 ſhillings; put thoſe 60 French Crownes, which make 18 pounds with the 4 ſhillings for the ſumme that anſwereth to the queſtion, and it is the juſt expentes of a yeare: and the work will be thus.

12	Z	14
52		6048.

A generall Rule.

And take this evermore for a generall rule touching this whole art, that the doubtfull or unknown number that you would be reſolved of, ſhall alwayes be ſet in the third place. Note alſo the firſt number & the third, muſt ever be of one nature and denomination, or elſe muſt in working be brought

to

to like Denomination, and then of neceſsity muſt the other number be in the ſecond place.

Remember alſo that the place of the firſt number is higheſt on the left ſide, and the place of the ſecond, right againſt it on the right ſide; the place of the third number is under the firſt, as by thoſe examples you have ſeen.

Scholar. This I truſt I can do.

Maſter. But and if the queſtion be asked thus: *In 8 weeks I ſpend 40 ſhillings, how long will 105 ſhillings ſerve me?* Here you ſee that 8 weeks anſwers himſelf, and ſaith 40 ſhillings. But how long time 105 ſhillings will ſerve you know not. Therefore you ſhall ſet 105 in the third place, according as I told you even now. And the firſt place muſt alwayes be of the ſame nature or Denomination that the third is of, which here is 40. Then muſt 8 needs be that other: Now multiply 105 by 1, and it will be 840, which if you divide by 40, it will yeelde 21, which is the fourth number, and ſheweth how many weeks 105 ſhillings will ſerve if you ſpend 40 ſhillings in 8 weeks.

The figure of this queſtion is this: as if you ſhould ſay: if 40 ſhillings ſerve for 8 weeks: 105 will ſerve for 21 weekes.

Shillings,	*Weeks.*
40	8
105	21

Other diverſities there bee of working by this Rule, but I had rather that you would learne this one well, then at the beginning

to

to trouble your minde with many formes of working, with this way can doe as much as all the other, and hereafter you shall learne the other more conveniently.

Note. ¶ And for your further aide and instruction, to make you better acquainted with this Golden Rule, I have here proponed sixe questions, and their answers, which I thinke most convenient and meet to preferre the desirous to perfect understanding. The first foure are all branches of one question sprung out of the best tree (for a young learner to taste of) that groweth in this *Ground* of *Arts*: for that no manner of question in the Rule of Three whatsoever it be, can be proponed, but it must be comprehended under the reason or stile of one of these foure.

The Questions.

If 15 *Els of Cloth cost* 7 pounds 10 shillings, *what comes* 27 *Elles to at that rate?* Answer; 13 pounds 10 shillings.

If 27 *Elles cost* 13 pound 10 shillings, *what are* 15 *Ells worth?* Answer; 7 pound 10 shillings.

If 27 *Elles cost* 13 pound 10 shillings: *how many Ells shall I have for* 7 pounds 10 shillings? Answer; 15 *Elles.*

If I sell 15 *Elles for* 7 pound 10 shillings: *how many Elles are to be delivered for* 13 pounds 10 shillings? Answer; 27 *Ells.*

If 8 *pound of any thing cost* 16 shillings 6 pence:

pence: *what mony is to be received for* 49 pound? Answer: 5 pound, 1 shilling od $\frac{1}{4}$.

If 4 pound *of any thing cost* 7 pence: *what mony will* 8765 *pound of that commodity cost?* Answer: 155 pound, 4 shillings, 3 pence, q.

Of all which questions, I omit the work of purpose, that you shall what your self thereby at convenient leisure, to climb each [illegible] and gather the fruit of them, and doe [illegible] before we make an end of this Rule, to give you some instructions of the backe Rule of Three, whose order is quite contrary to this that you have learned.

Scholar. I thanke you heartily for the six Questions, which I will (God willing) practise at convenient times: I pray you proceed therefore to the Backet or Reverse Rule.

The

The Golden Rule, or Rule of Proportion Backward, or Reverſe.

Maſter.

Note this well.

IN the former Rule evermore look how much the third number is greater then the firſt, ſo much the fourth number is greater then the ſecond. And contrariwiſe: look how much the firſt ſumme is greater then the third (if it do chance ſo) ſo much is the ſecond ſumme greater then the fourth.

The backer or reverſe rule of three.

But in this Rule, there is a contrary order, as this: That the greater the third ſumme is above the firſt, the leſſer the fourth ſumme is beneath the ſecond: and this Rule therefore you may call the Backer or Reverſe Rule, as in example.

Queſtion of cloth.

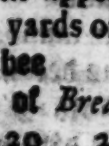
If I have bought 30 yards of Cloth of two yards breadth, and would have Canvas of three yards broad to line it withall, how many yards ſhould I need?

Scholar. Why, there is none ſo broad.

Maſter. I doe not care for that, I doe put this Example onely for your eaſie underſtanding: for if I ſhould put the Example in other meaſures, it would be harder to underſtand. But now to the matter: If you would know this queſtion, ſet your numbers as you

did

did before: but you ſhall multiply now the firſt number by the ſecond, and that ariſeth thereof, you ſhall divide by the third: which thing if you doe here, I mean if you multiply 30 by 2; it will be 60: which ſumme if you divide by 3 there will appeare 20. whereby I know, that if 30 yards of cloth of two yards broad, ſhould bee lined with Canvas of three yards broad, 20 yards of Canvas would ſuffice, as this figure ſheweth.



Breadth.	*Length.*
2	30
3	20

And now becauſe ye found fault with my Example, how ſay you, perceive you this?

Scholar. Yes Sir, I ſuppoſe.

Maſter. *Then anſwer me to this queſtion: how many Elles of Canvas of Ell breadth, well ſerve to line twenty yards of Say, of three quarters broad?*

Scholar. In good faith Sir I cannot tell, for I know not how to bring the ſummes to like Denominations.

Maſter. Then will I tell you, ſith there is mention here of quarters, and again every one of the meaſures both Els and yards may be parted into quarters, part them ſo both in the breadth and length, and then put forth the queſtion by quarters.

Scholar. Then ſhall I ſay thus. How many quarters of Canvas of 5 quarters broad will line 80 quarters of 3 quarters broad?

Maſter.

Maſter. Now anſwer to the queſtion.

Scholar. Firſt, I will ſet them downe in thoſe formes thus: for 5 is formed with the queſtion and is therefore the third number: then is 3 the number of the ſame Denomination, I meane becauſe they be both referred to breadth. Now I multiply 80 by 3, and it is 240, which I divide by 5, and it yeeldeth 48. Then ſay I that 48 quarters of 5 quarters broad, will ſuffice to line 80 quarters of three quarters broad.

Breadth.	*Length,*
3	80
5	

Maſter. Turn the quarters again into Ells and yards.

Scholar. Then I ſay, that 9 Ells and three quarters of a yard of ell-broad, will ſerve to line 20 yards of three quarters broad, as this figure ſheweth.

Breadth.	*Length.*
3	80
5	48

¶ Maſter. *Now what ſay you to this queſtion? I lent my friend* 400 pound *for* 7 *moneths, how much money ought he to lend mee againe for* 12 *moneths to recompence my courteſie ſhewed him? can you anſwer to this?*

Scholar. Yes Sir, I ſuppoſe, for I will ſet downe my Numbers thus: where I multiply 7 into 400, and it maketh 2803, which I divide by 12, and it yeeldeth

Months.	*Pounds.*
7	400
12	

yeldeth 233 pound, and there is 4 pound remaining of my Diviſion, what ſhall I do therewith?

Maſter. Turne the ſame 4 pound into ſhillings, & then divide it by 12 as you did before.

Scholar. Well Sir, it ſhall be done: ſo have I 6 ſhillings for my Quotient, and yet remaineth 8 ſhillings upon my Diviſion.

Maſter. You muſt alſo reduce that 8 ſhillings into pence, which maketh 96, and divide that alſo by your Diviſor.

Scholar. So have I done, and I finde 8 pence for my Quotient, and nothing is left.

Maſter. This muſt you alwaies doe when any thing remaineth upon your Diviſion, whether it be money, weight, meaſure, or any kinde of thing whatſoever. This Rule is ſo profitable for all eſtates of man, that for this Rule onely (if there were no more but it) all men were bound highly to eſteem Arithmetick.

By this rule may a Captain in war, worke manie things, as Maſter Digges in his Stratioços doth declare: Onely now in this my ſimple addition, for a taſte and incouragement, I will inlarge the Author with a queſtion or two more wiſhing you and every my Countreymen or Gentlemen whatſoever, that by nature be any thing given to Military affaires, to be familiar and acquainted with this Excellent Art, the which he ſhall finde not onely at the Sea, but alſo in the Campe and Field-ſervice, abounbantly to aid him, either in fortification, par-

ting of Souldiers wages, charges of Ordnance, Powder, Shot, Munitions, and Inſtruments, whatſoever, as for example.

Queſtion of an Armie.

If it ſhould chance a Captain which hath 40000 ſouldiers to bee incloſed with his enemy, that hee could have no freſh purveyance of victuals, and that the victuals which hee had would ſerve that Army but onely three moneths, how many men ſhould he diſmiſſe to make the victuall to ſuffice the reſidue 8 moneths.

Scholar. As you taught me, I ſet the numbers thus, ſaying: If three moneths ſuffice 40000, to how many will 8 ſuffice?

Moneths.	*Men.*
3	40000
8	

To know this; I multiply the firſt number 3 into the ſecond 40000, and it yieldeth 120000, which ſumme I divide by 8, and there will be in the quotient 15000, which if I doe ſubtract from 40000, the remainder will declare that hee muſt diſmiſſe 15000 as this figure ſheweth.

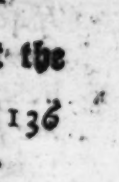
Moneths	*Men.*
3	40000
8	15000

A queſtion of a Fort.

Maſter. *Now anſwer mee to this queſtion: If 136 maſons in a moneth bee able to build a Fort to preſerve the ſouldiers from the Enemy, and ſuch expedition requireth that I would have the ſame fiinſhed in eight dayes: how*

how many workemen ſay you is there to be appointed?

Scholar. As you taught mee; I ſet the numbers thus, ſaying: If 28 dayes require 136 Maſons, what number of men by the like proportion will 8 dayes require?



To know this, I multiply the firſt number 28 into 136, and it yeldeth me 3808: which I divide by 8, and my Quotient is 476: which is the juſt number of Maſons that ſhall ſupply this woꝛke. And now me thinke theſe queſtions are very eaſie.

Maſter. Truly if you take delectation herein, you ſhall [illegible] this Art not onely eaſie, but wonderfull pleaſant and profitable. Now therefoꝛe one queſtion moꝛe I will propoſe, and ſo leave off this Rule in whole numbers, until we come to the uſe of it in broken numbers: foꝛ had you the underſtanding of broken numbers perfectly, not onely in this Rule, but in all other, the queſtion that in the ſight oꝛ appearance ſeemeth to bee 100 times moꝛe harder to abſolve, may thereby bee wꝛought as ſoon, oꝛ ſooner then this.

Scholar. Your woꝛds doe greatly incouꝛage me to bee ſtudious to attain whole numbers: but might I once attaine to be a Practitioner in broken numbers, I ſhould thinke my ſelfe happy.

Maſter. *VVhat ſay you then to this queſtion? If 48 Ioyners in two dayes make 200 light horſmen ſtaves (eſteeming they worke but 12 houres a day) and ſuch need requireth that 384 Joyners are ſet to the finiſhing of thoſe 200 ſtaves; in what time ſay you, will they make them up?*

Scholar. I ſee here that I muſt turn my 2 dayes into houres. And ſo doing, I ſet my numbers thus:

48 Z 24
384

ſaying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded upon an old Proverb, *many hands make quick ſpeed.*

I multiply 48 into 24, and it amounteth to 1159, which I divide by 384, and my quotient is 3 houres, which is my deſire.

Note. *I take this for a note worthy the marking, either in the Rule of Three, forward or backward, when the two numbers are multiplied together, the Product is of the ſame nature and determination that the ſecond number is of.*

The double Rule of Proportion direct.

Maſter.



Well, ſith you perceive now the uſe of this rule, I will ſhew other which inſue of the ſame, and firſt the *double Rule* which is ſo called, becauſe there is in it double working, by which thing onely it differeth from this. The double Rule.

Scholar. Then by an example I ſhall underſtand it well enough.

Maſter. *So ſhall you, and let this be the example: If the carriage of* 100 *weight (that is* 112 *pound)* 30 *miles doe coſt* 12 *pence, how much will the carriage of* 500 *weight coſt being carried* 100 *miles?* Queſtion of carriage.

Schol. I pray you ſhew me the working of it.

Maſter. You muſt make two workings of it: the firſt thus: If C weight coſt 12 pence, how much will five hundred weight coſt? Set your figures thus:

C weight.	*Pence.*
1	12
5	

And multiply 5 by 12, and thereof amounteth 60, which if you divide by one, the Quotient will be ſtill 60, that is the price of 500 weight for 30 miles.

Then begin the ſecond worke, ſaying: If

30 miles coſt 60 pence, how much will 100 miles coſt? Set your figure thus.

Miles. 30 100 *Pence.* 60



Then multiply 100 by 60 whereof amounteth 6000, which being divided by 30, will yield 200 pence. Then you may ſay, that ſo many pence ſhall coſt the cariage of 5000 pound weight 100 miles after the rate of 12 pence for the 100 carried 30 miles.

Scholar. Now I perceive it alſo.

¶ Maſter. Theſe and ſuch other like queſtions of the double Rule of Three, are to be anſwered much ſooner, at one onely working by the Rule of Proportion compoſed of five numbers, which anon I will ſhew you, and then when you have the uſe thereof, you may uſe it which way you think good.

Scholar. Sir, I thanke you much for your courteſie, And I long now till this Rule bee ended, that I may ſee how I may behave my ſelf with that new Rule of five numbers: for that I have ever ſince you taught me hitherto in the Golden Rule both forward and backeward, wrought but with three numbers onely.

Maſter. But yet a while we will go on forward with this Rule of Three, therefore anſwer to this queſtion.

Queſtion of ſowing. Thirty buſhels of wheat ſowed, yielded in one yeare 360, how many will 80 buſhels yielde in 7 yeares? I meane ſowing every

year

yeare of thoſe ſeven, ſtill foureſcore buſhels?

Scholar. First I ſay, that if 30 buſhels will yield 360 in one yeare, then 80 buſhels will yield 960 in one yeare. Then for the ſecond worke, I ſay, If one yeare yield 960, then 7 yeares will yield 6720; as theſe two figures doe ſhew.

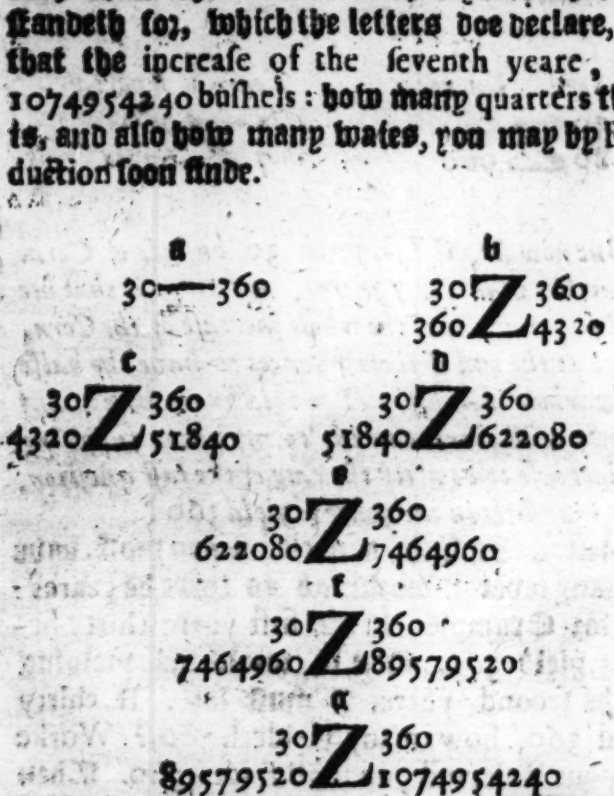
Seed.	*Encreaſe*	*Yeare.*	*Encreaſe.*
30	360	1	960
80	960	7	6720

Queſtion of Corn.

But now ſir, if I ſet forth 30 buſhels of Corne to another man for 7 yeares, agreeing ſo that hee ſhall ſow every year the whole increaſe of the Corn, and I at the end of theſe 7 yeares to have the halfe of the whole increaſe: I would know how many buſhels will there amount to my part, ſuppoſing the increaſe to be after the rate of the laſt queſtion, for 30 buſhels in one yeare to yield 360?

Maſter. In ſuch a queſtion you muſt have ſo many ſeverall workings as there be yeares: As for Example, in the firſt yeare thirty buſhels yield 360. then to know the yelding of the ſecond yeare, I muſt ſay, If thirty yield 360, how many yieldeth 360? Worke by your Rule, and you ſhall finde 4320. Then ſay for the third yeare. If thirty yield 360, How many will 4320 yield? you ſhall have 51840, and ſo every yeare multiplying the whole increaſe by 360, and dividing

it by 30, the increase of the next yeare will amount, as these 7 figures following do orderly declare: where I have set 7 letters for the 7 yeares, of which the first is set without Art, because that is the increase which you doe presuppose: and the last number of each other doth shew the increase of that yeare that it standeth for, which the letters doe declare, so that the increase of the seventh yeare, is 1074954240 bushels: how many quarters that is, and also how many wates, you may by Reduction soon finde.



Question of mowing

Now with one question more I will prove you. *If sixe Mowers doe mow 45 Acres in 5 dayes, how many mowers will mow 300 Acres in 6 dayes?*

Scholar.

Scholar. If 45 Acres require 6 Mowers, then 300 Acres require 40. Now again, if 5 daies require 40 mowers, then 6 daies need but 33 mowers.

Master. Why doe you not make mention of the 2 that remaineth in the last Division? for the last part of the question is wrought by the Backer Rule, where the first number 5 is multiplied into the second, that is 40, whereof amounteth 200, which if you divide by the third number 6, the Quotient will be 33, as you said: but then will there remain 2 which cannot well be divided into 6 parts: howbeit you may understand by the 6 part of 2, the third part of one mans worke, which you must put to the 33; or else you must say that 33 Workmen will end all the 300 Acres in 6 dayes save 2 mens worke for one day, or two dayes worke for one man. But such broken numbers called Fractions, you shall hereafter more better perceive, when I shall wholly instruct you of them.

Master. *Yet one question more of field matters I will propone, and so I will make an end of this double Rule of Three.*

Scholar. With all my heart. Sir I thanke you, and I will dispatch it as soon as I can, because I would faine see the order of the next Rule of 5 numbers.

Master. *If a Captain over a band of men did set 300 Pioners a worke, which in eight houres did cast a trench of 200 Rods: I demand how many labourers* Question of entrendchings.

Labourers will be able with a like trench in three houres, to intrench a camp of 3400 *Rods.*

Scholar. I thinke I am now in the Backe-house ditch: for I know not well which way to go about it. And besides that, truly I think I shall never come to preferment that way my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and service into the favour of your Prince, for the avail of your Countrey.

Example for Navigation: Sir *Francis Drake*, a man greatly honoured for his knowledge, was not the tallest man, and yet hath made as great an adventure for the honour of his Prince and Countrey, as ever Englishman did.

Scholar. Sir, I thanke you for your good incouragement. My minde, though I be little, is as desirous of knowledge, as any other: I have pondred now a little of it, and thus I set forth the worke.

Rod	*Men*
200	300
3400	Z

Saying, if 200 Rod require 300 men, what shall 3400 rods require? I multiply 3400 by 300, and it yieldeth 1020000, which I divide by 200, and my quotient is 5100 men.

Then must I say for the second worke, if in 8 houres 5100 men, be able to discharge it, how many shall performe the same in three houres? Now if I would worke by the Golden

Golden Rule of Proportion forward, I should finde a lesse number of men: because 3 houres is lesse then 8 houres: but because reason teacheth me, that the lesser the time is, wherein the trench must be made, the more Labourers I ought to have, thereupon I use now the Backer Rule, as in example. And I have in my Quotient 13600. So many Pioners must I have to intrench the Çampe in 3 houres.

Master. You have answered the question very artificially: And truely I commend you for your diligence and apt understanding: and now according to my promise, I will (in whole numbers) give you a little taste of the Rule of Proportion, compounded in 5 numbers.

The Rule of Proportion, compoſed of 5 Numbers.

The firſt part of the Rule of Proportion compound, direct.

THe Rule of Proportion compoſed, is diſtinct for moſt needfull queſtions, into ſeverall parts or workings: And there belongeth unto it alwayes five numbers, whereof in this Rule being the firſt part, the ſecond number and the fifth, are alwayes of one nature and like denomination, which Rule is to be wrought thus: you muſt multiply the firſt number by the ſecond, and that ſhall be your diviſor: Then again, multiply the other three numbers, the one by the other, and their Product ſhall be your dividend.

And now according to my promiſe, we will firſt work the queſtion of weight and carriage, which I delivered you in the double Rule of Three, to be abſolved by this Rule, which was this.

If the carriage of 1 *C. weight,* 30 *miles coſt* 12 *pence, what will the carriage of* 5 *C. weight ſtand me in, being carried* 100 *miles?*

C. weight.	*Miles.*	*Pence.*	*C. weight.*	*Miles.*
1	30	12	5	100

Now marke well how theſe five numbers ſtand: Then multiply the firſt number **by the** ſecond, **as** 30 **by** 1, **which maketh but** 30,

30. that number keep for your Diviſor. Then multiply the other 3 numbers, the one into the other: that is to wit, 12 by 5, which maketh 60: Laſtly 60 by 100, which as you ſee here in our Tables, ariſeth to 6000, which 6000 you ſhall divide by the Product of the two firſt numbers, which here is 30. And you ſee there is found 200 pence, which is the duttie that you ought to pay for the carriage of 500 weight 100 miles, after the rate of 12 pence a hundred, and agreeth with the concluſion of the double Rule of Three.

Scholar. Sir I thanke you, it is even ſo.

Maſter. *Yet note this in a generality in this Rule, looke what nature or denomination your middle number is of (which here are pence) and of the like denomination or nature is alwayes your quotient.* Note this.

Scholar. Well now and if it pleaſe you, by your patience I will ſee how I can end the queſtion next following of 30 Buſhels of wheat ſowed, which in one year yieldeth 360, how many then will 80 Buſhels yield in 7 year following every year of thoſe 7 will 80 buſhels, and according to your reaſons I ſet my numbers thus.

Buſh.	*Year.*	*Buſh.*	*Buſh.*	*Year:*
30	1	360	80	7
			80	
			28800	
			7	
			201600	

When

Where I multiply 30 by 1, and it maketh 30 my Diviſor; then multiplying the other 3 numbers the one into the other, as here appeareth in my Tables, they make 201600, which I divide by 30: and my Quotient is 6720 buſhels, my deſire, for ſo much alſo it came to at two workings by the Rule of Three.

Maſter. Yet one queſtion more I will propound unto you, and ſo leave this Rule, till it pleaſe God hereafter, that I may make you worke it in broken numbers.

Queſtion of Intereſt

What comes the intereſt of 258 pound, *for* 5 *moneths to, after the rate of* 8 pound *taken in the* 100 pound *for* 12 *moneths?*

Scholar. Sir, this is yet within the compaſſe of ſome reaſonable uſance. Therefore to miniſter equitie in this caſe, I will ſee how I can worke the ſame, which I ſet down thus, praying you if I have not done well, to ſhew me mine errour.

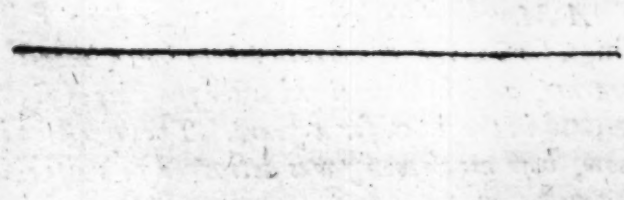
li. *moneths.* li. li. *men.*
100 — 12 — 8 — 258 — 5.

Maſter. Proceed, you have done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yeildeth 1200, and the three other numbers multiplied together produce 10320, which I divide by 1200: and my Quotient is eight pounds. Then according as you have taught mee heretofore, I turne the 720 pound that I left, into ſhillings: and dividing it by the firſt number my Quotient is 12 ſhillings,

lings. So I anſwer, that the loane of 258 pounds for 5 moneths, after the rate of 8 pound in the 100 pound for a yeare, comes to 8 pound 12 ſhillings.

Maſter, You ſay true, I commend your diligence: now behold the manner of the ſecond part of this rule.



The backer Rule, or the second part of the Rule of Proportion compound.

Master.

IN *the second part of this Rule of Proportion composed, the third number is like unto the first. And the Rule is to bee wrought thus: you shall now, contrary to the last Rule, multiply the third number and the fourth together, and that Product shall be your Divisor. Then multiply the fifth by the second, and the Product thereof by the first: and that is the number that shall be divided.* For example, I propound this question, for a proofe of my last question of Interest.

The proofe of the last question.

A Merchant hath received 8 pound 12 shillings, *for interest of certain money for* 5 *moneths tearme, which he received after the rate of eight* pound *in the* 100. *for a yeare. The question is now, how much money was delivered to raise this interest.*

Behold therefore the manner, how the question is set forth.

li. *moneths.*	li. *moneths.*	li. s.
100 —	12 —	8 — 5 — 8-12

Scholar. Sir I perceive it very well: and according

according to the doctrine which you prescribed for the working thereof: if please you now it is set downe, I thinke I can follow the worke.

Master. Nay, stay a while, and before you worke, marke well how I deliver a reason for the perfect understanding of this Rule, which is thus: *If* 8 *pound in* 12 *moneths doe yeeld mee* 100 *pound, to take* 8 *pound* 12 *shillings for five moneths, must needes yeelde a great deale more.* Note.

So upon the knowledge that I have in this Art, The first part of this Rule is answerable to the Rule of Three forward: and this latter part accordeth to the Rule of Three backward.

Schol. Sir, I yield you most heartie thanks for these your last instructions, they have given me great light into these two rules, whereby I may the better by deliberation conceive how to use them hereafter when occasion shall require.

Master. You say well, goe to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turn all your monie as you work into shillings, for your more ease in working. Note.

Scholar. If it please you to behold me a little, I will quickly end it: for I have but my first, my second, and my last number to be multiplied together for my Dividend: And my third

into my fourth for my Diviſor.

li.	*Moneths.*	*Moneths.*	li.	s.
100	12	8 — 5	8	12
20		20	20	
2000		160	172	
12		5		
4000		800		
2000				
24000				
172				

48000 4128000(
168000 8 00
24000
4128000

Which 4128000 I divide by 800, and my Quotient is 5160 ſhillings, which in pounds yieldeth 258. my deſire.

Maſter. I will here for this time in whole numbers end this Rule, and I will inſtruct you in the Rules of Fellowſhip. You may at your convenient leaſure for your exerciſe worke the ſame by the Rule of Three at twice. And for your aide and incouragement therein, I ſet down here a proffer how to apply it.

A			B		
Moneths.	li.	s.	*Pound.*	li.	
5	8	12	8	100	
12	412	4/5 c.	412 4/5	258	li.

The Rule of Fellowſhip.

The Rule of Fellowſhip without time.

BUt now will I ſhew you of the Rule of Fellowſhip or Company, which hath ſundry operations according to the divers number of the Company. This Rule is ſometime without difference of time, and ſometimes there is in it difference of time. Firſt I will ſpeak of that without difference of time, of which let this be an example.

A queſtion of company.

Foure Merchants of one Company made a bank of money diverſly: for the firſt laid in 30 pound, *the ſecond* 50 pound, *the third* 60 pound, *and the fourth* 100 pound, *which ſtock they occupy ſo long, till it was increaſed to* 3000 pound. *Now I demand of you what ſhould each receive at the parting of this money.*

Scholar. **I perceive that this Rule is like the other, but yet there is a difference which I perceive not.**

Maſter. **Then will I ſhew it to you: Firſt by** Addition, **you ſhall bring all the particular** ſummes **of the** Merchants **into one** ſumme, **which ſhall be the** firſt ſumme **in your working by the** Golden Rule, **and the whole** ſumme **of the gaines by that** ſtocke **ſhall bee the** ſecond ſumme. **Now for the** third ſumme **you ſhall**

shall set the portion of each man one after another, and then worke by the Golden Rule, and the fourth summe will shew you each mens gaines: as in example.

	30
	50
	60
	100
	240

The parcels of the foure Merchants make in one summe 240 pounds: set that in the first place, the gaines in the second, and the first mans portion of stocke in the third place, thus:

240 Z 3000
30 Z

Now multiplie the second by the third, and it will bee 90000, which you shall divide by 240, and there will appeare 375 pounds, thus:

And that is the gaine for the first man.

240 Z 3000
30 Z 375

Now for the second man, set the 50 pound that hee brought, in the third place, and worke as before: and his part will be 625 pound: as this figure sheweth.

Likewise for the third man, set his money which was 60 pounds, and his part of gains will be 750 pounds, as here appeareth.

240 Z 3000
50 Z 625

And so for the fourth man; if you set his sum which is 100 pound, his gaines will bee 1250 pound, as the worke will declare.

240 Z 3000
60 Z 750

Scholar. This I perceive: but is there any way to examine whether I have well done or no?

240 Z 3000
100 Z 1250

Note this common proofe.

Master. For the triall hereof, adde together all their foure portions, and if their addition make the whole summe of their gaines, then is the worke well done.

Scholar. That will I trie by and by; the foure parcels are those, which added together make 3000, which is the just summe of money that they gained, whereby I know the worke is well done.

375
625
750
1250
3000

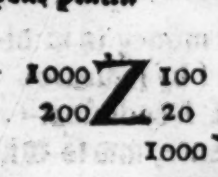
Master. Well, now another example will I put to you, not of gains, but of losse: for one reason serveth for both.

A question of losse.

If three Merchants in one ship, and of one fellowship, had bought Merchandise, so that the first had laid out 200 pound, the second 300 pound, the third 500 pound, and it chanced by tempest that they did cast over board into the Sea Merchandise of the value of 100 pound, how much should each man bear in this losse?

Scholar. If I shall do in this as you did in the

the other question, then muſt I ſome their 3 portions together, 200, 300, 500, which maketh 1000. Then ſay I, If 1000 loſe 100 then ſhall 200 loſe 20, and 300 ſhall loſe 30 and 500 ſhall loſe 50, as by the three figures it doth appear plain.



Maſter. Well, ſith now you have done theſe, I will propound a queſtion of more importance, which ſhall make you not onely the abler to underſtand this Rule, but alſo it will greatly aid you in the next Rule of Fellowſhip with time, if ſuch need be that your money be of diuers Denominations.

For this may not bee forgotten in all ſuch queſtions: If the number be of diuers kindes, you muſt by reduction bring it into one kinde, that is to ſay, to the leaſt value that is named in the queſtion. And likewiſe ſhall you doe, if the time bee of divers kindes, as ſome yeares, ſome moneths, weeks, and dayes, you ſhall make all moneths, weeks, or dayes, according as the leaſt name of time in the queſtion is, as for example.

Firſt in diverſity of money. Three companions bought 2000 *ſheep, and paid for them* 241 *pound* 13——4 *pence, of which ſumme one paid* 101 A queſtion of ſheepe.

pound 10 *ſhillings. The ſecond* 82 *pound* 17 *ſhillings* 10 *pence, And the third paid* 57 *pound,* 5 *ſhillings* 6 *pence: How many ſheepe muſt each of them have?* Anſwer. *The firſt ſhall have* 840. *The ſecond* 686. *And the third* 474. *And that muſt you worke thus:*

Solution. Firſt, conſidering that your money is of diuers Denominations, you ſhall (by reduction) bring it all into the ſmalleſt Denomination which is in it, that is to ſay, pence, and ſo will the Totall ſumme be 58000 pence.

Now if you turne each mans money into pence alſo, the firſt mans ſumme will be 24360 pence: The ſecond mans money will bee 19894 pence; and the third mans money will be 13746 pence.

Now to know how manie ſheep everie man ſhall have, let the whole ſumme of money, that is, 58000 pence be ſet in the firſt place, and in the ſecond place ſet the number of ſheep, and then orderly in the third place ſet each mans money, and then multiplying the third and the ſecond ſummes together, and dividing that that amounteth by the firſt, there will appeare the number of ſheep that each man ought to have; as theſe 3 figures do ſhew.

a b

58000 Z 2000 58000 Z 2000
24360 Z 840 19894 Z 686

c

58000 Z 2000
13746 Z 474

Scholar. Why doe you ſet the money in the firſt place, ſeeing in the queſtion you ſay 2000 ſheep coſt 58000 pence, and not thus: 58000 coſt 2000 ſheep?

Maſter. You remember I taught you at the beginning of the Golden Rule, that the firſt and third numbers muſt be of one name, and of like things: and evermore the number that the queſtion is asked of, muſt bee ſet in the third place.

Now is the queſtion plainly this: If foure men bought 2000 ſheep for 58000 pence, how many ſheep ſhall each man have?

But ſeeing in this queſtion, there ought more reſpect to be had to the ſumme of money, then to the ſum of the perſons; (for in the ſumme of money is their proportion toward the ſheep, and not in the number of perſons.)

If 58000 *pence bought* 2000 *ſheepe, how many did* 24360 *buy? Againe, how many did* 19894 *pence buy? And how many bought* 13746 *pence?*

Scholar. I perceive it reaſonablie, and ſo ſhall

ſhall I doe in all queſtions.

Maſter. Even ſo. But for eaſineſſe of the worke, marke this: Whenſoever the firſt and ſecond numbers have Cyphers in the firſt places, you may both in the multiplication and in the Diviſion leave out thoſe cyphers, ſo that you leave out like manie out of both ſummes, as in this queſtion, the firſt number 58000 hath 3 cyphers, and ſo hath the ſecond, that is 2000: therefore caſt away their cyphers, and ſo will the firſt number bee 58, and the ſecond 2: ſet them in their places, and worke according to the Rule, and you ſhall perceive that will bee all one, ſaving that this is the ſhorter and eaſier way, as theſe three figures doe ſhew.

Note.

a

58 Z 2
24360 Z 840

b

58 Z 2
2 9894 Z 686

c

58 Z 2
13746 Z 474

And this you ſee is both eaſier, and alſo the more certaine way to know the anſwer to this queſtion.

Scholar. Truth it is as you ſay: But Sir me ſeemeth I might aske a further queſtion here, not onely how many ſheep each man ſhould have, but alſo what everie ſheep coſt.

Maſter. That queſtion doth not onelie be-

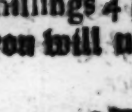
long

long to this Rule, but may alſo be diſcuſſed by Diviſion, eſpecially if the queſtions number bee one onely, as thus: Divide the totall ſumme 58000 pence by 2000 (or 58 by 2) omitting the cyphers, and the Quotient will be 29 pence, that is 2 ſhillings 5 pence. Howbeit, by this Rule you may do it, and beſt when the number of the queſtion doth exceede 1; as if I ſhould aske this queſtion,

2000 ſheep coſt 58000 pence, how much doe 20 coſt? Then ſhall I ſet my figures as before.



And doing after the Rule, there will amount 580 pence, that is, 2 pound 8 ſhillings 4 pence, the price of one ſheepe: but if you will uſe that eaſie way that I did teach you now, you may change the firſt and ſecond number thus.



Thus doe you perceive the uſe of the Rule without time.

Scholar. All this I underſtand very well: I pray you now inſtruct mee in the Rule of Fellowſhip with Time.

The

The Rule of Fellowſhip *with Time.*

Maſter.

The Rule of Fellowſhip with time.

TO the intent you may as well perceive the ſame Rule with diverſity of time, I propoſe this example.

Foure Merchants made a common ſtocke, which at the years end was increaſed to 35145 pound. *Now to know what ſhall be each mans portion of gain, you muſt know each mans ſtocke, and time of continuance.*

Queſtion of a Bank.

The firſt man of theſe foure laid in 669, l. which he did take from the ſtock again at the end of 10 moneths. The ſecond man laid in 810 pound, for 8 moneths. The third laid in 900 pound, for ſeven moneths. And the fourth laid in 1040, for 12 moneths.

Note. A generall Rule.

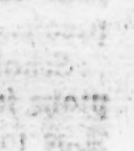
This queſtion ſhall you examine as you did the other before, ſaving that wherens in the third place of the Figure **you did ſet each** mans ſumme alone, **here you ſhall ſet the ſame being multiplied by the** number **of their** time: **and likewiſe in the** firſt place of the figure **you ſhall ſet the number which amounteth of their whole** ſummes **ſo multiplied by their time, and added into one whole** ſumme, **as thus.**

The

The firſt mans ſumme is 669 pounds, which I multiply by 10 (that was the number of his time) and it maketh 6690. The ſecond mans ſumme 810 pound, multiplied by 8 (which was his time) maketh 6480. The third mans ſumme 900 pound, multiplied by 7 (for that was his time) yeelds 6300. The fourth mans ſumme was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

The foure ſummes thus multiplied by their time, muſt be ſet orderly in the third place of the figure, and in the firſt place muſt be ſet the whole ſumme of all foure, which is 31950; and the gain muſt be in the ſecond place, which is 35145. Now to end the queſtion, I ſay firſt, If

31950 did get 35145, what did 6690 get? Anſwer. 7359



pounds, as by this figure appeareth.

Likewiſe, the ſecond man had to his part 7128 pound, the third muſt have 6930 pounds, and the fourth man ſhall have for his part 13728 pounds, as theſe figures doe partly declare.

b

31950 Z 35145
6480 Z 7128

c

31950 Z 35145
6300 Z 6930

d

31950 Z 35145
12480 Z 13728

Scholar. This I like very well: but what proof is there of this worke?

Another proofe.

Master. The same that I taught you for the other: howbeit, there is used both for this worke and the other also, this manner of proofe, to adde all the portions together, and if may agree to the whole summe, then seemeth your worke well done: but this is no sure proofe.

Scholar. Yet will I prove in this example: The foure parcels are these, which if I adde together, there will amount 35145, and that was the whole summe, whereby I perceive the worke is well done.

	7359
	7128
	6930
	13728
	35145

Master. If it fall out otherwise, be sure it is not well.

Scholar. Then do I understand this worke also very well: But what have I now to learne?

Master. There are many other excellent parts

parts behinde, of which I will not as now make mention, because that without the knowledge of Fractions they cannot bee duely taught, and much lesse understood. Therefore will I propose to you two or three questions more (that thereby you may better perceive the use of this Rule and all other the like) and so make an end for this Time.

¶ *Three Partners by some ill adventure sust ained the losse of* 160 pound, *whereof the first laid into the common stocke* 200 pound, *for tenne Moneths. The second laid in* 350 pounds, *and the third* 100 pound, *but for how long the two latter, is unknown: But breaking off their Partnership, the first found himselfe a loser* 80 pound, *the second* 56 pound, *and the third* 24 pound. *The question is, for how long time was the money of the two latter in company.* A question of losse.

For the solution hereof, and of such other like, you must also multiply the first mans 200 pound, that bee put into the stocke by his time of continuance, which was ten moneths; and it maketh 2000: wherefore now I affirme, if his money that lost 80 pound multiplied by his time make 2000: what shall his money make that lost 56 pound, and his that lost 24 pound, which two numbers I commit to the triall of the Rule of Three, at two workings, thus;

If 80 give 2000, what giveth 56? And again, if 80 give 2000, what giveth 24?

80

80 Z 2000 | 80 Z 2000
56 Z 1400 | 24 Z 600

To conclude, if you now divide 1400, the ſecond mans portion, by 350, which was his ſtock that he laid into company, you ſhall find in your quotient 4 moneths, and for ſo long time did the ſecond man put his money into the common ſtock.

Laſtly, if you divide the third mans new laying in, which was 600 by 100, which was his ſtock that he put into the company, the quotient declareth his time of continuance, which was ſix moneths. And thus is the queſtion reſolved.

Scholar. Sir, I have attentively beheld your working, and the more we travell herein, the more me thinke I am in love with this excellent Art.

Maſter. Then what ſay you to this Queſtion?

A queſtion of Canons

There is in a Cathedrall Church 20 Canons, and 30 Vicars, thoſe may ſpend by year 2600 pound, but every Canon muſt have to his part 5 times ſo much as every Vicar hath: how much is every mans portion ſay you?

Scholar. I pray you make the anſwer your ſelf alſo, ſo ſhall I perceive beſt the meanes to anſwer to ſuch other like.

Maſter. In this Queſtion you muſt doe as in thoſe beforeſaid, that have diverſitie of time,

time, for here is diuersity of portions. Therefore shall you multiply the number of the persons by their difference of portions: (as you did in the other by time:) Then must you multiply the 20, (which is the number of Canons) by 5, (for that is the number of their portion) so will it bee 100. Then 30, (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put these two summes together, and they make 130. Then say thus; If 130 spend 2600 pounds, what may 100 spend? The Rule sheweth 1000 pounds.

Again for Vicars: if 130 spend 2600 pound, what may 30 spend? Answer 600 pound, as these figures shew.

130 Z 2600	130 Z 3600
100 Z 2000	30 Z 600

But if every Canon should have so often times 4 pound as the Vicar should have 3 pound, then should I multiply 20 by 4, (that were 80) and 30 by 3, (that were 90) and then both were 170. Then should the figures be set as followeth.

li. s. d.	li. s. d.
170 Z 2600	270 Z 26000
80 Z 1223,-10, 7	90 Z 1376.-9, 5

But this sort is too hard for you, by rea-

ſon of the fractions, therefore I will let it reſt to that place.

And by this rule you ſee what the 20 Canons muſt ſpend; which ſumme if you divide by 20, you ſhall ſee each Canons proportion: and ſo of the Vicars, if you divide theſe ſumme by 30, the quotient will declare every Vicars portion.

The

The second Dialogue.

The accounting by Counters.

Maſter.

NOw that you have learned the kinde of Arithmetick *with the Pen, you ſhall ſee the ſame Art in* Counters: *which feat doth not onely ſerve for them that cannot write and read, but alſo for them that can do both; but have not at ſome time their pen or tables ready with them.*

This ſort is in two formes commonly. The one by lines, and the other without lines. In that that hath lines, the lines doe ſtand for the order of places: and in that that hath no lines, there muſt be ſet in their ſtead ſo many Counters as ſhall need for each line one; and they ſhall ſupply the ſtead of lines.

Scholar. By examples I ſhould better perceive your meaning.

Maſter. For example of the lines; loe here you ſee ſix lines, which ſtand for ſix places, ſo that the

Numeration by Counters.

nethermoſt ſtandeth for the firſt place, and the next above is for the ſecond, and ſo upward till you come to the higheſt, which is the ſixth line, and ſtandeth for the ſixth place.

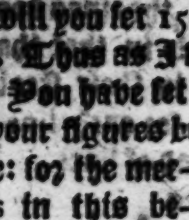
Now what is the value of every place or line, you may perceive by the figure which I have ſet on them, which is according as you learned before in Numeration of figures by the Pen, for the firſt place is the place of unites or ones, and every counter ſet in that line, betokeneth but one: and the ſecond line is the place of 10, for every counter there ſtandeth for 10: the third line the place of hundreds, the fourth of thouſands, and ſo forth.

Scholar. Sir, I doe perceive that the ſame order is here of lines, as was in the other figures by places, ſo that you ſhall not need longer to ſtand about Numeration, except there be any other difference.

Maſter. If you do underſtand it, then how will you ſet 1543?

Scholar. Thus as I ſuppoſe.

Maſter. You have ſet the places truly, but your figures be not meet for this uſe: for the meeteſt figures in this behalfe, is the figure of a counter round, as you ſee here, where I have expreſſed that ſame ſumme.

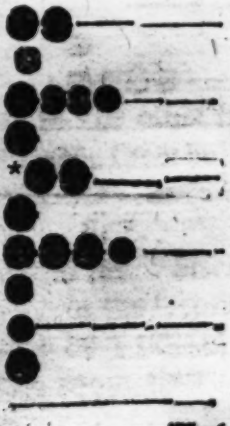


Scholar. So that you have not one figure

for 2, nor 3, nor 4, and so forth, but as many digits as you have, so many counters you set in the lowest line, and for every 10 you set one in the second line, and so of other. But I know not by what reason you set that one Counter for 500 between two lines.

Master. You shall remember this, that whensoever you need to set down 5, 50, or 500, or 5000, or set forth any number whose numerator is 5, you shall set one counter for it in the next place above the line that it hath his denomination of: As in this example of that 500, because the numerator is 5, it must be set in a void space, and because the denomination is a hundred, I know that the place is the void place next above hundreds, that is to say, above the third line.

And further you shall marke, that in all working by this sort, if you shall set downe any summe between 4 and 10, * ——— for the first part of that number you shall set down 5, and then so many counters more, as there rest numbers above 5. And this is true both of digits and articles. And for example, I will set down this summe 297965, which summe if you marke well, you need none other examples for to learne the numeration of this forme.



What this shall you marke, that as you do for other kindes of Arithmetick, set a prick in the places of thousandes; so this work you shall set a Scarre, as you see before.

Scholar. Then I perceive Numeration what, I pray you, doth shall I do in this Art? to adde two summes as many together?

Addition.

Addition.

Maſter.

THe eaſieſt way in this is to adde but two ſummes at once together: Howbeit you may adde more, as I will tell you anon.
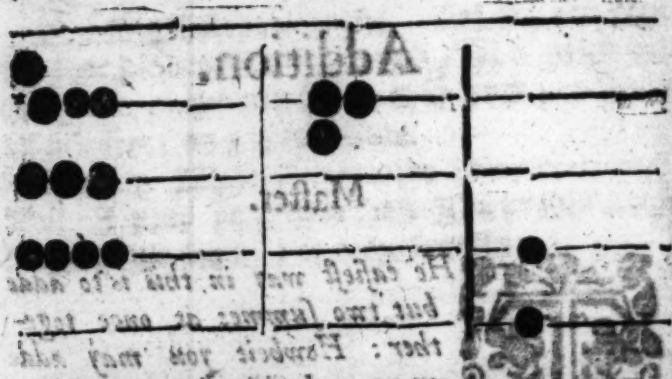
Therefore when you will adde two ſummes, you ſhall firſt ſet downe one of them, it forceth not which, and then by it draw a line croſſe the other lines. And afterward ſet downe the other ſumme, ſo that the line may be between them: as if you would adde 2659 to 8342, you ſet your ſummes as you ſee here.



Addition of two ſummes.

And then if you liſt you may then adde the one to the other in the ſame place: or elſe you may adde them both together in a new place: which way, becauſe it is moſt plaine, I will ſhew you firſt.

Therefore will I beginne at the Vnites, which in the firſt ſum is but 2, and in the ſecond ſum 9, that maketh 11: Thoſe do I take up, and for them I ſet 11 in the new room, thus:



Then doe I take up all the Articles under a hundred, which in the first fumme are 40, and in the second fumme 50, that maketh 90: or you may say better, that in the first fumme there are foure Articles of 10, and the second fumme 5, which maketh 9, but then take heed that you set them in their right lines, see here.



Where I have taken away 40 from the first fumme, and 50, from the second, and in their stead I have set 90 in the third room, which I have set plainly, that you might well

well perceive: howbeit seeing that 90 with the 10 that was in the third room, already, doth make 100. I might better for those 6 Counters, set 1 in the third line, thus:

For it is all in one summe, as you may see, but it is best never to set above counters in any line, for that may bee done with one counter in a higher place.

Scholar. I judge that good reason, for many are unneedfull where one will serve.

Master. Well, then will I adde forth of hundreds: I finde 3 in the first summe, and 6 in the second, which maketh 6000, them doe I take up, and set in the third room, where is 100 already, to which I put 900, and it will be 1000: therefore I set one counter in the fourth line for them all, as you see here.



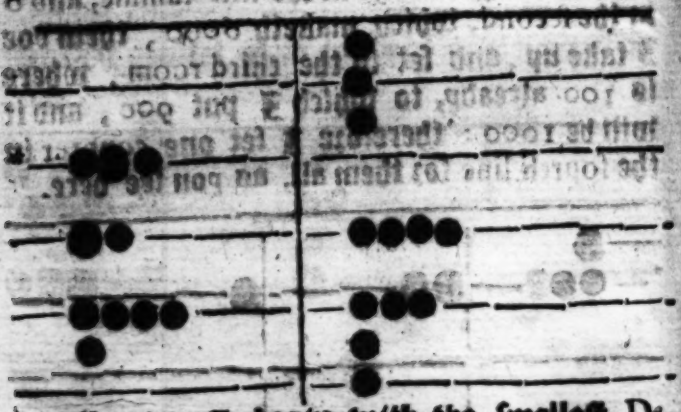
Then adde I the thousands together, which in the first summe are 8000, and in the

the second 2000, that maketh 10000, them doe I take up for these two places, and for them I set one counter in the fifth line, and then it appeareth as you see to bee 11001, for so many doth amount of the Addition of 8342 to 2659.

To adde summes together.

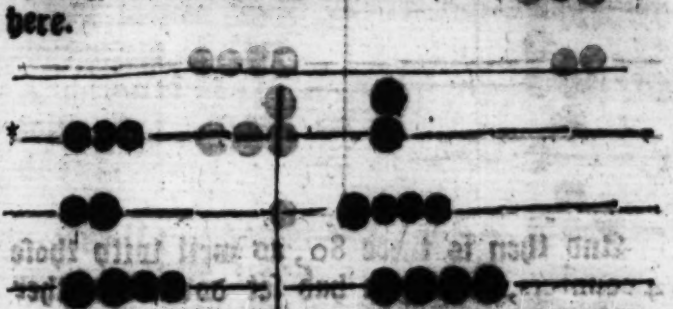
Scholar. Sir, this I do perceive: but how shall I set one summe to another, not changing them to a third place.

Master. Marke well how I doe it. I will adde together 65426 and 3245, which first I set down thus:

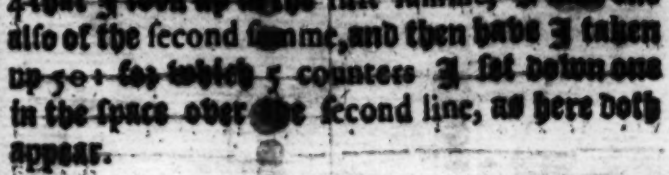


Then doe I begin with the smallest Denomination, which is 1 in the second summe, and set it in his place: then doe I finde 5 in the first summe, and 5 in the second, which put together, saving the two Counters, cannot be

be set in a void place of 5, but for them both I must set one in the second line, which is the place of 10, therefore I take up the five of the first summe, and the 5 of the second, and for them I set one in the second line, as you see here.



Then do I likewise take the 4 Counters of the first summe and second line, (which maketh 40) and adde them to the 4 counters of the same line in the second summe, and it maketh 80: but as I said, I may not conveniently set above 4 counters in one line, therefore to those 4 that I took up in the first summe, I take one also of the second summe, and then have I taken up 50: for which 5 counters I set down one in the space over the second line, as here doth appear.

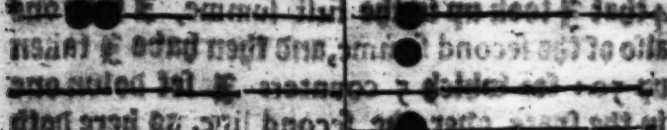


And



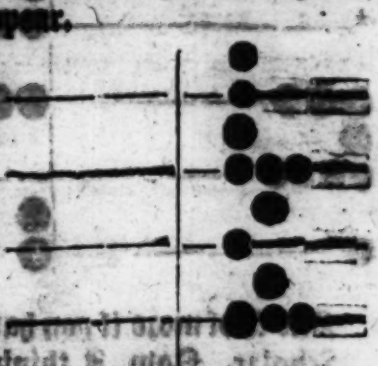
And then is there 80, as well with thoſe 4 counters, as if you had ſet down the other 4 alſo.

Now do I take the 200 in the firſt ſumme, and adde them to the 400 in the ſecond ſumme, and it maketh 600, therefore I take up the two counters in the firſt ſumme, and thoſe at them in the ſecond ſumme, and for them I ſet 1 in the ſpace above, thus



Then take I the 3000 in the firſt ſumme, unto
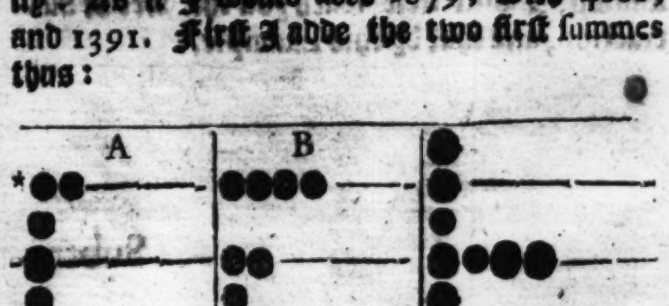
unto which there are none in the ſecond ſumme agreeing, therefore I doe onely remove thoſe three counters from the firſt ſumme into the ſecond, as here doth appear.



And you ſee the whole ſumme that amounteth of that Addition of 65436 with 3245, to bee 68681.

And if you have marked thoſe two examples well you need no further inſtruction in Addition of two onely ſummes: but if you have more then two ſummes to adde, you may adde them thus:

Firſt adde two of them, and then adde the third and fourth, or more, if there bee ſo many: As if I would adde 2679, with 4286, and 1391. Firſt I adde the two firſt ſummes thus:



And

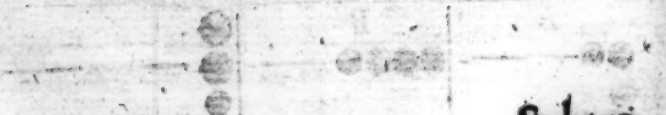
And then I adde the third thereto that



And so of more if you have them.
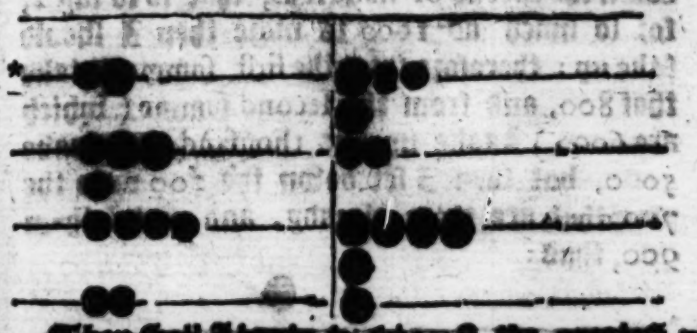
Scholar. Now I think it best that you passe forth to Subtraction except there be any way to examine this manner of Addition, then I thinke that were good to be known next.

Master. Where is the same proof here that is in the other Addition by the Pen, I meane Subtraction; for that only is a sure way, but considering that Subtraction, must be first known, I will first teach you the Art of Subtraction, and that by this Example,



Subtraction.

I would ſubtract 2892 out of 8746. Theſe ſummes muſt I ſet downe as I did in Addition: but here it is beſt to ſet the leſſer number firſt, thus:

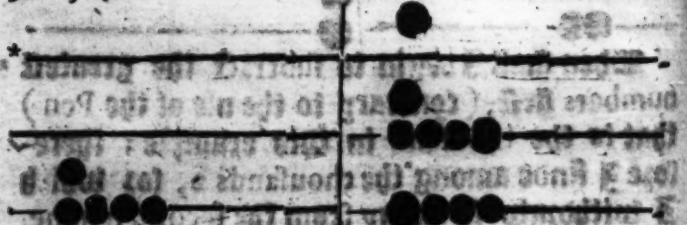


Then ſhall I begin to ſubtract the greateſt numbers firſt, (contrary to the uſe of the Pen) that is the thouſand in this example: therefore I finde among the thouſands 2, for which I withdraw to many from the ſecond ſumme, (where are 8) and ſo remaineth there 6, as this example ſheweth.



Then

Then doe I likewise with the hundreds, of which in the first summe I finde 8, and in the second summe but 7, out of which I cannot take 9, therefore this must I doe: I must look how much my summe differeth from 10, which I finde here to be 2, then must I abate for my summe of 800, one thousand, and set down the excesse of hundreds, that is to say 2, for so much as 1000 is more then I should take up: therefore from the first summe I take that 800, and from the second summe (which are 6000) I take up one thousand, and leave 5000, but then I set down the 200 unto the 700 that are there already, and make them 900, thus:



Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 cannot be abated from 40, I looke how much that 90 doth differ from the next summe above it, that is, 100 (or else which is all to one effect) I looke how much doth differ from 10, and I finde it to be 1: then in the stead of that 90, I doe take from the se-

cond

cond ſumme 100: but conſidering that is 10 too much, I ſet down 1 in the next line beneath for it, as you ſee here.



Saving that here I have ſet 1 Counter in the ſpace in ſtead of 5 in the next line.

And thus I have ſubtracted all ſave 2, which I muſt abate from 6 in the ſecond ſum, and there will remain 4. thus.

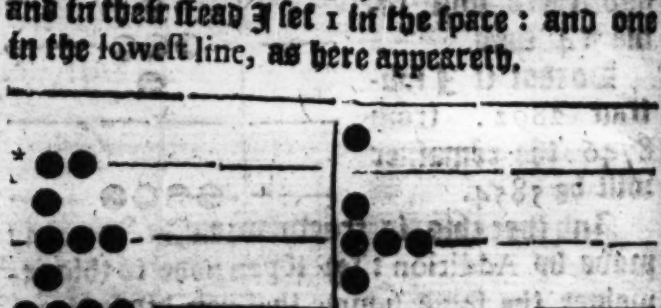
So that if I ſubtract 2892. from 8746 the remainer will be 5854.

And that this is truely wrought, you may prove by Addition: for if you adde to this remainer the ſame ſumme that you did ſubtract, then will the former ſumme 8746 amount again.

Scholar. That will I prove, and firſt I ſet the ſumme that was ſubtracted, which was 2892, and then the remainer 5854. thus: A proofe of Subtraction.

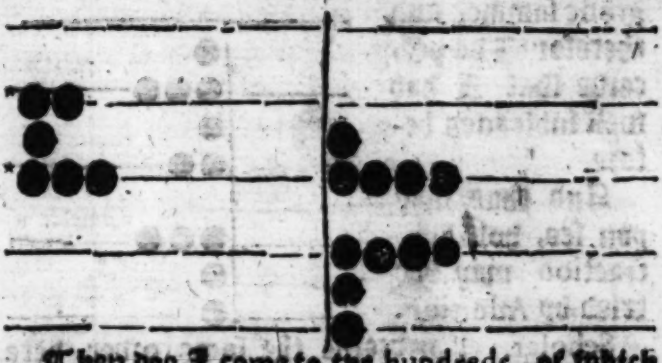


Then doe I adde the first 2 to 4, which maketh 6.: so take I up 5 of those Counters, and in their stead I set 1 in the space: and one in the lowest line, as here appeareth.

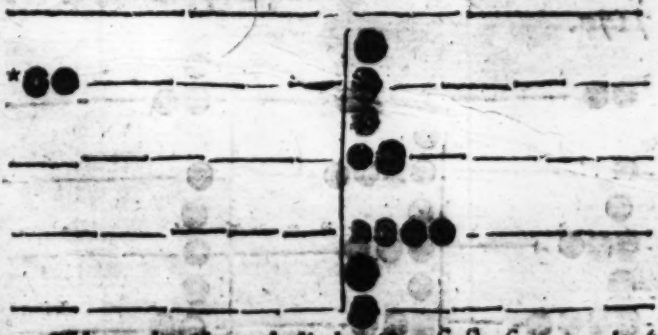


Then do I adde the 90 next above to the 50 and it maketh 140. Therefore I take up those 6 Counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus:

Then

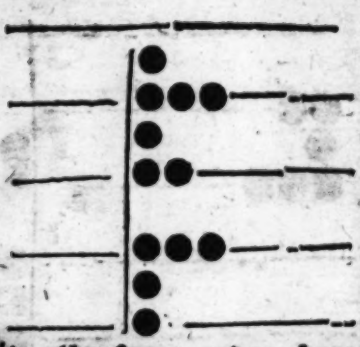


Then doe I come to the hundreds, of which I finde 8 in the firſt ſumme, and 8 in the ſecond, that maketh 1600, therefore I take up thoſe 8 counters, and in their ſtead I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and in the third line, as you may ſee here.



Then is there left in the firſt ſumme but onely 200, and in the ſecond 5000, which is 7000, which I ſhall take up from thence, and ſet in the ſame line in the ſecond ſumme to the one that is there already: and there will the whole ſumme appeare as you may well ſee, to bee 8746, which was the firſt

groſſe ſumme, and therefore I do perceive that I had well ſubtracted before.



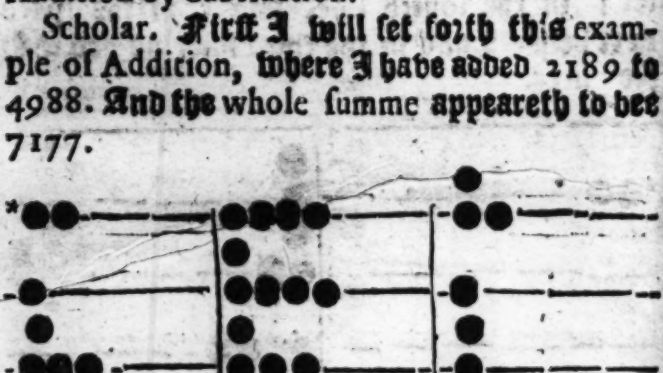
And thus may you ſee, how Subtraction may bee tried by Addition.

Scholar. I perceive the ſame order here with Counters, that I learned before in figures.

Maſter. Then let mee ſee how you can trie Addition by Subtraction.
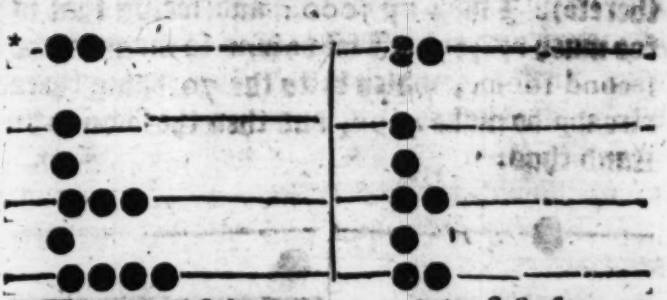
Scholar. Firſt I will ſet forth this example of Addition, where I have added 2189 to 4988. And the whole ſumme appeareth to bee 7177.

Proof of Addition by Subtraction.



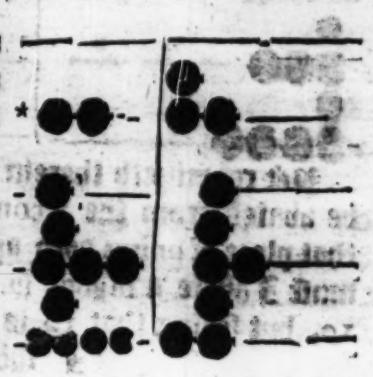
Now to try whether that ſumme be well added or no, I will ſubtract one of the firſt two ſummes from the third. And if I have well done, the remainer will be like that other ſum: as for example, I will ſubtract the firſt ſumme from the third, which I ſet thus in order.

Then

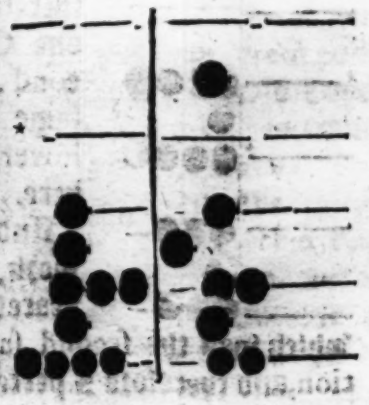


Then do I ſubtract 2000 of the firſt ſumme, from the ſecond ſumme, and then remaineth there 5000, thus:
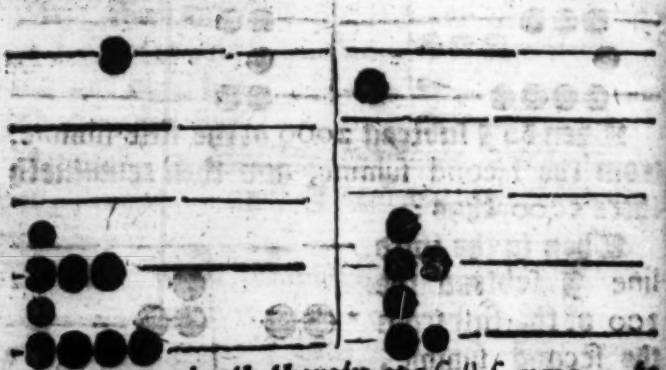
Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is onely 100. alſo: and then in the third line, reſteth nothing, as you may ſee in this example following.



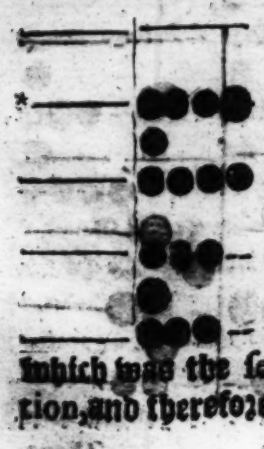
Then in the ſecond line with his ſpace over him I finde 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but onely 70, I muſt take it out of ſome higher ſumme, which is here onely 5000:



therefore I take up 5000: and seeing that is too much by 4920, I set downe so many in the second roome, which with the 70 being there already do make 4990, and then the summes do stand thus.



Yet remaineth therein the first summe 9, to be abated from the second summe, wherein that place of unites doth appear onely 7: then must I abate a higher summe, that is to say, 10, but seeing that 10 is more then 9 (which I should abate) by 1, therefore shall I take up one Counter from the second, and set downe the same in the first line, or lowermost line, as you see here.

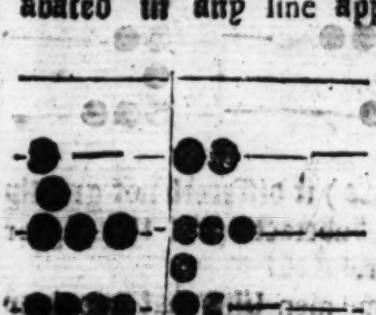


And so have I ended this worke, and the summe appeareth to be the same which was the second summe of mine Addition, and therefore I perceive I have wel done.

Master.

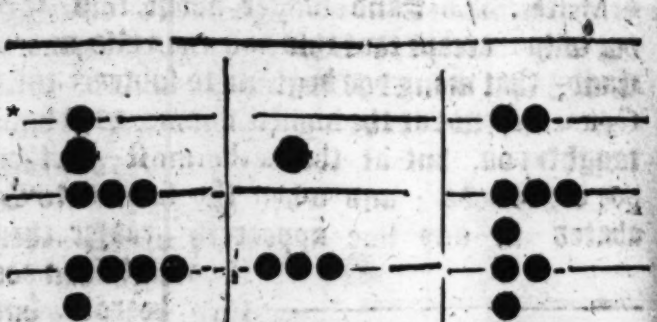
Another way of Addition.

Maſter. To ſtand longer about this, it is but folly: except that this you may alſo underſtand, that many doe beginne to ſubtract with Counters, not at the higheſt ſumme, as I have taught you, but at the nethermoſt, as they doe uſe to adde; and when the ſumme to be abated in any line appeareth greater then the other, then doe they borrow out of the next higher roome, as for example.



I ſhould abate 1846 from 2378, they ſet the ſums thus:

Firſt they take 6 which is the lower line, and his ſpace from 8 in the ſame roomes in the ſecond ſumme, and yet there remaineth two Counters in the loweſt line. Then in the ſecond line muſt 4 be ſubtracted from 7, and ſo remaineth there 3. Then 800 in the third line, and his ſpace, from 300 of the ſecond ſumme cannot be, therefore doe they abate it from a higher roome, that is, from 1000, and becauſe 1000 is too much by 200 therefore muſt I ſet downe 200 in the third line, after I have taken up 1000 from the fourth line. Then is there yet 1000 in the fourth line of the firſt ſumme, which if I withdraw from the ſecond ſumme, then doe all Figures ſtand in order thus: 532.



So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both, may use which you list.

Multi-

Multiplication.

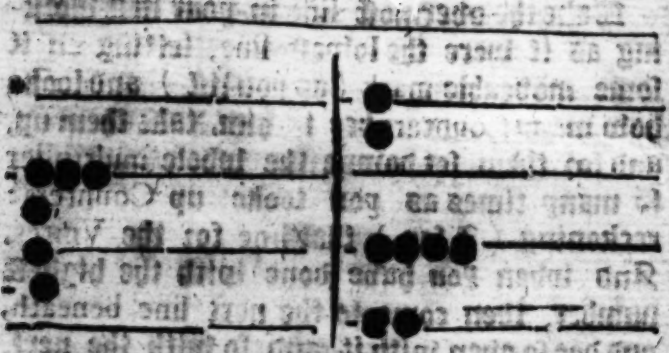
BVt now touching Multiplication: you ſhall ſet your numbers *into two* roomes (*as you did in thoſe other kindes*) *but ſo that the multiplier bee ſet in the firſt* roome: *then ſhall you begin with the higheſt numbers of the ſecond* roome, *and multiply them firſt after this ſort.*

Take the ouermoſt line in your firſt worke ing as it were the loweſt line, ſetting on it ſome moueable mark (as you liſt) and looke how many Counters bee in him, take them up, and for them ſet downe the whole multiplier ſo many times as you tooke up Counters: reckoning (I ſay) that line for the Vnites. And when you haue done with the higheſt number, then come to the next line beneath, and doe ſo euen with it, and ſo with the next, till you haue done all. And if there bee any number in a ſpace, then for it ſhall you take the multiplier five times, and then muſt you reckon that line for the Vnites, which is next beneath that ſpace. Or elſe after a ſhorter way, ye ſhall take onely halfe the multiplier, but then ſhall you take the line next above the ſpace for the line of Vnites. But in each working, if by chance your multiplier bee an odde number, ſo that you

you cannot take the halfe of it iuſtly, then muſt you take the greater halfe, and ſet down that, as if that it were the iuſt halfe: and further, you ſhall ſet one Counter in the ſpace betweene that line, which you reckon for the line of unites, or elſe one ly remoue forward the ſame that is to be multiplied.

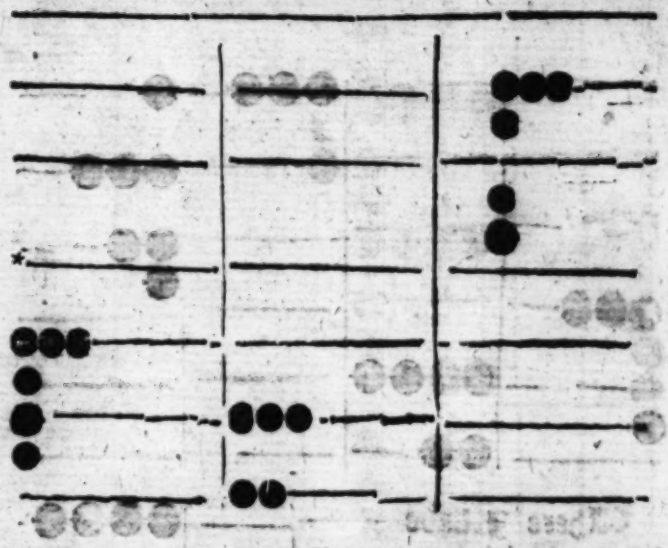
Scholar. If you ſet forth an example hereof, I think I ſhall perceiue you.

Maſter. Take this example: I would multiply 1542 by 365. therefore I ſet my numbers thus.



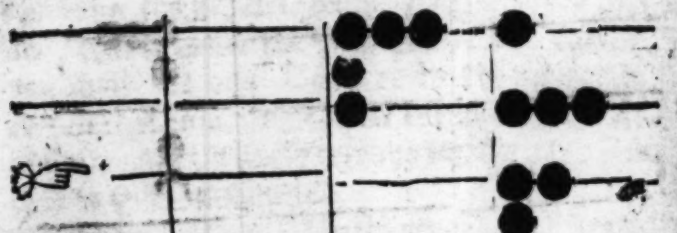
Then firſt I begin at the 1000 in the higheſt roome, as if it were the firſt place, and I take it up ſetting down for it ſo often (that is once) the Multiplier, which is 365, thus as you ſee here: where, for the one Counter taken vp from the fourth line, I haue ſet down other fiue which make the ſumme of the Multiplier, reckoning the fourth line, as if it were the firſt, which thing I haue marked by the Starre ſet at the beginning.

Scholar.



Scholar. I perceive well, for indeed this ſumme that you ſet downe, is 365000: for ſo much doth amount of 1000, multiplied by 365.

Maſter. Well then goe forth, in the next ſpace I finde one Counter, which I remove forward, but take it not up, but (as in ſuch a caſe I muſt) ſet downe the greater halfe of my Multiplier (ſeeing it is an odde number) which is 182, and here do I ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſee.



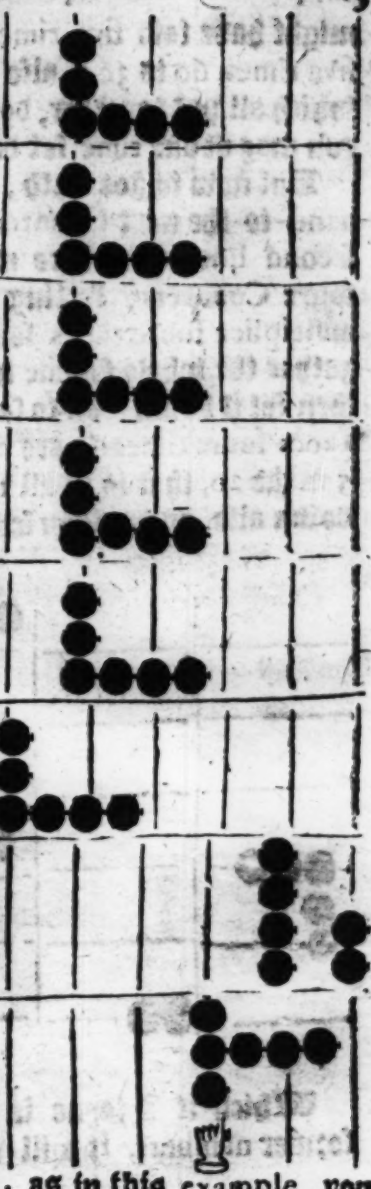
Where I have set the Multiplication with other, but for the ease of your understanding, I have set a little line between them. Now should they both in one summe stand thus.



How

Another forme of Multiplication.

Howbeit another forme to multiply ſuch Counters in ſpace, is this: firſt to remove the finger to the next line beneath the ſpace, and then to take up the Counter, and to ſet down the Multiplier five times, as here you ſee.

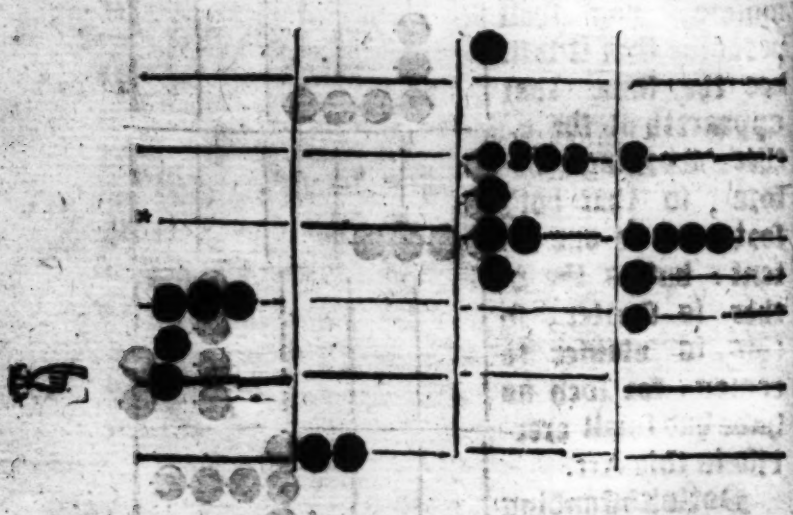


Which ſummes, if you doe adde together, into one ſumme, you ſhall perceive that it will bee the ſame that appeareth of the other working before, ſo that both ſorts are to one intent: but as the other is ſhorter, ſo this is plainer to reaſon, for ſuch as have had ſmall exerciſe in this Art.

Notwithſtanding you may adde them in your mind before you ſet them down: as in this example you might

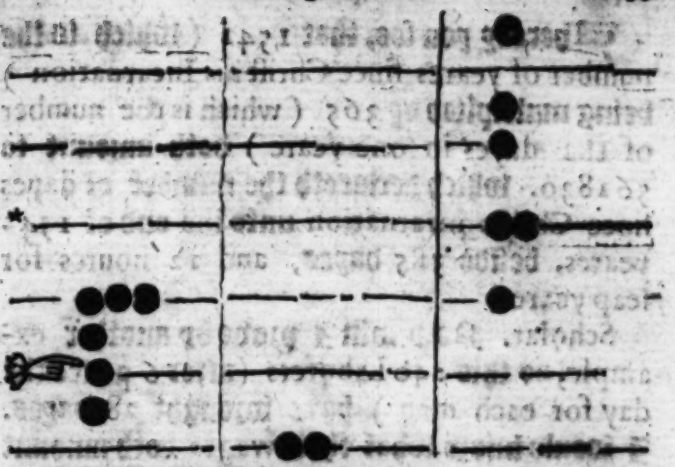
might haue ſaid five times 300 is 1500, and five times 60 is 300, alſo five times five is 25. which all put together, doe make 1825, which you may at one time ſet downe if you liſt.

But now to goe forth, I muſt remoue the hand to the next Counters which are in the ſecond line, and there muſt I take vp thoſe foure Counters, ſetting downe for them my multiplier foure times ſeuerally, or elſe I may gather the whole ſumme in my mind firſt; and then ſet it down: as to ſay, foure times 300 is 1200: foure times 60 are 240: and foure times 5 make 20, that is in all 1460: that ſhall I ſet down alſo, as here you ſee.

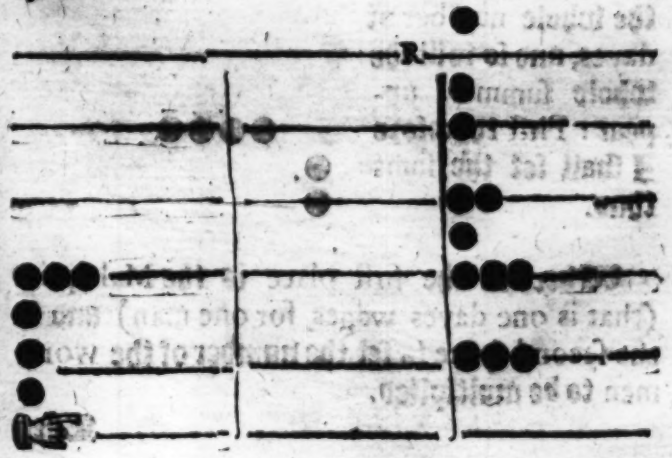


Which if I ioyne in one ſumme with the former numbers, it will appear thus.

Then



Then to end this Multiplication, I remove the finger to the lowest line where are onely 2, then doe I take up, and in their stead doe I set downe twice 365, that is 730, for which I set one in the space above the third line for 500, and 2 more in the third line with that one that is there already, and the rest in their order, and so have I well ended the whole summe thus:



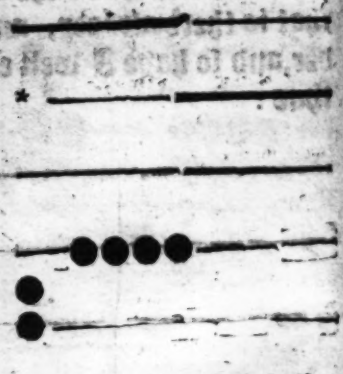
Whereby you ſee, that 1542 (which is the number of yeares ſince Chriſt his Incarnation) being multiplied by 365 (which is the number of the dayes in one yeare) doth amount to 562830. which declareth the number of dayes ſince Chriſts incarnation unto the end of 1542 yeares, beſide 385 dayes, and 12 houres for leap yeares.

Example of wages.

Scholar. Now will I proue by another example, as this: 40 Laborers (after 6 pence the day for each man) haue wrought 28 dayes. I would know what their wages doth amount unto.

In this caſe muſt I worke doubly: firſt I muſt multiply the number of the Laborers, by the wages of a man for one day, ſo will the charge of every day amount.

When ſecondly, ſhall I multiply the charge of one day by the whole number of dayes, and ſo will the whole ſumme appear: Firſt therefore I ſhall ſet the ſums thus.



Where in the firſt place is the Multiplier (that is one dayes wages for one man) and in the ſecond ſpace is ſet the number of the workmen to be multiplied.

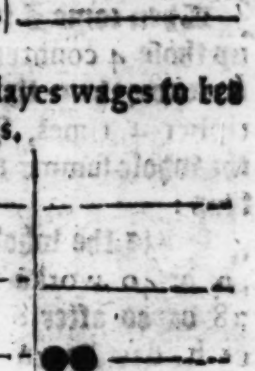
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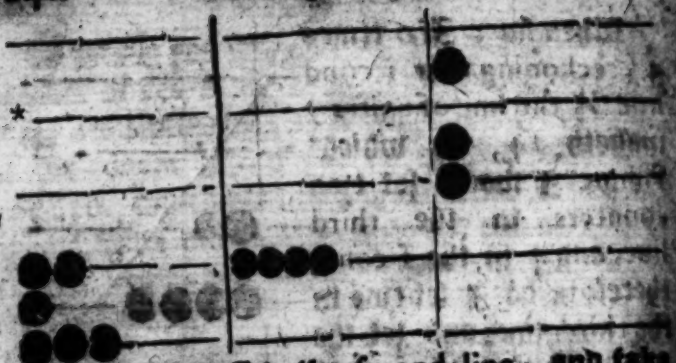
Then ſay: If 6 times 4 (reckoning that ſecond line as the line of unites) maketh 24, for which* ſumme I ſhould ſet two counters in the third line, and 4 in the ſecond, therefore do I ſet two in the third line, and let the 4 ſtand ſtill in the ſecond line thus.
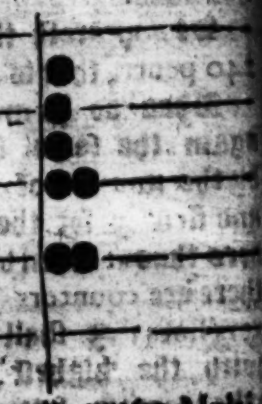


So appeareth the whole dayes wages to be 240 pence, that is 20 ſhillings.

Then do I multiply again the ſame ſumme by the number of dayes, and firſt I ſet the numbers thus: Then becauſe there are counters in divers lines, I ſhall begin with the higheſt, and take them up, ſetting for them the Multiplier ſo many times as I took up counters, that is ewice, then will the ſumme ſtand thus.





Then come I to the ſecond line, and take up thoſe 4 counters, ſetting for them the Multiplier 4 times, ſo will the whole ſumme appear thus:



So is the whole wages of 40 workmen for 28 dayes after 6 pence each day for a man, 6720 pence, that is, 560 ſhillings, or 28 pound.

Maſter. Now if you would proue Multiplication, the ſureſt way is by Diuiſion: therefore will I ouerpaſſe it till I haue taught you the Art of Diuiſion, which you ſhall worke thus:

Diuiſion

Division.

FIrſt, ſet down the Diviſor, for feare of forgetting, and then ſet that number that ſhall be divided at the right ſide, ſo farre from the Diviſor, that the quotient may be ſet between them: as for example.

An example of ſheep.

If 225 ſheep coſt 45 pound, what did every ſheep coſt? To know this, I would divide the whole ſumme, that is 45 pound, by 225, but that cannot be: therefore muſt I firſt reduce that 45 pound, into a leſſer Denomination, as into ſhillings, then I multiply 45 by 20, and it is 900; that ſumme ſhall I divide by the number of ſheep, which is 225, theſe two numbers therefore I ſet thus:



When begin I at the higheſt line of the dividend, and ſeek how oft I may have the diviſor therein, and that I may do four times:

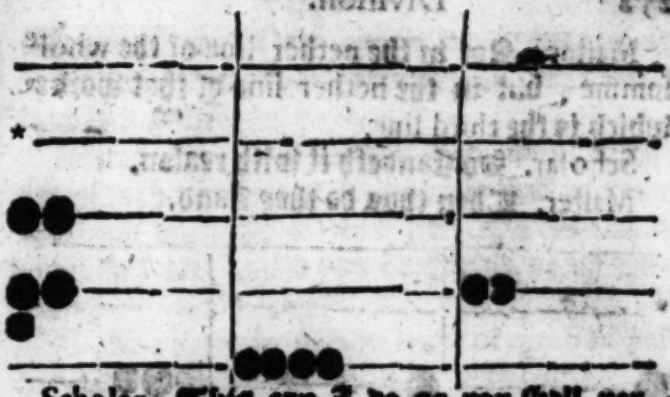
then ſay I, four times 2 are 8, which if I take from 9, there reſteth but 1, thus:



And becauſe I found the Diviſor 4 times in the dividend, I have ſet as you ſee, 4 in the middle room, which is the place of the quotient: but now muſt I take the reſt of the diviſor as often out of the remainder, therefore come I to the ſecond line of the diviſor, ſaying two times 4 make 8, take 8 from 10, and there remaineth 2, thus:

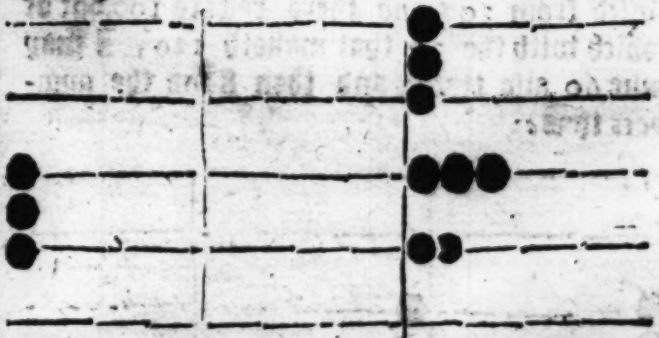
Then come I to the loweſt number, which is 5, and multiply it 4 times, ſo is it 20, that take I from 20, and there remaineth nothing, ſo that I ſee my quotient to be 4, which are in value ſhillings, for ſo was the dividend: and thereby I know that if 225 ſheep coſt 45 pound, every ſheep coſt 4 ſhillings.

Scholar.



Scholar. This can I do as you ſhall perceiue by the example. If 160 Souldiers do ſpend euery moneth 68 pound, what ſpendeth each man? Example of ſouldiers wages.

Firſt, becauſe I cannot diuide the 68 by 160, therefore I will turn the pounds into pence by Multiplication, ſo ſhall there be 16320 pence: now muſt I diuide the ſumme by the number of Souldiers, therefore I ſet them in order thus:



Then begin I at the higheſt place of the dividend, ſeeking my Diviſion there, which I finde once, therefore I ſet 1 in the nether line.

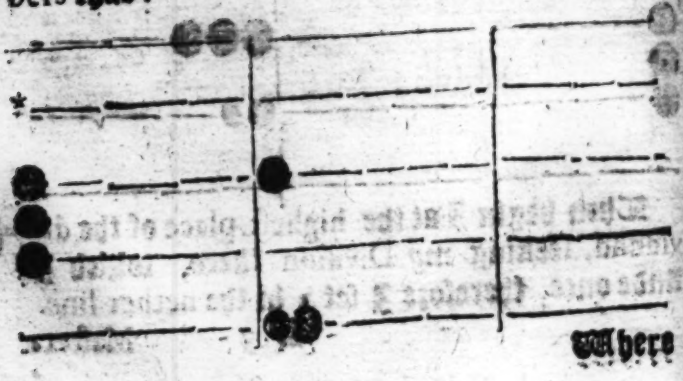
Master. Not in the nether line of the whole summe, but in the nether line of that worke, which is the third line.

Scholar. So standeth it with reason.

Master. Then thus do they stand.



Then seeke I againe the rest, how often I may finde my diviſor: and I ſee that in 300 I might finde 100 three times: but then the 60 will not be ſo often found in 20, therefore I take 2 for my quotient: then take I 100 twice from 300 and there reſteth 100, but of which with the 20 that maketh 120, I may take 60 alſo twice, and then ſtand the numbers thus:



Where

Where I have ſet the quotient 2 in the loweſt line: ſo is every Souldiers portion 102 pence that is 8 ſhillings 6 pence.
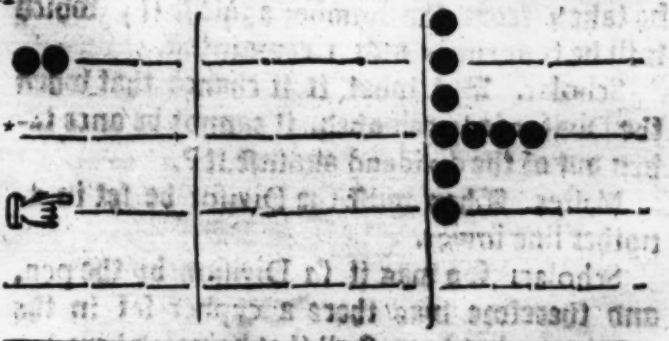
Maſter. But yet becauſe you may ſubtilly perceive the reaſon of diviſion, it ſhall be good that you ſet your diviſor ſtill againſt thoſe numbers from which you do take it, as by this example I will declare.

If the purchaſe of 200 acres of ground did coſt 260 pound, what did one acre coſt?

An example of purchaſe.

Firſt, will I turne the pounds into pence, ſo will there be 69600 pence. Then in ſetting downe theſe numbers, I ſhall do thus:

Firſt ſet the dividend on the right hand as it ought, and then the diviſor on the left hand againſt thoſe Numbers from which I intend to take him firſt, as here you ſee, where I have ſet the diviſor two lines higher then his own place.

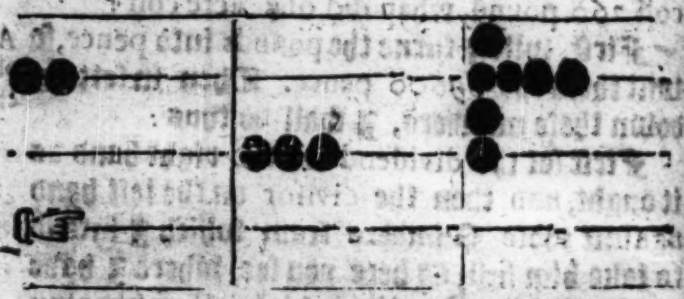


Scholar. This is like the order of Diviſion by the Pen.

Maſter. Truth you ſay, and now muſt I

set the quotient of this work in the third line, for that is the line of unites in respect of the Divisor in this work.

Then I see how often the Divisor may be found in the dividend, and that I finde 3 times, then set I 3 in the third line for the quotient, and take away that 60000 from the dividend: and further I set the Divisor one line lower, as you see here.



And then seek I how often the Divisor will be taken from the number against it, which will be four times and 1 remaining.

Scholar. But what if it chance that when the Divisor is so removed, it cannot be once taken out of the dividend against it?

Master. Then must the Divisor be set in another line lower.

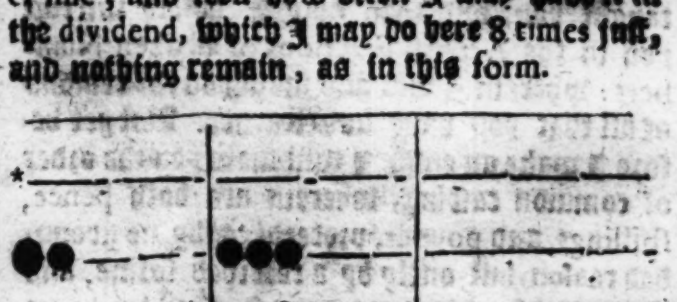
Scholar. So was it in Division by the pen, and therefore was there a cypher set in the quotient: but how shall that be noted here?

Master. Here needeth no token, for the lines do represent the places, onely look that you set your quotient in that place which standeth

deth for unites in reſpect of the Diviſor. But now to return to the example: I finde the diviſor four times in the dividend, and 1 remaining: for 4 times 2 make 8, which I take from 9, and there reſteth 1, as this figure following ſheweth: and in the middle ſpace for the quotient, I ſet 4 in the ſecond line, which is in this work the place of unites.



Then remove I the Diviſor to the next lower line, and ſeek how often I may have it in the dividend, which I may do here 8 times juſt, and nothing remain, as in this form.



Where you may ſee that the whole quotient is 348 pence, that is 29 ſhillings, whereby I know

know that so much cost the purchase of one Acre.

Scholar. Now resteth the proofs of Multiplication, and also Division.

Master. Their best proofs are each one by the other; for Multiplication is proved by Division, and Division by Multiplication, as in the work by the pen you learned.

Scholar. If that be all, you shall not need to repeat again that which was sufficiently taught already: and except you will teach me any other feat, here may you make an end of this Art, I suppose.

The reason of all the former rules.

Master. So will I doe as touching whole number, and as for broken number I will not trouble your wit with it, till you have practised this so well, that you be full perfect, so that you need not to doubt in any point that I have taught you, and then may I boldly instruct you in the Art of Fractions or broken Numbers: wherein I will also shew you the reasons of all that you have now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings, and pounds, proceeding by no grounded reason, but onely by a received forme, and that diversly, of divers men, for the Merchants use one form, and Auditors another.

Merchants

Merchants uſe.



But firſt for Merchants form, mark this example here, in which I have expreſſed this ſumme, 198 pounds 10 ſhillings, 11 pence. So that you may ſee that the loweſt line ſerveth for pence, the next above for ſhillings, the third for pounds, and the fourth for ſcores of pounds.

And further you may ſee that the ſpace betweeh pence and ſhillings, may receive but one counter, (as all other ſpaces likewiſe do) and that one ſtandeth in that place for 6 pence.

Likewiſe between the ſhillings and the pounds one counter ſtands for 10 ſhillings.

And between the pounds and 20 pounds, one counter ſtandeth for 10 pounds.

But beſide thoſe, you may ſee at the left hoe of ſhillings, that one number ſtandeth alone and betokeneth 5 ſhillings.

So againſt the pounds, that one counter ſtandeth for 5 pound. And againſt the 20 pounds, the one counter ſtandeth for five ſcore pounds, that is, 100 pounds: ſo that every hoe counter is five times ſo much as one of them againſt which he ſtandeth.

Auditors

Auditors Accompt.

Auditors Accompt.

NOw for the Accompt of Auditors, take this example.



Where I have expressed the same summe, 198 pound——19 shillings——11 pence.

But here you see the pence stand towards the right hand, and the other increasing orderly towards the left hand.

Again you may see, that Auditors will make two lines (yea and more) for pence, shillings, and all other values, if their summes extend thereto. Also you see that they set one Counter at the right end of each row, which so set there, standeth for five of that room, and on the left corner of the row, it standeth for 10 of the same row.

But now if you would adde, or subtract after any of both these sorts, if you mark the order of the other seat which I taught you, you may easily do the same here without much teaching: for in Addition you must first set down one summe, and to the same set the other orderly, and in like manner, if you have many; but in Subtraction, you must set down first the

the greatest summe, and from it must you abate the other, every Denomination from his due place.

Scholar. I do not doubt but with a little practise I shall attain these both: but how shall I multiply and divide after these forms?

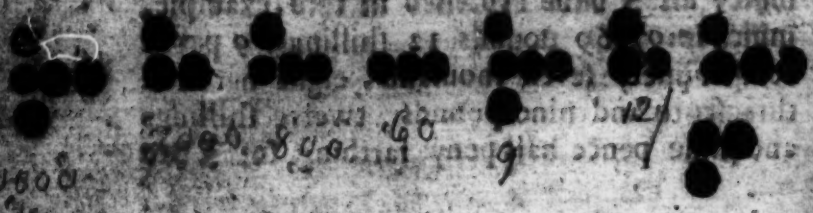
Master. You cannot duely do any of both by these sorts, therefore in such case you must resort to your other Arts.

Scholar. They that use such Accounts that it exceed 200 in the summe, they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in another farther row, and 500 at the left hand thereof, and the thousands they set in a farther row yet, and at the left side thereof they set the 5000, and in the space over they set the 10000, and in a higher row 20000, which all I have expressed in this example, which is 97869 pounds 12 shillings, 9 pence ob. q. Ninety seven thousand, eight hundred threescore and nine pounds, twelve shillings and nine pence half peny farthing, for I had

not

not told you before, where neither how you should set betwixt farthings, which (as you see here) must be set in a double place, standing beneath the penes, for 2 farthing, one counter ob. 3 counters, for ob, farthing 3 counters, and more there cannot be: for 4 farthings make 1 peny, which must be set in his due place.

And if you desire the same summe after Auditors manner, so here it is.



But in this thing you shall take this for sufficient, and the rest you shall observe as you may see by the working of each sort, for the divers wits of men have invented divers and sundry wayes, almost innumerable.

THE

THE SECOND PART OF ARITHMETICK, touching Fractions, briefly set forth.

Scholar.

ALheit I perceive your manifold businesses doth so occupie, or rather oppresse you, that you cannot as yet completly end the Treatise of Fractions Arithmeticall which you have prepared, wherein not onely sundry works of Geometry, Musick, and Astronomy be largely set forth, but also divers conclusions and naturall

Arithmeticall fractions.

naturall works, touching mixtures of Metals, and compositions of Medicines, with other ſtrange examples. Yet in the mean ſeaſon, I cannot ſtay my moſt earneſt deſire, but importunely crave of you ſome brief preparation toward the uſe of Fractions, whereby at the leaſt I may be able perfectly to underſtand the common works of them, and the vulgar uſe of thoſe rules, which without them cannot well be wrought.

Maſter. If my leaſure were as great as my will is good, you ſhould not need to uſe any importunate craving, for the attaining of that thing, whereby I may be perſwaded that I ſhall any way profit the Common-wealth, or help the honeſt ſtudies of any good Members in the ſame: wherefore while mine attendance will permit me to walk and talk, I am well willing to help you as I may.

Therefore, firſt to begin with the explication of this name Fraction, what take you it to be?

What a Fraction is

Scholar. Marry ſir, I think a Fraction (as I have heard it often named) to be a broken number, that is to ſay, to be no whole number, but part of a number.

Maſter. A Fraction indeed is a broken number, and ſo conſequently the part of another number, but that muſt be underſtood of ſuch another number as cannot be divided into any other parts then Fractions: for although I may take the third part of 60, or the fourth

part

part of it, and so of other parts diverslie, yet those parts be not properly, nor ought to bee called Fractions, because they may be expressed by whole numbers, for the third part of it is 20, the fourth part is 15. the twelfth part is 5, and so forth of other parts, all which bee whole numbers.

Wherefore properly a Fraction expresseth the parts or part onely of a unite, that is to say, that the number which is the whole or entire summe of any Fraction, may not bee greater then one: and therefore it followeth, that no one Fraction alone can be so great, that it shall make 1, as by example I will declare, as soon as I have taught you to know the forme how a Fraction is expressed or represented in writing. What a Fraction is properly.

Numeration.

The expressing of fractions.

But first to begin with expressing of a fraction, which is the Numeration of it: you must understand that a fraction is represented by two numbers set one over the other, and a line drawn between them, as thus, $\frac{1}{3}\ \frac{1}{4}\ \frac{2}{5}\ \frac{12}{17}$, *which four fractions you must prononnce thus,* $\frac{1}{3}$ *one third part,* $\frac{1}{4}$ *three quarters,* $\frac{2}{5}$ *two fifth parts,* $\frac{12}{17}$ *ten seventeen parts.*

Scholar. I understand this forme of their expression and pronunciation, but their meaning or valuation seemeth more obscure. Yet I think that by the two first Fractions: I understand the valuation of the two latter Fractions, and consequently of other.

Master. Name them then, that I may perceive your taking of them.

Scholar. $\frac{2}{5}$ betokeneth two fift parts, that is to say, if one be divided into 5 parts, that Fraction doth expresse two of those 5 parts: $\frac{12}{17}$ doth signifie, that if one be divided into 17 parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$, that is, one third part, doth easily declare, that if one thing bee divided into three parts, I must take out one of them: so $\frac{1}{4}$, that is three quarters, doth declare that one being divided into foure quarters, I must take (for this Fraction) three of

of those quarters.

If there be no more difficulty in the Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kindes of workes. For I understand that the same kindes of workes be in fractions, that bee in whole numbers.

Master. There are the same kindes of works in both, albeit the order of them is divers, as I will anone declare: but yet more in Numeration before we leave it. You must understand, that those two numbers which expresse a fraction, have severall names, the overmost, which is above the line, is called the Numerator, and the other beneath the line, is called the Denominator. Numerator. Denominator.

Scholar. And what is the reason of their divers names? For (in mine opinion) both bee Numerators, seeing both they doe expresse the numeration of the fraction.

Master. You are deceived; for one onely (which is the overmost) doth expresse the Numeration, and the Denominator doth declare the number of parts, into which the unite is divided, as in this example; when I say: divide a pound weight of gold between foure men, so that the first man shall have $\frac{3}{15}$, the second $\frac{4}{15}$, the third $\frac{2}{15}$, and the fourth $\frac{6}{15}$.

Now doe you perceive that by the Denominator (which is one in all foure fractions) it is intended that the pound weight should bee divided into so many parts, I meane 15,

and by the 4 ſeveral Numerators, is limited the divers portion that each man ſhould have, that is, that when the whole is parted into 15, the firſt man ſhall have two of thoſe 15 parts: the ſecond man three of them: the third man foure: and the fourth man ſix. And ſo may you ſee the ſeverall offices (as it were) of thoſe two numbers, I mean of the Numerator and the Denominator.

And hereby you perceive that a man can have no more parts of any thing then it was divided into, neither yet aptly ſo many: ſo that it were unaptly ſaid: You ſhall have $\frac{15}{15}$, that is 15, fifteen parts of any thing, ſeeing it were better ſaid, you ſhall have the whole thing.

Scholar. So doth it appear reaſonably, for the labour is vain to divide any thing, and then to apply the Diviſion to no uſe. And much leſſe reaſonable were it to ſay $\frac{16}{15}$: for if the whole be divided into 15 parts onely it is not poſſible to take 16 of them, that is to ſay, more then all together.

Maſter. This is true touching the proper and apt uſe of the name of a Fraction; yet improperly (and after a vulgar acceptation for eaſineſſe in worke) both thoſe formes bee called Fractions, becauſe they be written like Fractions, although they bee none indeed: for $\frac{15}{15}$ and generally in ſuch other, where the Numerator and Denominator be equall, are not Fractions, but the whole thing with all his parts: And ſo $\frac{12}{12}$ is not to bee called a Fraction,

ction, but a mixt number, of a whole number and a Fraction, for it is as $1\frac{3}{12}$, that is one whole and $\frac{3}{12}$ parts; as shall be declared in Reduction. Therefore they do abuse the names that call them Fractions, where the Numerator is either equal, or greater then the Denominator. An improper fraction of a mixt number.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in worke write some numbers after that sort like Fractions: but they needed not to call them Fractions, but (as they be (whole numbers, or mixt numbers, (that is, whole numbers with Fractions) expressed like Fractions, or as improper Fractions.

Now must you understand, that as no Fraction properly can be greater then one, so in smalnesse under one the nature of Fractions doth extend infinitely, as the nature of whole numbers is to increase above one infinitely, so that not onely one may be divided into infinite Fractions or parts, but also every Fraction may bee divided into infinite Fractions or parts, which commonly be called Fractions of Fractions: and they be expressed diversly: as for examples $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is, three quarters of two third parts of one halfe part. Whereby is signified, that if one bee divided into two halfes, and the one halfe into three parts, and two of those Fractions of fractions.

three parts be divided jointly into foure quarters, this fraction of fractions doth represent three of those quarters.

Scholar. I pray you let me prove by an example in common money, whether I doe rightly understand you or no. One Crowne which I take for an unite, doth contain 60 pence; therefore the halfe of it is 30 pence, $\frac{2}{3}$ of that half is 20 pence, whereof $\frac{3}{4}$ is fifteen pence, so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crowne; and so is 3 pence $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceive this well enough: yet this note I give you by the way, that the forme of expressing the fractions is voluntary, and hath no other reason then the will of the Divisor, which forme many follow: for some expresse them thus, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ without any figure of distinction between them, which forme also may follow. Some other doe make lines betweene every fraction, and adde words of distinction, after this sort, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, which forme is best.

Some other expresse them thus in slope forme, to distinct them from fractions of one whole numbers, for if they were in one right line thus, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ then ought it to bee pronounced, three quarters, and two third parts, and a halfe, which maketh almost two whole unites, lacking but one twelfth part. And so is it nothing agreeable

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{2}$

with

with the other fraction of fractions: wherefore it is a great oversight in certain learned men, which do expresse them so confusedly with such severall fractions, that a man cannot know the one from the other.

Therefore some men (*as* Stifelius) *do expresse without a line, numbers of proportion, being applied to* Addition *or* Subtraction, *because they must be taken as two, where the line in fractions maketh them to be taken for one: for of the* Numerator *and* Denominator *is made one number.*

Scholar. Then I perceive there be three severall varieties in fractions: First, when one onely fraction is set for one number, as $\frac{4}{5}$, that is, foure fifth parts. The second is, when there be set two or more severall fractions of one number, as $\frac{4}{9}$. $\frac{2}{5}$, that is, foure ninth parts, and two fifth parts. The third sort is fractions of fractions, as $\frac{4}{9}$, of $\frac{2}{5}$, that is, foure ninth parts of two fifth parts. Three severall varieties.

Master. You have said well, if you understand well your own words.

Scholar. If it shall please, I will by an example in the parts of an old English Angel, expresse my meaning.

Master. Let me hear you.

Scholar. The old English Angel did contain 7 shillings 6 pence, that is, 90 pence: Now $\frac{4}{5}$ of it is 72 pence: And of the same 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is foure ninth parts, and two fifth parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{2}$ of $\frac{2}{5}$, that

is foure nine parts of two fifth parts, seeing $\frac{2}{3}$ is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16, for $\frac{1}{9}$ of 36 is but 4, and that taken foure times maketh 16.

Master. This is plainly expressed, and truly, and hereby (I doubt not) but you doe perceive, that as great a difference, as is between 16 and 76, so much difference is between those two Fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that ye understand these varieties, I will proceed to the rest of the workes: First, admonishing you, that there is another order to be followed in Fractions, then there was in whole numbers: for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Division, and Reduction: but in Fractions, (to follow the same aptnesse in proceeding from the easiest workes to the harder) wee must use this order of workes, Numeration, Reduction, Addition, Subtraction, Multiplication, and Division.

The order of works in fractions.

Scholar. That Addition and Subtraction should go together, and Division to follow Multiplication, naturall order doth perswade: but why Reduction should be first in order here, next to Numeration, and Addition, and Subtraction in the middle, I desire to understand the reason.

Master. As in the Art of whole numbers, Order would reasonably begin with the easiest, and so goe forward by degrees to the hardest;

hardeſt: even reaſon teacheth in Fraction the like order. And conſider that Addition or Subtraction of Fractions, can very ſeldome bee wrought without Reduction: and contrariwiſe, Reduction may be wrought without this form of Addition or Subtraction: therefore was it orderly required, that Reduction ſhould goe before Addition and Subtraction, and this reaſon ſerveth for the placing of Reduction before the other.

Scholar. Then, if Reduction be the eaſieſt, I pray you declare the forme of it, firſt by rule, and then by example.

Maſter. Your requeſt is good.

Redu-

Reduction of Fractions.

Of Reduction of Fractions, there are five varieties.

THerefore will I now declare the diversities of reduction of fractions, which commonly hath five varieties, or formes.

1 *First, when there bee sundry Fractions of one intire unite, they must bee reduced to one Denomination, and also into one Fraction.*

2 *Secondly, when there be proponed fractions of fractions, they must be reduced likewise into one fraction: for otherwise they cannot bee brought into one Denomination.*

3 *Thirdly, when an improper fraction is proponed, that is to say, a fraction in form, which indeed is greater then an unite: it must be reduced into apt form, expressing the unite or unites of it, and the proper fraction distinctly. And sometimes also it shall be needfull to convert such a mixt number of unites with fractions, into the form of a fraction, that is, into an improper fraction: which two forms I esteem but as one, because they work one kinde of number.*

4 *Fourthly, there happeneth sometimes fractions to be written in great numbers, which might be written in lesser numbers: therefore is there a meane to reduce such great numbers into their smallest termes.*

5 *Fifthly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certain*

certain parts, but none of like Denomination *with that* fraction, *then may you reduce the ſaid* fractions *into another, whoſe* Denomination *ſhall expreſſe the common parts of that whole thing.*

Scholar. This diſtinction in Doctrine delighteth mee much, but more with hope then preſent fruit: for as yet I doe not underſtand ſcarcely the varieties, and much leſſe the practiſe and uſe of their workes.

Maſter. Reduction is an orderly alteration of Numbers out of one forme into another, which is never done orderly but for ſome needfull uſe, as in every of the ſaid five ſeverall formes, I will diſtinctly declare.

First therefore, when two, or more ſeverall fractions of any unite be proponed: as for example $\frac{3}{16}$ and $\frac{4}{63}$; becauſe it is hard to tell what proportion of the intire number thoſe two fractions do expreſſe, therefore was Reduction deviſed to be a mean whereby theſe ſeverall fractions might be brought into one Denomination and fraction. The firſt forme of Reduction

And in theſe fractions, this is the Art for bringing them to one Denomination,

Multiply firſt the Denominators *together, and the totall thereof you ſhall ſet twice down under two ſeverall lines for two new* Denominators, *or rather for one common* Denominator. *Then multiply the* Numerator *of the firſt fraction, by the* Denominator *of the ſecond, and ſet the totall thereof for the* Numerator *over the firſt line. Likewiſe multiply the* Numerator *of the ſecond fraction* How to reduce fractions of divers denominations into one denomination.

by

by the Denominator of the first, and set that totall over the second line for the Numerator of that fraction: and so are these two first Fractions of severall Denominations, brought to one Denomination.

Scholar. If I understand you, as I thinke I doe, my Example shall declare the same. The Fractions, which you proponed were these, $\frac{3}{16}$, and $\frac{4}{6}$, whose Denominators (being 16 and 6) I multiply together, and there amounteth 26, which I set under two lines, thus : $\frac{\quad}{96}$ $\frac{\quad}{96}$

Then I multiply the Numerator of the first Fraction by the Denominator of the second, saying, 3 into 6 maketh 18, that I set over the first line for a new Numerator, and it will be thus, $\frac{18}{96}$.

Likewise I multiply the Numerator of the second Fraction, by the Denominator of the first, saying, 4 times 16 maketh 64, that I set for the second Numerator, and the Fraction will appear thus, $\frac{64}{96}$.

So that both Fractions brought to one Denomination, must stand thus, $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You have done well.

Scholar. I beseech you let me examine it after my accustomed forme, by common parts of coyne or other measure.

Master. Go to.

Scholar. I have a peece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is the sixteenth part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that is

is $\frac{12}{96}$ Againe $\frac{1}{6}$ of the ſame peece of gold is 16 pence, ſo that $\frac{4}{4}$ parts maketh 64 pence, that is $\frac{64}{96}$. And ſo I finde the ſummes to agree with the other before.

Maſter. *So have you now the Art to bring two ſuch fractions into one Denomination: And if there be more then two, then muſt you multiply all the Denominators together, and ſet the totall thereof ſo many times down as there be fractions; and then to get for each one a new Numerator, multiply the Numerator of the* firſt, *by the Denominator of the* ſecond, *and the totall thereof, multiply by the Denominator of the* third, *and ſo forth, if there be more. Likewiſe multiply the Numerator of the* ſecond, *by the Denominator of the* firſt, *and the totall thereof by the Denominator of the* third. *And in the ſame ſort multiply the Numerator of the* third *into the Denominator of the* firſt, *and the totall thereof into the Denominator of the* ſecond, *and ſo forth if there were moe. So theſe three fractions* $\frac{2}{5}\frac{2}{4}\frac{1}{3}$ *doe make by Reduction theſe other three fractions of Denomination.* $\frac{24}{60}\frac{40}{60}\frac{20}{60}$. *All which you may bring into one fraction by adding the Numerators together, and putting the totall for the totall Numerator, reſerving ſtill that ſame common Denominator. And thoſe three fractions make one improper fraction, thus:* $\frac{192}{60}$.

Note the Reduction of three fractions, or more, into one.

Scholar. All this I perceive, and alſo that this laſt Fraction is more then an unite, and therefore you did call it an improper Fraction.

Ma-

Maſter. There be certain other formes of working in this Reduction, which I will briefly touch alſo, to give you an occaſion to exerciſe your wit therein.

The firſt variety of Reduction

The firſt variety is this: When you have made and written down your common Denominator (*as I have taught before*) *then to get a* numerator *for the firſt, do thus: Divide the* common denominator *by the* denominator *of the firſt fraction, and the quotient multiplyed by the* numerator *of the ſame, yeeldeth a new* numerator *for the firſt new fraction. So likewiſe do with the ſecond and the third, and with all the reſidue, if there be more.*

Scholar. That will I prove in your laſt example of theſe three fractions, $\frac{2}{5}\frac{1}{4}\frac{2}{3}$. When the denominators be multiplied they make 60. for 5 into 4 maketh 20, and 20 by 3 yeeldeth 60, that I ſet down three times thus: $\overline{60}\ \overline{60}\ \overline{60}$ then to have a numerator, for the firſt, I muſt divide 60 by 5 (the denominator of the firſt) and the quotient is 12, which I muſt multiply by 2 (the numerator firſt) and that maketh 24, and ſo have I for the firſt Fraction, $\frac{24}{60}$.

Likewiſe for the ſecond Fraction: I divide 60 by 4, and there cometh 15, which I multiply by 3, and ſo have I 45, for the ſecond Fraction $\frac{45}{60}$. Then for the third in like ſort will come $\frac{40}{60}$.

The ſecond variety.

Maſter. *Another way is this: If it happen ſo, that the leſſer* denominator, *can by any* multiplication *make the greater, then note the multiplier, & by it multiply the numerator over that leſſer denomina-*

tor,

tor, and for the leſſer Denominator put the greater, as thus in theſe two fractions $\frac{1}{12}$ and $\frac{2}{3}$, three being the leſſer Denominator multiplied by 4, will make 12, which is the greater Denominator: therefore by the ſame 4 I do multiply 2 which is the Numerator over 3, and that maketh 8: under which I do put 12, being the greater Denominator which is alſo made by multiplication of 4 into 3, and ſo have I theſe two fractions $\frac{1}{12}$, $\frac{8}{12}$, thus ſhortly reduced, without altering the one Fraction.

Scholar. This I underſtand.

Maſter. Then mark this third way: If the Denominators doe not happen ſo, that one by Multiplication may make the other; then look whether they both may be parts of any other one number; as in $\frac{1}{12}$ and $\frac{7}{18}$, although the leſſer taken but twice, be too much to make 18, yet they both may be parts unto 36, therefore look how many times twelve is in 36, and that quotient being multiplied by the Numerator over 12, the totall ſhall be put in ſtead of the Numerator over 12, and for 5 put 15, thus, $\frac{11}{36}$. So likewiſe look how often is 18 in 36, becauſe it is twice, therefore by 2 multiply 7, which is over 18, and it will be 14: ſet that for the Numerator, and in ſtead of 18 put 36; and then your Fractions reduced ſtand thus, $\frac{11}{36}$ $\frac{14}{36}$, in ſtead of $\frac{1}{12}$, and $\frac{7}{18}$. The third variety.

And if you will prove whether you have wrought well or no, that may bee proved by Reduction of them again to their former Denominations, which Art ſhall be taught in the fourth kinde of Reduction, where greater termes of Fractions be reduced into ſmaller in Proofe.

number

number, but no ſmaller in proportion. And if in ſuch Reduction the ſame termes or numbers come again that were before, then is the worke good, elſe not.

Scholar. Sir, I hear your words, but I doe not underſtand many of them: which if it pleaſe you, declare.

Maſter. With a good will, when convenient place ſerveth, but that muſt be in the ſaid fourth kind of Reduction, which teacheth how to reduce Fractions of Fractions into one Fraction, and ſo to one Denomination.

The ſecond form of Reduction of fractions of fractions into one fraction and Denomination.

When Fractions of Fractions be propoſed, you ſhall multiply the numerators of each into other, and ſet the totall for the new Numerator, and then multiply all the Denominators likewiſe, and take their totall for the new Denominator, and ſo are they ſpeedily reduced.

Scholar. If that be all, then I underſtand it already, as by this example I will declare. Theſe be the Fractions $\frac{2}{4}$ of $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{7}{9}$ which I would reduce to one Denomination, and proper ſimple Fraction.

Therefore begin I with the Numerators, and multiply them together, ſaying, 3 by 2, maketh 6: and 6 by 6, maketh 36, which multiplyed by 7, yeelds 252: that I ſet over a line for the Numerator thus:

$$\frac{252}{\quad}$$

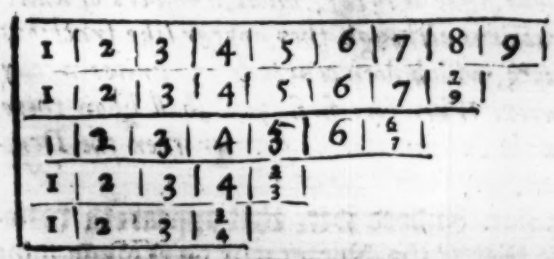
Then I multiply the Denominator, 4 by 3 maketh 12, and that by 7 bringeth 84, which multiplied by 9, yeelds 756, the new

new Denominator. And so the whole reduced fraction is this, 252/756 which is too hard a fraction for me to understand yet.

Master. You think so, and no marvell, but anon you shall learn to judge it easily, for this fraction is no more indeed then 1/3, although it be in greater termes, and therefore more strange, and more obscure.

And this sufficeth for this Reduction, save that I will shew you by a figure of measure the just rate and reason of this kinde of fractions, and also the due understanding of their Reduction.

The entire measure parted into 9.



Here you see the longest measure, (which standeth for the whole and entire quantity) first parted into 9 divisions, whereof 7 are severed by the second measure: and thereof againe are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being divided into foure parts, the lowest

meaſure doth contain $\frac{1}{4}$, ſo that the ſame $\frac{1}{4}$ muſt be named, not $\frac{1}{4}$ of the whole meaſure, but indeed is $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar. This example is ſo ſenſible, that I cannot chooſe but ſee it. And furthermore ſee alſo, that the ſame Fraction is equall to $\frac{1}{9}$ of the entire meaſure, as the lines which run up and down do expreſſy ſet forth. Alſo I ſee here that $\frac{2}{3}$ of $\frac{6}{7}$ is equall to $\frac{4}{9}$. And further yet, that $\frac{2}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$.

Maſter. I am glad that you ſee it ſo well, not doubting but you will gather greater light of knowledge hereby.

The third forme of Reduction of improper fractions.

But now it is time that wee come to the third forme of Reduction, which teacheth of improper Fractions, that is to ſay, mixt numbers of unites and fractions although they appear like fractions, as this $\frac{26}{5}$, which doth conclude 25 unites wholly, and $\frac{1}{5}$ over. Wherefore firſt you ſhall know them, by that the Numerator is greater then the Denominator.

Scholar. Indeed Sir, that appeareth reaſonable, that if the Numerator do expreſſe more parts to be taken of any unite, then the Denominator doth ſignifie that unite to be divided into, it muſt needs follow, that ſuch a Fraction importeth more then the whole, that is to ſay, the whole with certain parts over: but what Reduction is there in it?

Two ſeverall wayes in this Reduction.

Maſter. There bee two ſeverall kindes of Reduction, concerning ſuch Fractions. Sometimes

times it ſhall bee needfull to convert theſe Fractions into unites, and the proper fraction, that will remaine. And ſometimes contrariwiſe, it ſhall be meet to reduce mixt numbers, that is unites written with fractions into the forme of one ſimple fraction, and ſo bee there two wayes.

Scholar. **What is the mean of the firſt way to turn improper Fractions into unites with their proper Fractions?**

Maſter. *That is thus*; *Your* numerator *being greater then the* denominator, *muſt be divided by the ſame* denomination, *and the* quotient *thereof expreſſeth the* unites: *the remainer ſhall be put for the* numerator *of the* fraction *that reſteth, and the* denominator *muſt be the ſame that was before.*

The fifth way.

Reduction of improper fractions into unites, with their proper fractions.

Scholar. **For example, I take $\frac{17}{5}$, and dividing 17 by 5, the quotient will be 3, and there will remain 2.**

Maſter. **That you muſt write thus, $3\frac{2}{5}$, where (you ſee) I have written 3 without any line, as entire numbers ought to bee written, and the 2 that remained, I have ſet over the former** denominator, **with a line, as a proper fraction. And this number doth ſignifie now three unites, and $\frac{2}{5}$ of one.**

Scholar. **Then if I would by unites here underſtand Crownes, ſo it were 3 Crownes, and $\frac{2}{5}$, that is 2 s.**

Maſter. Even ſo, and therefore $\frac{17}{5}$ did ſignifie the ſame: But this happeneth ſometimes

that when the Reduction is ſo wrought, there remaineth nothing: And then it is not a mixt number, but a ſimple intire number, repreſented like a Fraction.

The ſecond way.

Scholar. As $\frac{15}{5}$ will make 3 juſt, and $\frac{18}{3}$ will make even 6. This I will remember. But now, what is the ſecond form of Reduction that you ſpeak of for theſe ſorts of fractions?

Reduction of whole numbers either alone, or joyned with fractions into improper fractions.

Maſter. *Whenſoever you have any of theſe two ſorts of* numbers, *that is to ſay whole* numbers *without* fractions, *or whole* numbers *with* fractions, *and you would turn them into the form of a* fraction, *you muſt multiply the whole* number *by that* denominator *which you will have to remain ſtill, and to the totall thereof adde the* numerator, *which you have already, and all that ſhall you ſet for the new* numerator, *keeping ſtill the former* denominator: *As if you have* $6\frac{1}{4}$ *which you would convert into an improper* fraction, *you muſt multiply* 6 *by* 4, *whereof cometh* 24, *& thereto adde the* numerator, *which is* 3, *and ſo have you* 27 *for the* numerator, *and* 4 *ſtill for the* denominator.

Scholar. Then is $\frac{27}{4}$ equall to $6\frac{1}{4}$.

Note.

Maſter. Even juſt, and ſo backward (as appeareth by the former Reduction) $6\frac{1}{4}$ maketh $\frac{27}{4}$. And thus one of their reductions may be the proof of the other work.

Scholar. This I perceive: But now if you would turn whole numbers without fractions into any fractions, I ſee not how that may be done, becauſe there is no Denominator to make the multiplication by.

Maſter:

Maſter. That is well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication muſt be made: for the proofe whereof I ſet downe 7, which is a whole number. And if you will have this number converted into any certain fraction, will mee to do it.

Scholar. I pray you reduce 7 into a Fraction.

Maſter. Then you care not what the Fraction be, ſo it be ſome Fraction.

Scholar. No, I paſſe not for the ſort of the Fraction.

Maſter. Then how can you thinke that you require me to do any thing certain, when you leave me to do as I liſt? And ſeeing you ſtand at that ſtay, whether thinke you that I muſt firſt intend in minde what Fraction I will make of it before I can do it indeed?

Scholar. Elſe you ſhould do ignorantly.

Maſter. Then will I limit my ſelf (ſeeing you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the denomination by quarters) and there amounteth 28 to be ſet for the Numerator, and the 4 muſt be ſet for the denominator, and the fraction will be thus $\frac{28}{4}$.

Scholar. Indeed I perceive this to be reaſonable, for without much triall I underſtand that $\frac{28}{4}$ of any thing doth make 7. And ſo then

if I would turn 8 into 5 parts, it will make $\frac{40}{5}$ which is all one with 8: for 8 Crownes turned into 5 parts, (that is into shillings) will make 40 shillings, that is, $\frac{40}{5}$ of a Crown.

The fourth forme of Reduction

Master. *Seeing you understand now these three kinds of* Reduction, *I will declare unto you the fourth kind, that is, when* fractions *be written in greater termes then they need, how they may bee brought to lesser termes.*

Scholar. To write any thing in greater terms then needeth, seemeth to be a fault, and so this Rule seemeth to amend that fault.

Master. It were a fault to doe any thing without need, which after must bee redressed: but in this case it is not so, neither did I say absolutely (as you doe) that it needeth not to expresse those Fractions in so great terms, but that the Fractions doe not need, I meane for their value to be understood: but yet it may be needfull for the ease of these workes whereto they be applyed; as for example, In the first kind of Reduction this was your own example, $\frac{1}{16}$ and $\frac{3}{6}$ which when you would reduce, you were fain to turn them first into one denomination, and so appeared they thus: $\frac{18}{96}$ and $\frac{48}{96}$ where the Fractions (for their own understanding) needed not to be turned out of smaller termes into greater, but yet the easinesse of working needed it.

Scholar. Sir, I understand now, not onely the difference of this need (for the Fractions might better be understood as Fractions severall,

ſeverall, each in his value, when they were in leſſer termes, although they could not ſo well be reduced) but alſo I underſtand what you mean by greater terms, and leſſer terms, whereof before I was in doubt: for I ſee you call the Numerator and Denominator, the terms of the fraction. Termes of Fractions.

Maſter. *I am glad you underſtand it ſo well: now when then you would value any* fractions (*becauſe they may beſt be done when the terms are ſmalleſt*) *you ſhall reduce them to the ſmalleſt that you can, which thing you may do thus: Divide the greateſt of any ſuch two terms by the leſſer, and if any thing remain, by that remainer divide the laſt* diviſor: *and if any thing remain now, by that divide the firſt* diviſor (*which was before the remainer of the laſt* diviſion) *and ſo continue ſtill, till nothing do remain in the* diviſion: *and then marke your laſt* diviſor, *for it is the* number *that will eaſily reduce your* fractions, *if you divide both the* numerator *and the* denominator *by the ſame* number, *and put for the* numerator *the* quotient *of his* diviſion, *and for the* denominator *alſo his* quotient, *that riſeth by his* diviſion.

Scholar. I take for example $\frac{18}{96}$, and becauſe 96 is the greateſt number, I divide it by 18, and the quotient is 5, and there reſteth 6, what ſhall I do with this quotient?

Maſter. Nothing in this worke, but now ſeeing there remaineth ſomewhat by that remainer muſt you divide the laſt diviſor.

Scholar. If I ſhall divide 18, (which was

the laſt Diviſor) by 6, that was the remainer, ſo is the Quotient 3, and nothing reſteth.

Maſter. As for the Quotient, I omit him yet: but becauſe there doth remain nothing, therefore is 6 (which was your laſt Diviſor) that number by which you may reduce the fraction proponed.

Scholar. Then as you taught me, I muſt divide the Numerator 18 by 6, and the quotient is 3, which I muſt put for the numerator over a line, thus: And then by the ſaid 6 muſt I divide alſo the Denominator 96, and the Quotient will be 16, which I muſt take for the Denominator, and ſo is the Fraction $\frac{3}{16}$. And ſo me thinketh this rule doth prove the work of the firſt Reduction.

$$\frac{3}{16}$$

Maſter. That is true, if the firſt Reduction were made of fractions into their leaſt terms, and elſe not, without ſome help, as the ſecond number in that place will declare.

Scholar. The ſecond number was $\frac{2}{3}$, which was turned into $\frac{64}{96}$ by that Rule. Now if I ſhall by this Rule reduce it againe into the leaſt terms, I muſt divide 96 by 64, and there remaineth 32, wherefore I muſt take that 32 for the Diviſor, to reduce the ſaid Fractions. Then do you divide 64 by 32, and the Quotient is 2, which I ſet for my numerator. Again, I divide 96 by 32, and the Quotient will be 3, and ſo I have but $\frac{2}{3}$.

Maſter. Muſe not at the matter, for you have

have done well enough: but you think you have not the fraction that you looked for, that is, $\frac{4}{6}$, yet have you one equall to it, as by the parts of a shilling you may prove.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{8}{3}$ be all three equall. And now I perceive that because $\frac{4}{6}$ was not written in the least termes that it might be, therefore this Reduction brought forth not it, but that other which is written in the least termes. Now understand I this Rule well. But is there any other way to work this Reduction?

Master. Yes, but first note this, that if you finde no such Divisor, to reduce the fraction till you come to 1, because one doth make no division, therefore that fraction is already in his least termes, as by $\frac{71}{100}$ you may prove, and so $\frac{85}{98}$, and many other like. Another way to work this Reduction

But now for your better aid to find the due proportion in least terms, with more ease for a yong learner, you shall mediate or take the half of the Numerator, and also of the Denominator as long as you may upon a line, alwayes parting them with a right down dash of your pen as you work, which may easily be done, if the numbers be even: as 2.4.6.8, or 10, but if they be odd (though it be but one of them) then must you abbreviate them by 3.5.7, or 9. &c. Note that to mediate any number is to divide by two.

And because examples doe most instruct, I have here set downe the manner of two or three, whose last number at the end of the line shew-

sheweth the least terme of valuation of that Fraction.

As for example: I would reduce $\frac{288}{576}$ into his least terme or value, whereupon I set forth $\frac{288}{576}$ with a long line drawn from it, thus:

298	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And because both the Numerator and the Denominator end in even numbers, I see this may be abbreviated by 2, or 4 or 6, &c. Therefore on the other side of the right down dash toward the right hand, I first take the half of the Numerator: saying, the half of 2 is 1, the half of 8 is 4: and again, the half of 8 is 4: which 144 is now a new Numerator, and therefore I part it with a right down dash as before.

Then do I also take the half of 576, in saying, the half of 5 is 2, and the half of 17 is 8, and the half of 16 is 8, and so have I 288 for a new Denominator.

Then beginning again; saying, the half of 144 is 72, and the half of 288 is 144: thus continuing the mediation or division by 2, untill you come to the last worke, as appeareth here in the example, where the same is reduced to $\frac{1}{2}$ which is equall $\frac{288}{576}$.

So the second example $\frac{28}{112}$ first abbreviated by 2, and again by 2, and last by 7, is reduced to $\frac{1}{4}$ which is equall to $\frac{28}{112}$.

28	14	7	1
112	56	28	4

Again,

Again, $\frac{1465}{4395}$ abbreviated first by 5, then by 293.

1465	293	1
4395	879	3

Scholar. Sir, I thanke you much, this is very easie and good for a young learner.

Master. So it is, but yet notwithstanding, if you can without that division by memory, espy the greatest number that may divide exactly both termes of your Fractions proponed, then need you not to use that division, as in this Fraction $\frac{42}{96}$, I see that 12 is the greatest number that can divide them both: and therefore without any work, by memorie onely, I turn that into $\frac{7}{8}$; but this ability in knowledge is got by exercise.

Yet one other way of easie Reduction in this kinde there is, when your Fraction hath any cyphers in the first places of both termes, then may you by casting away the Cyphers, make a briefe Reduction as thus: $\frac{100}{400}$ here take away the cyphers, and it will be $\frac{1}{4}$, which is the same in value with $\frac{100}{400}$.

Scholar. And so if I have $\frac{402}{650}$, it will be $\frac{4}{65}$.

Master. You are deceived, for you take away more cyphers from the Numerator then you do from the Denominator, which you may not do.

Scholar. I confesse my fault, which came of too much hastе, I was more gladder of the

Rule

Rule then wise in using it: but now I understand it I trust.

Master. Then may I goe in hand with the fifth or last kinde of Reduction, which teacheth how to turn any Fraction proponed into any other Denomination that you list, or into any part of common coynes, weights, or measures, or such like.

The fifth kinde of Reduction.

For declaration whereof, first you shall marke whether your fraction *be a simple* fraction, *either else a* fraction *of sundry parts, I mean of more terms then two. And if your* fraction *be a* fraction *of* fractions, *or otherwise compound, you must reduce it to one simple* fraction: *And then mark well the* Denomination *of that other* fraction, *into which you would turn this: for by that* denominator *you must multiply the* numerator *of your first* fraction, *and the totall* Product *thereof shall you divide by the* Denominator *of your first* fraction, *and that* quotient *shall be the* Numerator *of the* Denominator *proponed*: as for example, *I have this* fraction $\frac{3}{5}$, *which I would turn into ten parts: therefore I multiply this* 10 *by* 3, *that is the* numerator *of my* fraction, *and there ariseth* 30, *which I divide by* 5, *and the* quotient *is* 6, *which must be the* numerator *to* 10, *and so* $\frac{3}{5}$ *will be* $\frac{6}{10}$.

To reduce fractions to a Denominati o appointed.

Scholar. This is easie enough to do.

Master. Then shall you see another example of the same fraction that is not so easie: as if I would turn $\frac{3}{5}$ into 8 parts, prove you that work.

Scholar, I must multiply 8 by 3, and there amount-

amounteth 24, which I divide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Maſter. And ſee you nothing doubtfull in this worke?

Scholar. I ſee that when 24 was divided by 5, there remained 4, which I did not paſſe of, becauſe ye ſpake nothing of any remainer, but onely of the quotient.

Maſter. By likelihood you remember what I ſaid to you in Diviſion of whole numbers, that you would not paſſe of the remainer there but onely note it as a ſumme that could not be divided without knowledge of Fractions. Wherefore now mark this, that in all diviſions of whole numbers, when there is any remainer, you ſhall ſet it over a line as a Numerator, and ſet the diviſor for the denominator, and that Fraction doth make the Diviſion compleat, and is part of the quotient: As if I would divide 48 by 5, the quotient will be $9\frac{3}{5}$: ſo in your former worke when 24 was divided by 5, the quotient ſhould be $4\frac{4}{5}$, and ſo the new Fraction ſhould be thus, $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, that is $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may prove by example of ſome Coyne.

Scholar. Then I take a Crowne, whoſe $\frac{1}{5}$ is 3 s. Now if I would prove whether the 3 s. be $\frac{4}{8}$ add $\frac{4}{5}$ of $\frac{1}{8}$, I ſhall have a cumbrous worke to do.

Maſter. Indeed for whole pence, your example is a little troubleſome: yet turning the crown

crown into halfe pence, it is eafie enough.

Scholar. That will I try.

¶ Firſt, I ſee that $\frac{1}{5}$ of a Crown is 3 ſhillings, which is 36 pence, or 72 halfe pence. Now if I can finde that this Fraction $\frac{4}{8}$ and $\frac{1}{5}$ of $\frac{1}{8}$ be equall unto 3 ſhillings, then am I fully anſwered.

Becauſe I cannot take $\frac{4}{8}$ of a Crowne, I turn the Crown into halfe pence, as you willed me, which makes 120, which I divide by 8, my Quotient is 15, which taken foure times, make 60 ob. Now reſteth me to have $\frac{1}{5}$ of the $\frac{1}{8}$ part of a Crown, whereof $\frac{1}{8}$ part is 15 ob. the 15 being parted into 5 parts, the Quotient is 3, which taken fours times maketh 12 ob. which with my 60 before amounteth to 72, which are then equall to $\frac{1}{5}$, my deſire.

Maſter. I commend you for your diligence, you might have wrought it thus: either $\frac{4}{8}$ being abbreviated as before I taught, is $\frac{1}{2}$. Now half a Crown is 2 ſhillings 6 pence. Now $\frac{1}{5}$ of $\frac{1}{2}$ is a Fraction of Fractions, which if you doe reduce into one entire Fraction, as before you have learned in ſaying, 5 times 8 is 40, for a new Denominator, and once 4 is 4, for a new numerator: it maketh $\frac{4}{40}$, and abbreviated alſo make $\frac{1}{10}$. Now the tenth part of a Crown is 6 pence, which put to 2 ſhillings 6 pence, make alſo 3 ſhillings your deſire.

But now one example more for this Rule, and then we ſhall end it. If I have $\frac{7}{15}$ of a Soveraign (accounting the Soveraigne 20 ſhillings)

lings) how many ſhillings is that $\frac{7}{15}$?

Scholar. I muſt multiply 7 by 20 and that maketh 140, which I ſhall divide by 15, and the quotient will bee 9 $\frac{1}{15}$, or in leſſer termes $\frac{1}{3}$

Maſter. That is 19 ſhillings, and one third part of a ſhilling, that is 4 pence, as by the ſame Rule you may prove. And this for this time ſhall ſuffice for Reduction. And now I will proceed to Addition.

Addition.

Whenſoever you have any Fractions *to be added, you muſt conſider whether they be of one* denomination *or not, and if they bee of one* denomination, *then adde the* Numerators *together, and ſet that that amounteth for the* numerator *over the common* denominator, *and ſo have you done: The reaſon is, becauſe that ſuch differ little in* Addition *or* Subtraction *from the worke of vulgar* denominations, *where the* denominators *be of the* number, *as* 3 pence, *and* 5 pence, *make* 8 pence, *where the* denomination *is not altered. But if the* fractions *be not of one* denomination, *or any of them be mixt of whole* numbers *and* fractions, *then muſt you firſt reduce them to one* denomination, *and after adde them. And if they be many, then adde firſt two of them, and ſo the* ſumme *that doth amount of the* Addition, *and the third, and then the fourth,* &c. *if you have ſo many.*

Addition of fractions of one Denomination.

Scholar.

Scholar. This seemeth easie enough, now that I have already learned to reduce, without which I could never have wrought this: And therefore now I see good reason why you did place Reduction before Addition.

Master. It is well considered, but yet refuse not to expresse your understanding of it by an example.

Scholar. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$ and because the denominators are like (and so needeth no Reduction) I adde 7 to 5, which maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbers, being abbreviated $\frac{2}{3}$.

To adde fractions of divers Denominations.

Now if you have many numbers to be added, as $\frac{1}{5}$ $\frac{2}{10}$, first I must reduce them (because they have divers denominators) into one denomination, and then they will bee thus, $\frac{112}{400}$ $\frac{280}{400}$ $\frac{168}{400}$, or in lesser termes $\frac{11}{40}$ $\frac{14}{40}$ $\frac{16}{40}$, which by Addition do make $\frac{41}{40}$, that is $2\frac{1}{40}$.

Master. Now may we go to Subtraction.

Sub-

Subtraction of Fractions.

Subtraction of Fractions.

Subtraction *hath the ſame precepts that* Addition *had, for if the* denominators *bee like, then muſt you ſubtract the one* numerator *from the other, and the reſt is to be ſet over the common* denominator, *and ſo your* Subtraction *is ended: but and if you have many* fractions *to be ſubtracted out of many, then muſt you reduce them to one* denomination, *and into two ſeverall* fractions, *that is, all that muſt be ſubtracted into one* fraction, *and the reſidue into another* fraction, *and then work as I ſaid before.*

Scholar. For the firſt example I take $\frac{11}{12}$ to be ſubtracted out of $\frac{17}{12}$, and the reſt will bee $\frac{6}{12}$ or $\frac{1}{2}$.

For another example I take $\frac{1}{4}$ to be ſubtracted out of $\frac{7}{8}$, which I muſt reduce, and it will be thus $\frac{8}{32}$ and $\frac{28}{32}$.

Then doe I ſubtract 24 out of 28, and there reſteth 4, which I ſet over the common denominator for a Remainer, thus $\frac{4}{32}$: that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to bee ſubtracted from $\frac{7}{8}$ and $\frac{9}{10}$: and becauſe their Denominators be divers, I doe reduce them into one Denomination thus $\frac{1440}{192}$ $\frac{1692}{192}$ —— $\frac{1728}{972}$ $\frac{1820}{182}$.

Then do I adde the two first, and they make $\frac{1242}{1920}$. Also I adde the two last, and they yeeld $\frac{1428}{1920}$. Then do I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$, that is in smallest termes $\frac{23}{120}$. And thus have I done with Subtraction, except you have any more to teach me.

Master. Prove one example or more out of Fractions of divers Denominations.

Scholar. I take the two Fractions $\frac{7}{8}$ to bee subtracted from $\frac{9}{24}$ which being reduced, will stand thus $\frac{168}{192}$ and $\frac{72}{192}$: How would I subtract 168 out of 72, but I cannot.

168	72
8	24
192	

Master. Then may you perceive that you mistook the Fractions: for you can never subtract the greater out of the lesser, although you may adde, multiply or divide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his termes lesser then $\frac{9}{24}$, yet is $\frac{7}{8}$ the lesser Fraction: for generally if you multiply the Numerator and the Denominators of two Fractions crosse-wayes, that Fraction is the greatest of whose Numerator commeth the greatest summe, as in this example: 7 multiplied by 24 maketh 168, and 9 being multiplied by 8 yeeldeth but 72, therefore is the first Fraction 7 the greatest of these two, so can you not subtract it out of a lesser Fraction.

The greatest of two Fractions.

But if you should subtract a Fraction out of a whole number, what should you doe?

Scholar.

Scholar. Marry I would reduce the whole number into a Fraction of the same Denomination that my Fraction is, and then work by Subtraction.

Master. So may you doe, but it is much easier, if your Fraction be a proper Fraction, that is to say, lesse then an unite, to take an unite from the whole number, and then turn it into an improper Fraction, and so work your Subtraction. As if I would subtract $3\frac{2}{4}$ from 4, I may take 1 from 4, and turn it into $\frac{4}{4}$, from which I bate $2\frac{2}{4}$, there will remain $\frac{2}{4}$. And if the first Fraction be an improper Fraction, then may I take so many unites from the whole number, that they may make an improper Fraction, greater then that first, and then worke by Subtraction. As if there bee proponed $\frac{10}{3}$ to bee subtracted from 6, because $\frac{10}{3}$ is more then 3, and not so much as 4, I must take 4 from 6, and turn them into thirds thus $\frac{12}{3}$, then abate $\frac{10}{3}$ and from $\frac{12}{3}$, there resteth $\frac{2}{3}$: so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$, which is $\frac{10}{3}$: from 6 whole: then set 1 under 6, as thus $\frac{6}{1}$: And then to reduce these two Fractions into one Denomination, as here appeareth $\frac{18}{3}$ from $\frac{6}{1}$: Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$, which maketh $2\frac{2}{3}$ your desire. And thus will I make an end of the worke of subtraction of Fractions, and proceed to Multiplication.

$$\frac{10}{3} \times \frac{18}{3} \quad 8 \quad 3$$

Multiplication of Fractions.

Multiplication of Fractions.

THerefore when any two fractions be proponed to be multiplied together, the numerator of the one muſt bee multiplied by the numerator of the other: and the ſum that amounteth thereof muſt be ſet for a new numerator: likewiſe the Denominator of the one muſt be multiplied by the Denominator of the other, and that that amounteth ſhall be ſet for the Denominator, and this new third Fraction expreſſeth the Product of the Multiplication of the two firſt Fractions proponed, whereof take this Example, $\frac{1}{5}$ multiplied by $\frac{1}{12}$ doth make $\frac{1}{60}$.

$$\frac{1}{5} \quad \frac{1}{12} \quad 15 \quad 60$$

Schol. I perceive then that 3 being the Numerator of the firſt Fraction, is multiplyed by 5 being the Numerator of the ſecond Fraction, whereof amounteth 15, the Numerator of the third Fraction. And ſo likewiſe 5 being the Denominator of the firſt Fraction, is multiplied by 12 the Denominator of the ſecond Fraction, whereof amounteth 60 the new Denominator, ſo that I perceive how the work is done, but I do not perceive how $\frac{1}{60}$ is greater then $\frac{1}{5}$, for if I ſhal uſe my former manner of examination by the parts of ſome coine, I ſee that $\frac{1}{5}$ of a Crowne is 36 pence, and $\frac{1}{12}$ of a Crowne is 25 pence, whereof the one multiplied by the other, doeth make 900 pence, which is 15 Crownes, but by your multiplication

tiplication there amounteth $\frac{11}{20}$., which is but 15 pence, and that is much leſſe then any other of both the firſt Fractions.

Maſter. That difference is between Multiplication in whole numbers, and Multiplication in broken numbers, that in whole numbers, the ſumme that amounteth is greater then both the other whereof it came: but in Fractions it is contrariwiſe: for the ſumme that amounteth is leſſer then any of the other two Fractions whereof it is produced.

Scholar. I deſire much to underſtand the reaſon thereof.

Maſter. Although I purpoſed to reſerve the reaſons of works Arithmetical for the perfect Book of Arithmetick, yet I will ſhew you this, becauſe of the ſtrangeneſſe of the work.

You ſee in whole numbers, that of two numbers being multiplyed together, is made the third number, which third number doth beare the ſame proportion to the number multiplyed, that the multiplyer doth beare to an unite. And ſo in Fractions the third number which amounteth of Multiplication, beareth the ſame proportion to each of the two firſt Fractions, that the other of thoſe two Fractions doth bear to an unite.

Scholar. Sir, I underſtand your words thus: when 40 is multiplyed by 12, there doth amount 480, which 480 doth contain 40 ſo many times in it, as 12 doth containe Unites, that is to ſay, twelve times. And ſo it

appeareth that 480 doth contain twelve so many times also as 40 doth contain unites, that is 40 times. But now I see not how the third number in this example of Fractions can contain any of the two former (as it happened in whole numbers) seeing it is lesser then either of them.

Master. No marvell if you cannot see that thing which is not possible to be seen of any man, how the third number in Multiplication of Fractions should be greater then any of the two former Fractions: but yet this may you see (which I said) that the third number in Fractions so multiplyed, doth beare the same proportion to any of the two former Fractions that the other of those two Fractions doth bear to an unite, as in your example, $\frac{1}{5}$ being multiplyed by $\frac{1}{12}$ doth make $\frac{12}{60}$. Now I say that $\frac{12}{60}$ doth beare the same proportion to $\frac{1}{5}$ that $\frac{1}{12}$ doth bear to an unite, as you may in your own formes of examination by Coine, try it: for in an old angell (which in times past was currant for 7 shilling 6 pence, are 180 half pence) which I set for the intire unite, whose parts (according to the Fractions aforesaid) are these, for $\frac{12}{60}$ set 45 halfe pence, for $\frac{1}{5}$ take 108 halfe pence, and for $\frac{1}{12}$ put 75 halfe pence. Now doth 45 bear the same proportion to 108, that 75 doth bear to 180, for 45 is $\frac{1}{12}$ of 108, and so is 75 also $\frac{1}{12}$ of 180.

That these reasons may be better reserved till another time, when the knowledge of pro-

Propoſitions in due order ſhall be taught: yet in the meane ſeaſon I will ſhew you how it commeth to paſſe, that in Fraction the third ſumme muſt needs be leſſer then any of the other two.

Conſider this, that when a Fraction is preponed, as in the former example $\frac{1}{5}$ if it be multiplied by more then 5, it will make more then one entire number. As if I multiply $\frac{1}{5}$ by 5, that is to ſay, if I take it 5 times, it will make three entire unites. Example: in a Crown $\frac{1}{5}$ of it maketh 3 ſhillings, which if I take five times, it will amount to 15 ſhillings, that is, three entire Crowns; ſo if I take the ſame $\frac{1}{5}$ but twice, it will yeeld 6 ſhillings, that is one entire Crowne, and $\frac{1}{5}$. Now if I take it but once, it cannot be more then it was before, that is 5 ſhillings. And if I take it leſſe then once, it cannot bee ſo much as it was before. Then ſeeing that a Fraction is leſſe then one, if I multiply a Fraction by another Fraction, it ſhilleweth that I doe take the firſt Fraction leſſe then once, and therefore the ſumme that amounteth, muſt needs be leſſe then the firſt Fraction. Note.

Scholar. Sir, I thank you much for this reaſon. And I truſt I doe perceive the thing, as by example of this ſame Fraction $\frac{1}{7}$ I will appeale. If I take $\frac{1}{7}$ of a Crown once, that is to ſay, if I multiply $\frac{1}{7}$ by 1, it will be as it was before, but 3 ſhillings: ſo if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then

will it be but half so much: likewise if I multiply it by $\frac{1}{3}$ that is, if I take but the third part of one, it will yeeld but 12 pence, that is, the third part of the first Fraction.

And so to make an end: if I take but the twelfe part of one, that is, if I doe multiply it by $\frac{1}{12}$ it will yeeld but the twelfth part of the first Fraction, which is but 3 pence. And it followeth, that if $\frac{1}{12}$ make 3 pence, then $\frac{5}{12}$ must needs make five times so much, that is, 15 pence, which was the summe that hath given the occasion of all this doubt.

Master. Now I perceive you have sufficient understanding in this sort of Multiplication for this time, wherefore I will proceed to the rest.

To multiply a whole number into a fraction.

In Multiplication *it happeneth sometime, that there bee whole numbers to bee multiplyed with Fractions, and may be in two sorts: for either the whole number is severall from the Fraction, and is the* Multiplier, *or else the whole number is joyned with one, or both of the Fractions, and so maketh a mixt number thereof. If it bee in the first sort, then needeth there no* Reduction, *but onely multiply the* numerator *of the Fraction by that whole number, and the totall thereof set for the new* numerator.

Scholar. I understand you thus. If I have $\frac{6}{7}$ to bee multiplyed by 16, then must I multiply that 16 with 6, which is the Numerator, whereof commeth 96, and that must I set for the new Numerator: keeping still 23 for

for the Denominator, and so the Fraction will be $\frac{22}{35}$ that is $4\frac{4}{34}$.

Master. And in this sort of work you may abridge the labour thus. If it happen the Denominator to be such a number as may evenly be divided by the said whole number proposed, then divide it thereby, and set the Quotient of that division for the former Denominator, but reserve still the numerator, and so is the multiplication ended.

Scholar. Then fain this example $\frac{1}{20}$ to bee multiplyed by 5, and because 5 will justly divide 20, therefore I take the Quotient of that division, which is 4, and set in stead of 20, and so the Fraction will be $\frac{1}{4}$ that is $1\frac{1}{4}$.

Master. Which is all one with $\frac{22}{20}$ that would have followed of the other sort of work.

Scholar. I perceive it very well.

How to multiply mixt numbers.

Master. *Now then for the other sort, where the number is mixt, take this way: first to reduce the said whole number and fraction into one improper Fraction (as I shewed you in* Reduction*) and then multiply them together, as if they were proper Fractions.*

Scholar. $13\frac{1}{5}$ being set to bee multiplyed by $\frac{1}{5}$ first I must reduce the mixt number, as in this Example appeareth, by multiplying 13 by 5, and that maketh 65, whereto I must adde the numerator 3,

$13\frac{1}{5}$ by $\frac{1}{5}$

68

5 40

340

5

8

and

and so the Fraction will be $\frac{68}{5}$ which two Fractions now I shall multiply after the accustomed forme, and it will be $\frac{140}{40}$, or $\frac{14}{4}$.

Master. You have done well: and so may you see, that although most part of the formes of multiplication may bee wrought without Reduction, yet some cannot, as namely, mixed numbers.

Duplation. *And yet one note more I will tell you of* Multiplication *before we leave it: That is, whensoever you would multiply any* fraction *by* 2, *which commonly is called* duplation, *you may do it not only by doubling the* numerator, *but also by parting the* denominator *into halfe, if it be eaven.*

Scholar. Then if I would double $\frac{5}{12}$ I may chuse whether I will make it $\frac{10}{12}$ or else $\frac{5}{6}$. And indeed I see that is all one, but that the dividing of the Denominator seemeth the better way to make smaller termes of the Fraction, and so they shall need the lesse Reduction.

Master. It is so: and now I shall not need to tell you that multiplication is proved by Division, and Division likewise by multiplication: but the like worke that I shewed you in multiplication, will I shew you in Division

Division

Diviſion of Fractions.



Diviſion of Fractions.

Henſoever two fractions *bee proponed, that one ſhould be divided by the other, I muſt ſet down firſt the* fraction *that ſhall be divided (which is called the* dividend*) and then after it the other which is the* diviſor: *Then ſhall I multiply the* numerator *of the* dividend *by the* denominator *of the* diviſor, *and that which amounteth I muſt put for a new* numerator. *Againe, I ſhall multiply the* denominator *of the* dividend *by the* numerator *of the* diviſor, *and the* number *that amounteth thereof I muſt put for the new* denominator. *And this third* fraction *is the* Quotient *of the ſaid* diviſion.

Scholar. This ſeemeth eaſſe in forme, as by example thus: If I would divide $\frac{5}{8}$ by $\frac{2}{6}$. firſt I multiply 5, (being the Numerator of the Dividend) by 6 which is the Denominator of the Diviſor, and thereof riſeth 30: then I multiply 8 (being the Denominator of the Dividend) by 2, being the numerator in the Diviſor: and ſo riſeth 16, the which I muſt make a third Fracter, thus $\frac{30}{16}$.

	30	
5		2
8		6
	16	

Maſter. Me ſeemeth you are quicker in underſtanding now, then you were when I taught

taught you the Art of whole numbers, but that is no marvell: for the more knowledge that a man getteth the readier shall he finde his wit, and quicker in understanding: but yet of two things I will admonish you, which you might have observed here for the ease of work and lightnesse of understanding, the nature of the Quotient.

Whensoever you divide one Fraction by another, either they be both equall together, or else the one is greater then the other: if they be equall, their Quotient shall be such, that the numerator and the Denominator of it shall be equall also. And if the two first Fractions bee inequall, their Quotient shall declare the same by the inequality of the Numerator and Denominator, as in these Examples following shall appear.

First, if equall Fractions $\frac{4}{7}$ and $\frac{12}{21}$ bee equall together, and if the one be divided by the other, the Quotient will bee $\frac{252}{168}$, as you may perceive by that Rule aforesaid.

Now in the unequall Fractions, as $\frac{4}{9}$ and $\frac{4}{3}$, the Quotient will be $\frac{12}{36}$, where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Note how to know the proportion between two numbers.

Master. The reason is this: when any Fraction is divided by another, the Quotient declareth what proportion the Dividend beareth to the Divisor. So $\frac{1}{2}$ divided by $\frac{1}{4}$, maketh 2, which must bee founded, not 2, but twice,

twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the Quotient representeth the Dividend, and the Denominator representeth the Divisor. And this is always true, whether the greater Fraction be divided by the lesser, or the lesser by the greater. But this proportion will not be exactly known, till you have learned the Art of proportions: notwithstanding some light of it I have declared in the Rule of Reduction. But now for the easie remembrance of the Quotient in Division, as soone as you have set downe your two Fractions the one against the other, then make a straight line for the Quotient: and as soone as you have multiplyed the Numerator of the Dividend, by the Denominator of the Divisor, set the Number that amounteth over the said line, and then multiply the other two Numbers, and set their totall under the same line.

Scholar. I perceive you would not have me trust to memory till I were better expert, lest oftentimes I happen by misse-remembrance to bee abused. This Example I take for that declaration.

If I would divide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one against the other (as here doth appear) and then make another line for the Quotient in some good distance,

$$\frac{2}{3} \text{ by } \frac{3}{4}$$

distance, where I may set the numbers of the Quotient, as soon as any of them is multiplied. So then as soone as I have multiplyed 2 by 4, which maketh 8, I shall set that 8 over that line, thus: And then multiply 3 by 3, which yeeldeth 9: and that 9 must be set under the same line, and then will the whole Quotient appeare thus $\frac{8}{9}$. whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$ as 8 is to 9, but how may I perceive that?

Master. Although you might better perceive it by the Rule of Reduction, yet this example may be declared in common coines, as in a common shilling of 12 pence, of which $\frac{2}{3}$ maketh 8 pence, and $\frac{3}{4}$ doth make 9 pence, and so you may easily see that their proportions doe agree. And if you had taken this Example before when you tooke the Example of $\frac{2}{3}$ and $\frac{3}{4}$, your Quotient would appeare (as this doth) more easie to understand; whereas that Quotient being $\frac{12}{16}$, is not an easie proportion for you to perceive, being yet little acquainted with proportions.

Scholar. If there be whole Numbers to bee divided by a Fraction, how shall I performe it?

To divide a whole number by a fraction.

Master. *When any whole number shall be divided by a fraction, you must multiply the said whole number with the Denominator of the Fraction, and set the totall thereof for the new Numerator, and for the Denominator set the*

the Numerator of the fraction.

Scholar. Then 20 divided by $\frac{1}{4}$ will make $\frac{20}{3}$, as here appeareth $\frac{80}{3}$.

$\frac{20}{1}$ by $\frac{1}{4}$

Master. *Even so: but if you would divide the Fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall for the Denominator, without changing the Numerator.* To divide the Fraction by the whole number.

Scholar. Then to divide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth $\frac{20}{23}$ by $\frac{4}{1}$ in this Example $\frac{20}{92}$.

$$\frac{20}{23} \quad \frac{20 \text{ by } 4}{1} \quad \frac{}{92}$$

Master. You say well. And by the same Example you give me cause to remember another briefe way to doe the same: for if you had divided the said Numerator by 4, and set the Quotient for the Numerator, keeping still the old Denominator, it would have been not only as well done, but also in a Fraction of lesser terms. Another briefe way.

Scholar. I guesse it to be even so, by a like worke that you taught me in multiplication: And for proof thereof $\frac{20}{23}$ being the Dividend, and 4 the Divisor, I divide the Numerator 20 by 4, and the Quotient is 5, which I set for 20 over 23, thus $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$, as by dividing or abbreviating both these termes by 4, and so reducing them

to

to their least Denomination, I may easily prove: as appeareth by this example $\frac{12}{39} \frac{1}{13}$.

Master. You conceive it well. And if there be mixt numbers, (either one or both) you must first reduce that mixt number into an improper Fraction, and then work as you have learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Division: if the Denominators be like, then divide the numerators, as if were in whole numbers, and the Quotient, whether it be Fraction, whole number or mixt, is a good Quotient for that Division. And generally, if one of the numerators may justly divide the other by that Quotient, multiply the Denominator of the lesser numerator, and set it that both amount in the rooms of the same Denominator, and then for a numerator to it, set the Denominator of the other Fraction.

Scholar. Then if I would divide $\frac{1}{4}$ by $\frac{12}{17}$ I see that 3 will divide 12, and the Quotient will be 4, by which I must multiply the other 4, that is the Denominator under 3, and then it is 16, which is set for the Denominator 4, and over it in stead of 3 I must set 17 the other Denominator, and so it is thus $\frac{17}{16}$.

Master. And so is $\frac{7}{16}$ in stead of $\frac{11}{48}$. which would have risen by the common worke, as here appeareth.

$$\begin{array}{r} 51 \\ 3 \quad\quad 12 \\ \hline 4 \text{ by } 17 \\ 48 \end{array}$$

And

And now for Mediation (*which is to divide by* 2) *marke this, if the* Numerator *be an even number, set the half of it in his place without the* Divisor, *and so have you done: and if the* numerator *be not even, then double the* Denominator.

Scholar. That is, if I would mediate $\frac{14}{11}$, I may make the Quotient $\frac{7}{11}$ and if I would mediate $\frac{1}{11}$, I must make it $\frac{2}{22}$.

Master. And thus will I make an end of the workes of common fractions for this time, not doubting but you can apply them both to the Rule of Progression, and also to the Golden Rule, without any other teaching then you have learned before, which might seem tedious to repeat, in regard you have sufficient knowledge in Reduction, Addition, Subtraction, Multiplication, and Division: And therefore will I goe in hand with the Rule of Proportion, or Golden Rule, which now will appeare easie enough.

The Golden Rule direct in Fractions.

Master.

THerefore as touching the Golden Rule for the placing of the three numbers proponed in the question whereby to finde the fourth, and for the form of their worke, with other like notes, I refeire you to that which you have already learned. The rule of proportion, in Fractions.

Note this for a generall Rule.

But this eaſie forme of working by Fractions ſhall you note, that if your three numbers be Fractions, for an apt work and certaine, multiply the numerator of the firſt number in the queſtion, by the Denominator of the ſecond: And all that againe multiply by the Denominator of the third number, and the totall thereof ſhall you keep for to bee the Diviſor. Then multiply the Denominator of the firſt number by the numerator of the ſecond, and the whole thereof by the numerator of the third, and the totall thereof ſhall be your Dividend.

Now diuide this dividend by the Diviſor which you found out before, and that number ſhall be the fourth number of the queſtion which you ſeek for, as in this example.

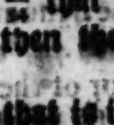
A queſtion of velvet.

If $\frac{2}{4}$ of a yard of Velvet coſt $\frac{2}{3}$ of a ſoveraign, eſteemed at 20 ſhillings, what ſhall $\frac{3}{8}$ coſt?

Scholar. If it pleaſe you to let mee make the anſwere, I would firſt place theſe three numbers as I learned in whole numbers, thus: $\frac{1}{4}$ $\frac{1}{6}$ Z $\frac{2}{3}$

And then according to your new rule, I muſt multiply 3, being numerator in the firſt number, by 3. the denominator of the ſecond: and thereto commeth 9, which I multiply againe by 6, the denominator of the third number, and ſo have I 54, which I keep for the diviſor. Then multiply I 4 the denominator of the firſt, by 2 the numerator of the ſecond and there ariſeth 8, which againe I multiply by

by 5 the numerator of the third, and it maketh 40. Then muſt I divide 40 by 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$ in leſſer termes, and then the figure will ſtand thus:



But what that is in money I cannot tell, except I ſhall worke it by Reduction, as you taught me.

Maſter. It ſeemeth not now, you may reduce it when you liſt, but it were diſorderly done here to mingle divers workes together, where we do not ſeek the value of the thing in common money, but in apt number, which you have well done: and therefore will I yet ſhew you another like way of eaſineſſe in worke, how you may change your three Fractions into three whole numbers, by which you ſhall worke, as if the queſtion were propoſed in whole numbers. The firſt number you ſhall finde as I taught you: now to finde the Diviſor of the ſecond number, take the numerator of the ſecond Fraction, and for the third number, take that that ariſeth of the multiplication of the Denominator of the firſt, by the numerator of the third, and then worke your queſtion.

Scholar. *For example hereof, I put this queſtion. If $\frac{11}{12}$ of 1 pound weight of ſilver, be worth $\frac{12}{13}$ of a Soveraigne, what is $\frac{1}{12}$ of 1 pound weight worth?* A queſtion of ſilver.

For the anſwer, firſt I place the Fractions in order thus:



Then to turne these Fractions into whole numbers, I multiply 11, which is the Numerator of the first by 4 (the denominator of the second) and there commeth 44, which I multiply by 2 the denominator of the third, and so amounteth 88, which I set for the Divisor in the first place. Then in the second place I set 12, which is the Numerator of the second Fraction, and in the third place, I set the sum that amounteth of 12 being the denominator of the first number, multiplyed by one, being Numerator in the third number, and so the figure will stand as here you see.

88 Z 12
12

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I divide by 88, and the quotient will be $1\frac{56}{88}$, or in lesser termes, $1\frac{7}{11}$, and then the figures will stand thus:

$\frac{11}{12}$ Z $\frac{12}{4}$
$\frac{1}{3}$ $1\frac{7}{11}$

The proof of the golden rule.

Master. These two formes now you understand well enough, and as for any other at this time I will not repeat, onely this shall you mark for the proof of this Rule, whether your worke be well wrought or no. Multiply the first number by the fourth, and note what amounteth; then multiply the second by the third, and mark what amounteth also. Now if these two numbers so amounting be equall, then is your worke well done, else you have erred. And this shall suffice for the former Rule.

The

The Backer Rule, or Reverſe Rule in Fractions.

RUle in the Backer Rule, this ſhall you note for eaſe of worke, that you multiply the numerator of the firſt by the numerator of the ſecond, and the whole thereof by the Denominator of the third, and that amounteth thereof, ſhall bee the dividend. Then multiply the denominator of the firſt, by the denominator of the ſecond, and that whole by the numerator of the third, and that that ariſeth thereof, ſhall be the diviſor. Example of this.

The backer Rule in Fractions.

Note this alſo for a generall Rule.

I did lend my friend $\frac{1}{4}$ *of a* Porteguiſe, *ſeven Moneths upon premiſe that he ſhould do as much for me again; and when I ſhould borrow of him, he could lend me but* $\frac{1}{12}$ *of a* Porteguiſe: *now I demand how long time I muſt keep of his money in juſt recompence my loane, accounting* 13 *Moneths in the yeare?*

A queſtion of Loan.

Scholar. The firſt number muſt be the firſt money borrowed, that is $\frac{1}{4}$ of the Porteguiſe: the ſecond number the 7 moneths, that is $\frac{7}{13}$ of a yeare: and the third number the money that was lent in recompence, that is $\frac{1}{12}$ of a Porteguiſe: then I ſet the numbers thus:

$\frac{1}{4}$ $\frac{7}{13}$ $\frac{1}{12}$

39 3 Then

Then (as you taught me) I multiply 3 (being Numerator in the firſt number) by 7 the numerator of the ſecond number, and it maketh 21, which I multiply by 12 the Denominator of the third, and ſo have I 252 for the dividend: then I multiply 4, the Denominator of the firſt, by 13 the Denominator of the ſecond, and it yeelds 52, which I multiply again by 5, the Numerator of the third, and it will make 260, that is the Divisor. Then I divide 252 by 260, ſo it will be in the ſmall fraction $\frac{252}{260}$ of a year.

Maſter. And thus do you ſee ſome eaſe in working, better then to multiply and divide tediouſly ſo many Fractions.

Statute of Aſſiſe of bread and ale.

Another queſtion yet will I propoſe, to the intent you may ſee thereby the reaſon of the Statute of Aſſiſe of Bread and Ale, which in all Statute Bookes, in Latine, French and Engliſh is much corrupted for want of knowledge in this Art; for the right underſtanding whereof, I propone this queſtion.

Queſtion of bread.

When the price of a quarter of Wheat is 2 ſhillings, the farthing white loafe ſhal weigh 68 ſhillings; then I demand what ſhall ſuch a loafe weigh, when a quarter of Wheat is ſold for 3 ſhillings.

Scholar. This queſtion muſt be wrought as it is proponed in whole numbers: and not in Fractions.

Maſter. You ſeem to ſay reaſonably, howbeit in the Statute of Aſſiſe, the rate is made by

by the proportion of parts in a pound weight Troy, else could it not be a Statute of any long continuance, seeing the shillings doe change often, as all other moneyes doe: but this Statute being well understood, is a continuall Rule for ever, as I will anon declare by a new Table of Assise, converting the shillings into ounces, and parts of ounces.

Wherefore here by a shilling you must understand $\frac{1}{20}$ of a pound weight, and so by a penny $\frac{1}{240}$ of an ounce: wherefore although you might worke this question proposed by whole numbers well enough, for that thus when the Statute was made, yet to apply it to your time, and make it serve for all times generally, it is best to worke it by Fractions, setting for 2 shillings $\frac{2}{20}$, and for 68 shillings $\frac{68}{20}$, and so for three shillings $\frac{3}{20}$, and then will the figure of the question stand thus.



In which question, because all the Denominators be like, you shall worke onely with the Numerators.

Scholar. Then shall I multiply 68 by 2, whereof commeth 136, which if I divide by 3, the quotient will be $45\frac{1}{3}$: but how shall I make a Fraction of that, to stand with the other?

Master. Have you so soon forgotten what was taught you so lately? this is his forme.

$45\frac{1}{3}$
20

Scholar. I remember it now, and then it signifieth 45 twenty parts, and the third deale of one twenty part.

Note what a shilling is.

Master. So is it that maketh in shillings 45 shillings 4 pence, whereby you may note one great errour in the Statute Books; which have constantly 48 shillings in that Assize. And by this Rule, if you examine the Statute, you shall finde many summes false. Wherefore for the true understanding of that Statute, and such like, as I have made mention of it, and somewhat recognized it, so do I wish that all Gentlemen and other Students of the Lawes would not neglect this Art of Arithmeticke, as unneedfull to their studies. Wherefore to encourage them thereto, and to gratifie both them and all other in generall, I will exhibite a Table of that part of the Statutes in two Columnes, and in a third Columne, I will adde the correction of those errours which have crept into it.

Here followeth the Table.

The price of a quarter of Wheat.		The weight of a farthing white loafe by the Statute Bookes.			The Correction by just Assise.		
s.	d.	l.	s.	d.	l.	s.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4	2	14	5 4/5
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 2/5
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/5
5	0	1	8	2 1/2	1	7	2 2/11
5	6	1	4	5	1	4	8 1/22
6	0	1	2	8 1/2	1	2	8
6	6	1	0	11	1	0	11 2/11
7	0	0	16	1 1/2	0	19	5 4/7
7	6	0	15	0	0	18	1 2/5
8	0	0	14	0 1/4	0	17	0
8	6	0	13	0 1/4	0	16	0
9	0	0	12	7 1/2	0	15	1 1/5
9	6	0	12	11 1/2	0	14	3 11/19
10	0	0	12	4 1/4	0	13	7 1/7
10	6	0	11	10	0	12	11 1/8
11	0	0	11	4	0	12	4 4/17
11	6	0	11	0	0	11	9 11/21
12	0	0	10	8	0	11	4

In the common Bookes there is no further rate of Aſſiſe made, then into 12 s. the quarter of wheat, but in an ancient Copy of 200 years old (which I have) there is added the rate of Aſſiſe unto 20 s. the quarter, but yet was that Aſſiſe alſo either wrong caſt at the firſt penning, or elſe corrupt ſince that time, for lacke of juſt knowledge in the Rule of Proportion, *which I will adde here alſo to gratifie ſuch as be deſirous to underſtand truth exactly.*

The price of a quarter of Wheat.		The weight of a farthing white loafe by the Statute Bookes.			The Correction by juſt Aſſiſe.		
S.	D.	l.	S.	D.	l.	S.	D.
12	6	0	11	0	0	10	10 $\frac{14}{25}$
13	0	0	11	0 $\frac{3}{7}$	0	10	9 $\frac{2}{13}$
13	6	0	10	1 $\frac{1}{3}$	0	10	8 $\frac{2}{9}$
14	0	0	9	7	0	9	8 $\frac{4}{7}$
14	6	0	9	2 $\frac{1}{2}$	0	9	4 $\frac{16}{29}$
15	0	0	9	1 $\frac{1}{3}$	0	9	0 $\frac{4}{7}$
15	6	0	9	1 $\frac{1}{4}$	0	8	9 $\frac{7}{31}$
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	2 $\frac{10}{11}$
17	0	0	8	5	0	8	0
17	6	0	7	10	0	7	10 $\frac{2}{35}$
18	0	0	7	6	0	7	6 $\frac{2}{3}$
18	6	0	7	3	0	7	4 $\frac{8}{37}$
19	0	0	7	2	0	7	2 $\frac{12}{19}$
19	6	0	5	10	0	6	11 $\frac{2}{13}$
20	0	0	5	6	0	6	3 $\frac{1}{3}$

These two Tables I have set severall, because no man should thinke that I would either adde or take away from any Law those parts which might of right seeme either superfluous, either diminute: but yet I may not bee so curious as to neglect manifest errours, which is not onely my part, but every good Subjects duty with sobriety to correct. And for abundance of assurance, I have rather done it in this private Booke, than in any Booke of the Statutes it self, trusting that all men will take it in good part.

Scholar. I would wish so, but I dare not so hope, sith never good man that would reforme errour, could escape the venemous tongues of envious detractors, which because they either cannot or list not to doe any good themselves, doe delight to bark at the doings of other, but I beseech you to stay nothing for their perverse behaviour.

Master. I consider many things that some may object, whereunto I am not unprovided of such answers, but I will not seem so hasty to make the answers before I heare their Objections, but as I trust that men are of a better nature, and more gratefull now then some have beene in times past. As I have done in the Statute of Assise for bread in rate of shillings, so will I set forth the like Table in pounds and ounces, and the parts thereof, that it may be easily applyed to all times: But I meane not by this to alter any word of the Statute.

Concerning the following Tables.

Statute, (being ſo good an Ordinance, and of ſo great continuance) but onely to make it as a kinde of expoſition and declaration of the ſaid Statute, truſting that thereby the Statute may be better underſtood, and conſequently better put in execution. And here you ſhall note, that I have accounted the ſhillings after the rate of 60 ſhillings to the pound weight, becauſe I eſteem it the moſt apt for our time. Wherefore in the firſt Columne you ſette the price of Wheat directly againſt it; in the ſecond Columne, you may finde the weight of a farthing white loafe, in this our time: and if you double the number (as I have done in the third Columne) then have you the weight of the halfe penny white loafe; and ſo in the fourth Columne) to ſet the weight of a penny white loafe. It needeth not to tell, that the ſight doth ſettle, how that every Columne is parted into three ſmaller pillars, whereof the firſt Columne hath theſe three titles, pounds, ounces, and penny weight. And as in the firſt Columne 12 pence make a ſhilling, and 20 ſhillings make a pound, ſo in the other three Columnes 20 pence weight maketh an ounce, and 12 ounces doe make a pound.

A pound weight.

Gentle

Gentle Reader, touching the underſtanding of the *Table* following, wherein according to our time, *Maſter Record* alloweth 60 *pence* to the *ounce*, and 3 *pound* or 60 *ſhillings* to the *pound*, and thereupon after the rate of 60 *ſhillings* to the *pound Troy*, doth he frame or produce this his *Table*, beginning at 3 *ſhillings the quarter*, till he come to 40 *ſhillings* 6 *pence the quarter*. And this his proportion (for that he hath not ſet down any one *Example* to continue the work) hath been hurd for many to conceive or comprehend, and therefore the onely chief cauſe why I have written this *digreſſion*, for the better underſtanding of him therein.

The firſt thing therefore that is ſought for in this *Table*, as in the other aforeſaid, is a *Maxime* grounded upon the *Statute*, which is this. When the *quarter of Wheat* is ſold for *two ſhillings*, then the *farthing white loafe* ſhall weigh 68 *ſhillings*, whereby a *ſhilling* is meant $\frac{1}{20}$ of a *pound*, and by a *penny*, $\frac{1}{20}$ of an *ounce*. Now therefore for a generall Rule, to finde what weight the *farthing white loafe* ſhall weigh at 3 *ſhillings the quarter*, till you come to 40 *ſhillings* 6 *pence the quarter*, is thus to be wrought. Comming to the *firſt ground*, and working by the *Backer Rule*, ſay; If *two ſhillings the quarter* give, or allow the *farthing white loafe* to weigh 68 *ſhillings*, what weight ought the *farthing white loaf* to weigh at 3 *ſhillings the quarters*? Worke, and you ſhall find 45 *ſhillings* 4 *pence*, as before in the correction of the *firſt*

Table

Table is noted. Then for the *second work*, say by the *Rule of Three Direct*, if 20 *pence* giue one *ounce*, what giueth 45 *shillings* 4 *pence*? multiply and diuide, and you shall finde 544 *ounces*, which 544 *ounces* being multiplyed by 3, for 3 *pounds*, or 60 *shillings*, yeeldeth 1632 *ounces*, which diuided by 20, produceth 81 *ounces*, and $\frac{12}{20}$ or rather $\frac{1}{2}$ of an *ounce*, equall unto 12 *penny weight*, which is *halfe an ounce*, and 2 *penny weight*, and so maketh in all 6 *pounds* 9 $\frac{1}{2}$ *ounces* and 2 *penny weight*. Now the next way to continue this *Table*, to know the weight of the halfe *penny white loafe*, is thus, multiply 1632 *ounces* by 2, and it bringeth forth 3264 *ounces*, and diuided by 20, it yeeldeth 163 *ounces* and $\frac{1}{5}$, which is equall to 13 *pounds* 7 *ounces*, and 4 *penny weight*, as M. *Record his Table* noteth.

Thirdly, for the weight of the *penny white loafe*, multiply 1632 *ounces* by 4, and diuide by 20, and after by 12 as before, and you shall finde 27 *pounds* 2 *ounces*, and 8 *penny weight*, &c. This *Method*, or else by doubling the *farthing white loafe*, for the *weight of the halfe penny white loafe*, and so doubling the *halfe penny white loafe*, for the *weight* of the *penny white loafe*, is the order to continue the *Table* to the end thereof.

The

The price of a quarter of Wheat.

li.	s.	d.
0	3	0
0	4	6
0	6	0
0	7	6
0	9	0
0	10	6
0	12	0
0	13	6
0	15	0
0	16	6
0	18	0
0	19	6
1	1	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	0	0

The weight of a farthing white loafe.

Po.	ounc.	Penny weight.
6	9	12
4	6	8
3	4	16
2	8	12 $\frac{8}{5}$
2	3	4
1	11	6 $\frac{6}{7}$
1	8	8
1	6	2 $\frac{4}{3}$
1	4	6 $\frac{8}{9}$
1	2	16 $\frac{16}{11}$
1	1	12
1	0	11 $\frac{11}{13}$
1	0	4 $\frac{8}{5}$
1	0	17 $\frac{4}{3}$
1	0	4
1	0	12
1	0	1 $\frac{1}{2}$
1	0	11 $\frac{22}{19}$
1	0	3 $\frac{2}{5}$
1	0	15 $\frac{6}{7}$
1	0	8 $\frac{8}{11}$
1	0	1 $\frac{12}{21}$
1	0	16
1	0	10 $\frac{1}{25}$
1	0	5 $\frac{1}{13}$
1	0	0 $\frac{2}{45}$



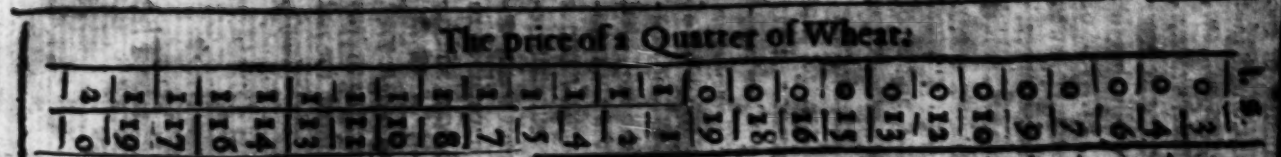
The price of a quarter of Wheat.

The weight of a half penny white loafe.

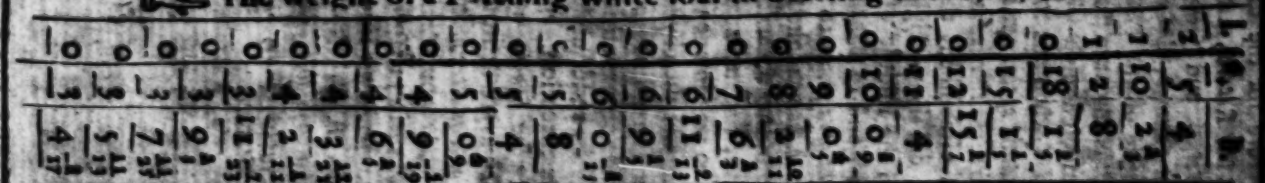
The weight of a penny white loafe.

HAving spoken before for the understanding of the Table placed by M. Record, a man indued with rare knowledge in Arithmeticall and Geometricall proportions, touching the Statute of Coynage, and the Standard thereof, as appeareth in his Epistle of this Book dedicated to King Edward the sixth, intimating unto his Highnesse that the Standard of Coyne is much alsered from the 14. yeare of King Edward the third (when this Statute and Assise was confirmed) to the Standard of this our time. But it appeareth that in King Edward the thirds time, when the Assise of Bread and Drinke was established, that a Sterling penny, round without clipping, did then weigh 32 cornes of wheat drie, and taken out of the middle of the eare, and 20 of these pence made an ounce, and 12 ounces made a pound Troy. And so from the weight of a penny to 20 shillings sterling, which then weighed 12 ounces, tooke Bread his weight and proportion? And now finding 60 pence is an ounce. This onely cause (I perceive, for the zeale of a Common-wealth) moved him to set downe the same Table in this private booke, meaning not thereby to alter any word of the Statute being so good an ordinance and of so long continuance, but as a kinde of exposition by the way, that thereby the Statute may be better understood, and so consequently better put in execution. Which Assise of his, is three times greater then the Statute now alloweth. Therefore also, to gratifie such as are desirous of knowledge, according to these prices of a quarter of wheat) I have added to this Author these three other new Tables following, and reduced their prices into their just proportions of Sterling money, and also reduced the money into known weight Troy, according to the Statute. And thereafter according to proportion in my other three Tables, have I noted the just weight that a Farthing, Half penny, and Penny white-Loafe, ought to weigh by the Statute.

The price of a Quarter of Wheat:



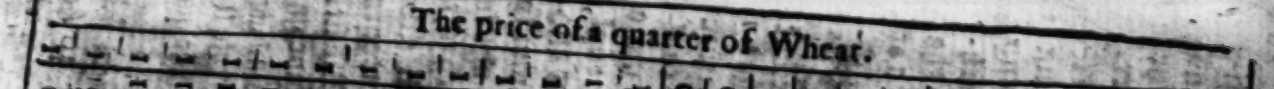
☞ The weight of a Farthing white loaf in Sterling money by Assise.



The weight of a farthing white loaf in Troy weight by Assise.



The price of a quarter of Wheat.



The price of a quarter of Wheat.

li.	s.	d.
0	3	0
0	4	0
0	5	0
0	6	0
0	7	0
0	8	0
0	9	0
0	10	0
0	11	0
0	12	0
0	13	0
0	14	0
0	15	0
0	16	0
0	17	0
0	18	0
0	19	0
1	0	0
1	1	0
1	2	0
1	3	0
1	4	0
1	5	0
1	6	0
1	7	0
1	8	0
1	9	0
1	10	0
1	11	0
1	12	0
1	13	0
1	14	0
1	15	0
1	16	0
1	17	0
1	18	0
1	19	0
2	0	0

The weight of a half penny white loafe in Troy weight by Affife.

lb.	ounc.	penny weight.	grain.
4	6	8	
3	0	5	1/3
2	3	4	
1	9	15	1/3
1	6	2	2/3
1	3	10	2/7
1	1	12	
1	0	1	1/9
0	10	17	1/3
0	9	17	8/11
0	9	1	1/11
0	8	7	1/11
0	8	3	1/3
0	7	5	4/11
0	6	16	
0	6	8	
0	6	0	8/9
0	5	14	40/99
0	5	8	8/11
0	5	3	22/33
0	4	19	
0	4	14	16/33
0	4	11	1/3
0	4	7	1/33
0	4	3	9/33
0	4	0	16/33

The weight of a half penny white loafe in Troy weight by Affife.

lb.	ounc.	penny weight.	grain.
9	0	16	
6	0	10	2/3
4	6	8	
3	7	10	2/11
3	0	5	1/3
2	7	1	1/7
2	3	4	
2	0	3	2/9
1	9	15	1/3
1	7	15	2/9
1	6	3	2/11
1	4	14	4/11
1	4	0	2/3
1	2	10	4/11
1	1	12	
1	0	16	
1	0	1	2/9
0	11	9	4/11
0	10	17	1/3
0	10	7	4/11
0	9	18	
0	9	9	4/33
0	9	2	1/3
0	8	14	8/11
0	8	7	4/33
0	8	1	3/33

Scholar. Sir, I doe thanke you most heartily for this, not onely in mine owne name, and in the name of all students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in Bread, which long time hath been abused. And if you know any like thing more, wherein you would vouchsafe to declare the errours, and set forth the truth, you cannot but obtain great thankes of all good hearted men that love the Commonwealth.

Maister. I have sundry things to declare, but I have reserved them for a private Booke by it self, yet notwithstanding because the statute of the rate of measuring of grounds is so common, that it toucheth all men, and yet no more common then needfull, but so much corrupt, that is, too farre out of all good rate; not onely in the English Bookes of statutes, commonly Printed, but also in the Latine Bookes, and in the French also, (for I have read of each sort, and conferred them diligently) I will give you a Table for the restitution of those errours, as may suffice for this present time. And first I will propose one question to you touching the use of that statute, whereby you may perceive the order how to examine the whole statute, and every parcell thereof, and the question is this.

A question of measuring of ground.

When the Acre of ground doth containe foure Perches in breadth, then must it contain 40 Perches

ches in length. Then do I demand of you, how much shall the length of an Acre be, when there is in the breadth of it 13 Perches. But before you shall answer to this question, I will declare unto you another Statute, which is the ground of the former Statute. And that Statute is this.

It is ordained, that three Barly-cornes dry and round, shall make up the measure of an inch: 12 inches shall make a foot, and 3 foote shall make a yard, (the common English books have an Eloe) five yards and a halfe shall make a Perch, and 40 Perches in length, and 4 in breadth, shall make an Acre. This is that Statute, wherebye you may perceive, that the intente of the Statute is, that one Acre should containe 160 square Perches. Now let me hear you answer to the question. Statute measure. An Acre.

Scholar. As I perceive by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an Acre, so could I make those numbers all in fractions, and so worke the question: but seeing I may doe it also in whole numbers, I take that forme for the most safe: therefore have I set the question in forme.

Then doe I multiply 40 by 4, and it maketh 160, which I divide by 13, and the Quotient is $12\frac{4}{13}$.

$$\begin{array}{cc} 4 & 40 \\ 13 & Z \\ & 12 \end{array}$$

Maister. Now turne that $\frac{4}{13}$ into the common parts of a Perch, as they be named in the former Statute: Howbeit it shall bee best to take one of the least parts in Denomination

for avoyding of much labour, as Feete in breed the Perch containeth 16 ½.

Scholar. Then to return $\frac{1}{12}$ into Feet, I multiply 16 ½ by 4, and it maketh 66. which I must divide by 12, and the Quotient is 5 $\frac{6}{12}$.

Master. So I find, that if the Acre hold in breadth 12 Perches, it shall containe in length 12 Perches, 5 Foot, and $\frac{6}{12}$ of a foote, which is not fully an Inch, for the Inch is $\frac{1}{12}$ of a foote. But here all the Statute Bookes in Latine and English (that I have seene) doe note it to be 13 Perches, 5 foote, and one Inch, which maketh above 13 Perches, too many in the Acre: so that I would have thought the errour to have crept into the Printed Bookes, by the great negligence that Printers in our time doe use, save that in written copies of great antiquity, I doe finde the same; yet have I one French copy which hath 12 Perches ½ and one foote, and that misseth very little of the truth.

Note this errour

Scholar. Then I see it is true that I have often heard say, that the truest Copies of the Statutes, be the French copies.

Master. That is often true, but not generally, as I have by conference tried diversity: but in this Statute the French Booke is most corrupt: in all other places lightly.

But now to performe my promise, I will set forth the Table for measuring of an Acre of ground, onely by such parts as the Statute doth mention, because at this time I doe of purpose

purpose to) ſite it for the better underſtanding of that Statute, and hereafter with other things intend to ſet forth this ſame more at large.

In this Table *following, I have not done as in the other* Statute *before compared by reſtitution with the faults crept into the* Statute, *but onely have written that true meaſure, which the equity of the* Statute *doth pretend. For it were vile to judge of ſo noble Princes and worthy Counſellors, as have authoriſed and ſet forth this* Statute, *that they would make one* Acre *in any forme greater then another, but every one to be juſt and equall with each other, which is the ground alſo of my worke: and hereby may all men perceive how needfull* Arithmetick *is to the* Students *of Law. But now I think beſt to make an end of theſe matters for this preſent time, ſith the* Table *hath in it none obſcurity that I ſhould need to declare.*

The breadth of the Acre.	The length of the Acre			
Perches	Perches	Feete.	Inches.	Parts of an Inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	12/13
14	11	7	0	6/7
15	10	11	0	0
16	10	0	0	0
17	9	6	9	2/17
18	8	14	8	0
19	8	6	11	12/19
20	8	0	0	0
21	7	10	2	4/7
22	7	4	6	0
23	6	15	9	2/23
24	6	11	0	0
25	6	6	7	1/5
26	6	2	7	6/13
27	5	15	3	1/3

The

The bredth of the Acre.	The length of the Acre.			
Per che	Perches	Feet.	Inches.	Parts of an Inch.
28	5	11	9	$0\frac{8}{7}$
29	5	8	6	$0\frac{12}{29}$
30	5	5	6	0
31	5	2	7	$0\frac{28}{31}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$0\frac{17}{17}$
35	4	9	5	$0\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$0\frac{1}{37}$
38	4	3	5	$0\frac{12}{19}$
39	4	1	8	$0\frac{4}{13}$
40	4	0	0	0
41	3	14	10	$0\frac{28}{41}$
42	3	13	4	$0\frac{8}{21}$
43	3	11	10	$0\frac{24}{43}$
44	3	10	6	0
45	3	9	2	0

Scholar.

Scholar. Indeed Sir, I understand the Table (as I thinke) by those other which you set forth before. For in the first columne to set the Perches of the breadth of an Acre, and then in the two Columnes following appeareth how many Perches, and how many foote that same Acre must have for his length.

Master. You take it well: howbeit to speake exactly of breadth and length, and the first Columne doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limit his length, and the shorter side doth betoken the breadth, yet it is no great abuse in such Tables, where a man cannot well change the Title, to let the name remaine, although the proportions of the numbers doe change: for still by the first Columne is expressed the measure of the one side, and by the two other Pillars in one Columne, to set forth the measure of the other side. And this shall be sufficient now for the use of the Golden Rule.

The Rule of Fellowship.

NOw somewhat will I touch certaine other Rules, which for their severall names may seeme divers Rules, and distinct from this, but indeed they are but branches of it: yet because they have severall workings in appearance, but also

also pleasant in use, I will give you a taste of each of them. As for the Rule of Fellowship, both single and double, with time and without time, I shall need to say little more then I have already said in teaching the workes of whole numbers: yet an example or two will we have to refresh the remenbrance of the same, and to declare certain proper uses and applications of it, as this for one.

The Rule of Fellowship without time.

Foure men got a booty or prize in time of warre, the prize is in value of money 8190 pound, and because the men be not of like degree, therefore their shares may not be equall: but the chiefest person will have of the booty the third part, and the tenth part over: the second will have a quarter, and the tenth part over: the third will have the sixth part: and so there is left for the fourth man a very small portion, but such is his lot (whether he be pleased or wroth) he must be content with one 20 part of the prey: Now I damand of you what shall every man have to his share?

A question of Inequall society.

Scholar. You must be fain to answer to your owne question, else it is not like to be answered at this time.

Master. The forme to understand the solution of this question, and all such like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be parts of some other of them, for all such parts you may overpasse and take for them all those numbers, whose parts they be: as in this example the shares be these $\frac{1}{3}\ \frac{1}{12}\ \frac{1}{4}\ \frac{1}{10}\ \frac{1}{6}\ \frac{1}{19}$ (if

I multiply all the Denominators together, beginning with 3, and so goe on unto 20, it will make 144200: but considering that 3 is a part of 6, I will omit that 3, and likewise 10, which is a part of 20, I may overpasse also, and then is there but 3 Denominators to multiply, that is 4, 6, and 20 which make 480, which summe I take for my worke, because all the Denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ and $\frac{1}{10}$, the $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48, which I put in one summe for the first mans share, & it maketh 208. Then for the second mans share, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must have $\frac{1}{8}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole summe: so that if the whole prey had beene but 480 pound, then were the question answered: but because the summe was of greater value, by this meanes now shall I find the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number of that I found by multiplying the Denominators, and in the second place the summe of the booty, And look what proportion is betweene the first number and the second, the same proportion shall be between the parts of that first number, and the parts of the second, comparing each to his like, Therefore I must put in the third

The reason of this rule.

third place, one of the parts or shares, and then worke by the former Rule of Proportion or Golden Rule. And because I have 4 severall parts of the first number, by which I would finde out foure like parts of the second number, therefore must I make foure severall figures.

Scholar. Now I trust I can answer to your question, as by your leave I will prove:

And to trie it, I set the foure figures thus, marked with A, B, C, D, to shew their order.

A

480 / 208 Z 8190

B

480 / 160 Z 8 90

C

8 0 / 80 Z 8190

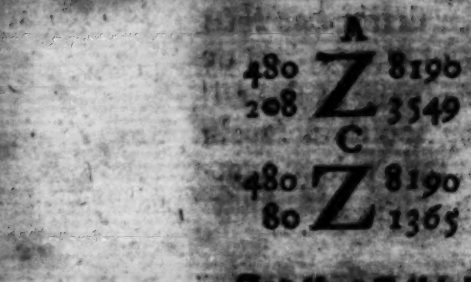
D

480 / 24 Z 8190

And then in each of them I multiply the second number by the third, and divide their totall by the first, and so amounteth the fourth summe which I seek for: For if I doe multiply 8190 by 208, it maketh 1703520, which being divided by 480, maketh in the Quotient 3542 for the first mans portion.

And so working with the other three figures, I finde for the second man 2866 $\frac{1}{2}$, and for the third man 1365, and then for the fourth man 409 $\frac{1}{2}$, and so every mans share is set forth in the figure here annexed.

A



And thus I thinke I have done well.

The proof by Addition.

Maſter. If you miſdoubt your working, and liſt to prove it, adde all the ſhares together; and if they make the totall, then ſeemeth it well done.

Scholar. I may ſet them thus: and then by Addition the juſt ſumme doth amount, that is, 8190, and therefore (as you ſay) it ſeemeth to be well wrought.

	3549
	2866½
	1365
	409½
	8190

But I beſeech you, is there any doubt in this triall, that you uſe that word, ſeemeth?

Maſter. You may eaſily conjecture, that if you did aſſigne the firſt mans ſhare to the laſt, and ſo change all the reſt, and one had anothers ſhare, yet would the Addition appeare all one, and therefore is not the proofe exact.

But if you will make a juſt proofe for the firſt mans part, take $\frac{1}{3}$ and $\frac{1}{10}$ of the whole ſumme, and if it agree with the number in the figure, then it is well done. And ſo doe for the ſecond, third, and fourth ſummes, and this proofe faileth not. Now will I propound certaine other queſtions, which have beene ſet forth

The juſt proofe.

forth by certain learned men, albeit not without some ouersight, which questions I purpose heartily, I do not repent to depraue those good men, whose labours and studies I much praise and greatly delight in. But onely according to my profession, to seeke out truth in all things, and to remoue all occasions of errour as much as in me lieth: and for that cause I will onely name the Questions without hurting the Authors name.

The first question is this.

Foure men did build an house, which cost them 3000 crownes, their shares were such that one man should pay ½ of the summe, and sixe crownes ouer: the second should pay ⅓ and 12 crownes ouer; the third man must lay out ¼ abating 8 crownes: and the fourth man should pay ⅕ and 20 crownes more. Can you answer to this question? A question of building

Scholar. No, I cannot sir, and that you know best of any man, for I know no more then you have taught me.

Master. Then I dare say you cannot doe it, neither yet the best learned man that ever did propose it: for the question is impossible. For declaration whereof, I will be bold to vse first the representation of the Numbers in their aptest forme, (although I have not yet taught that manner of worke) because it may appeare plainly that the question is not possible. For here I have set the parts, and added them, and they make the whole summe, and ½ and 30 more. Now, how is it possible An impossible question.

to

to divide truely either gaines either charges. So that the particulars $1\frac{1}{2}$ shall be more then the totall?

$\frac{1}{2}$ + 6
$\frac{1}{3}$ + 12
$\frac{1}{5}$ 8
$\frac{1}{4}$ + 20
$1\frac{1}{2}$ + 30

Scholar. It is against the forme of proofe by addition of parts.

Maister. You say truth. And (because you shall perceive it the better) I will trie it after the vulgar forme, as in this figure you see where the $\frac{1}{2}$ with 6 over, is 1506, for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{1}{5}$ would be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one summe, doe make 5280, where the totall summe should be but 3000, which summe of 3000 if you divide by $\frac{1}{4}$, you shall have $\frac{1}{4}$ of it, that is 1250, and thereto adde 30 more, then will both these sums make 5280: whereby you may see how this forme (as well as the other) doth declare that the particulars in that question would make more than the whole summe by $\frac{1}{4}$ and 30 more, and therefore can that question not be accepted as a possible thing, but yet doe certaine learned men propound such questions, and answere to them:

	1506
	1012
	1992
	770
	5280
	2000
	2250
	5280

There-

Therefore somewhat to say to their excuse (rather of their good meaning, then for their doing) I will anone declare what may bee said for their defence: but in the mean season, I will propound the Question as it may bee wrought by good possibility.

As if foure men build a house together, and it cost them 3000 Crowns, and then for the partition they agree thus: that as often as the first man doth pay 6 Crowns, so often the second must pay 4, the third man 8, and the fourth man 3. Or else thus; that the first man shall pay double so much as the fourth, and the second man shall pay ⅔ of the first mans charge: the third man shall pay double so much as the second: (and these two may agree to one end) but further for their encouragement it is appointed also, that the first shall give 6 Crowns overplus, and the second 12, and the fourth shall give 20, but the third man shall give no overplus, but shall have 8 Crowns abated of his charge.

The former question of building, now possible.

Now is the Question possible to be assoiled, and this is the way to doe it. Marke the proportion of the severall charges, and set out small numbers in that Rate, by which you may reduce the worke to the Golden Rule, as here in the first forme the numbers are already named, 6, 4, 8, 3: and in the second forme (although they be but plainly named, yet they may bee the same numbers) for 6 is double to 3, and 4 is ⅔ of 6: and againe 8 is double to 4. Now adde these together, and they make 21, which 21 must be set for the first number in the Golden

den Rule: for if it with the overplus of each mans charge would make the totall summe of the charges, then were those severall summes the charges of each man, besides his overplus: but now it is not so.

But yet this is true: (so excellent are conclusions Arithmeticall) that looke what proportion each of their severall summes doth bear to 21, the same proportion doth the just charges of every man (besides his overplus) beare to the totall of the charges, the overplus being deducted: wherefore this may you note, that before you doe apply the totall of his charges to the Golden Rule, you must deduct the overplus, which is 6, 12, and 20, that is in the whole 38: but then 8 must be restored for the abatement of the third man, and then remaineth to be deducted 30: take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden Rule, for the second summe: and for the third summe, I must put each of the small numbers before mentioned, which although they be not severall charges, yet they represent them in proportion. And so making for every mans charge a severall question, the figures will be 4, which I marke with foure letters, A, B, C, D, thus.

A

21 Z 2970
6 Z 848 4/7

B

21 Z 2970
4 Z 565 5/7

C

21 Z 2970
8 Z 1131 3/7

D

21 Z 2970
3 Z 424 8/12

Where I have set for briefnesse the summe of every mans charge in the fourth place, yet supposing that you can tell how to try out the fourth summe by so many Examples as ye have had.

Scholar. As I trust that I understand this former, so I desire much to know what may bee said for them that mistooke this Question.

Master. You seem so desirous to know this errour, that you have forgotten to examine, whether this work be without fault.

Scholar. It seemeth this worke to be well done, because the Addition of the fourth several numbers doth make the totall summe of 2970, which was to be divided into such foure parts.

Master. But then have you forgotten that the first man must pay sixe crownes more because his share, and the second man 12 Crowns more, the third man 8 Crownes lesse, and the fourth man 20 Crownes more; for without these, your first totall of 3000 Crownes will not be made.

Scholar. Then must I adde to the first mans summe 6 more, and it will bee 854 $\frac{4}{7}$ and to the seconds summe, I must adde 12, and it will be 577 $\frac{1}{7}$: from the thirds summe I must abate 8, and then will the summe bee 1123 $\frac{4}{7}$: then adding unto the fourths summe

20, it will be 444 $\frac{2}{7}$, and these
foure ſumes will make 3000,
which is the whole charge,
as in this example it may
appear, where firſt I gather
the 6 $\frac{1}{2}$ that maketh 2, and ſo
proceed I in the Addition
to the end.

	854 $\frac{4}{7}$
	577 $\frac{1}{2}$
	1123 $\frac{1}{7}$
	444 $\frac{2}{7}$
	3000

Maſter. Now have you well done, and this worke in the ſame ſummes is brought of other Learned men for the true ſolution of the queſtion, as it was proponed, which (as I ſaid) was impoſſible: and now examine by theſe ſeverall ſummes, and ſee whether it doth agree with the ſummes in the Queſtion proponed.

The firſt man muſt pay $\frac{1}{2}$ and 6 over of the totall ſumme: help think you, is 854 $\frac{4}{7}$ the halfe and 6 more of 3000?

Scholar. No that it is not, for it ſhould be 1506: and for the ſecond man 1012: and for the third man 1992, and for the fourth man 770: whereof not one ſumme agreeth to this worke. But I marvell, that ſo wiſe men could bee ſo much over-ſeen.

Maſter. It is commonly ſeene, that when men will receive things from elder Writers, and will not examine the thing, they ſeeme rather willing to erre with their Ancients for company, then to bee bold to examine their workes or writings. Which ſcrupulosity hath ingendred infinite errors in all kindes of know-

knowledge, and in all civill adminiſtration, and ſo in every kinde of Art. But theſe Learned men did not mean any other thing by this queſtion, then to finde ſuch numbers as ſhould beare the ſame proportion together, as thoſe numbers in the queſtion proponed did bear one to another: which thing you ſhall perceive more plainly by another queſtion of theirs, that is this.

A queſtion of a Teſtament.

A man lying upon his Death-bed, bequeathed his goods (which were worth 3600 Crownes) in this ſort: becauſe his Wife was great with childe, and hee yet uncertaine whether the Childe were Male or Female, hee made his bequeſt conditionally, that if the Wife bare a Daughter, then ſhould the Wife have halfe his goods, and the Daughter $\frac{1}{3}$, but if ſhe were deliuered of a Sonne, then that Sonne ſhould have $\frac{2}{3}$ of the goods, and his Wife but $\frac{1}{3}$. Now it chanced her to bring forth both a Sonne and a Daughter; the queſtion is, How ſhall they part the goods agreeable to the Teſtatour his Will?

Scholar. If ſome cunning Lawyers had this matter in ſcanning, they would determine this Teſtament to be quite voide, and ſo the Man to die inteſtate, becauſe the Teſtament was made inſufficient, ſith this condition was not expreſſed in it, and alſo it might have chanced that ſhee ſhould have brought forth neither Sonne nor Daughter, as often hath beene ſeene: ſo is the Will inſufficient in that point alſo.

Master. Such Scanners should seem too cunning, and yet not so cunning as cruell: for the minde of the Testator is to be taken favorably for the aid of the Legatories, when there ariseth such doubt. But let us try this work, not by force of Law, but by proportion Geometricall, seeing the Testator did minde to provide for each sort of them.

Scholar. If the Sonne shall have $\frac{1}{2}$ by force of the Testament, so must the Mother have $\frac{1}{3}$. Again, because she hath a Daughter also, therefore ought she to have $\frac{1}{2}$, and the Daughter $\frac{1}{3}$, that is both wayes $\frac{11}{23}$. and $\frac{11}{32}$, which commeth to the whole goods, and $\frac{1}{3}$ more.

Wherefore it seemeth also impossible.

Master. In this matter the minde of the Testator is to be understood that such proportion should be between the portion of the Wife, and the Sonne, as is between $\frac{1}{2}$, and $\frac{1}{3}$, that is, the Sonne must have $\frac{1}{2}$. for $\frac{1}{3}$ to his Mother, so shall he have 3 to 2, that is as much as his Mother, and half as much more; and the Mother must have the like rate in comparison to her Daughter. Then must I finde out 3 numbers in such proportion, that the first may have as much as the second, and half as much more (that is) in proportion sesquialtera, and the second to the third, in that same proportion: such numbers be 9, 6, 4.

Scholar. I pray you Sir, how shall I finde out these numbers?

Master. That will I gladly tell you.

What

Whatsoever the proportion be of any three numbers, multiply the Termes of that prportion togeather, and the number that amounteth, shall be the middle number of the three: then multiply that middle number by the lesser term, and divide that totall by the greater, and the least number of the three will amount. So if you multiply that middle number by the greater extreame, and divide the totall by the lesser extream, then will the greatest number of that Progression amount.

To finde three numbers in any proportion.

Scholar. Then in this example to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must divide (as you taught me in Division) $\frac{1}{2}$ by $\frac{1}{3}$, and the Quotient will be $\frac{1}{2}$, that is, $1\frac{1}{2}$. whereby I perceive that the proportion in this question, is as 3 to 2. Therefore as you taught me even now, I multiply 3, by 2, and the summe is 6, which must be the middle number: then I multiply the middle number 6 by 2, which is the least tearme, and the summe is 12, that I doe divide by 3, being the greater tearme, and the Quotient is 4: so is 4 the least number of the three. Then I multiply 6 by 3, whereof cometh 18, and that I divide by 2, and so have I 9 which is the greatest number of the three.

To finde the proportion between 2 numbers.

Master. Another way yet may you finde the third number in any Progression, if you have two of them: for if the middle number be one of them which you have, then multi-

ply it by it self (as in this example, 6 by 6, maketh 36) and that totall divide by the other number which you have, and the third number will be the Quotient.

Scholar. When I divide 36 (which commeth of 6 multiplied by it self) by 4 the Quotient will be 9: and if I divide 36 by 9, the Quotient will be 4. But what if I know the first number, and the third, and would have the middle number?

Master. Multiply the 2 numbers together, and in their totall you must seek the root of that number, and it shall be the middle number: but because as yet you have not learned to extract Roots, therefore use the first forme which I have taught you, till I teach you to extract Roots. And now go forwards with the answer to the same question.

Note.

Scholar. I perceive then, that the Sonne must not have ½ of the goods, neither the Mother ⅓, nor yet the Daughter ⅓, but yet must the goods be divided into such proportion, that the Sonne shall have 9 Crownes for 6 to his Mother, and the Mother shall have 6 Crownes for every 4 to her Daughter. Then I apply it to the Golden Rule in three examples, as followeth.

Where the first number is the Addition of those three numbers 9, 6, 4: and the third is one of them severally: the second is the totall of the goods in that Testament: and then by

by the worke of the Golden Rule, I find out the fourth number in every worke: that is for the Sonne 1705 $\frac{5}{19}$, for the Mother 1136 $\frac{16}{19}$, and for the Daughter 757 $\frac{12}{19}$: the which summes added together, doe make the summe of the whole goods as may be seen by this Example.

19 Z 3600
9 Z
16 Z 3600
9 Z
19 Z 3600
4 Z

1705 $\frac{5}{19}$
1136 $\frac{16}{19}$
757 $\frac{12}{19}$
3600

And this (me thinketh) I doe perceive, because in this Case there is a necessary remedy deuised against an vrgent inconuenience: therefore those learned men thought they might vse the like liberty in that other question.

Master. Your guesse is good, but they had so good reason for them in the one, as they haue in the other: As in another example of theirs, it may better appeare, as in this.

Another question of a Testament.

A man left vnto his three Sonnes 7851 crowns to be parted in such sort, that the first Sonne should haue $\frac{1}{2}$, the second Sonne $\frac{1}{3}$, and the third Sonne $\frac{1}{4}$ which is not possible: for $\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{1}{4}$ doe make $\frac{26}{24}$, or $\frac{13}{12}$, that is, 1 $\frac{1}{12}$. so it is more then the whole: but reduce these Fractions *into one denomination, the least that they will come to, and they will*

be

be $\frac{6}{11}, \frac{4}{12}, \frac{3}{12}$, *and so may you part the goods into such proportion as these three Numerators beare together, that is, the first to have 6 for every 4 to the second, and the second to have 4 as often as the third hath 3: and so their portions will bee for the first,* 3623 $\frac{7}{13}$, *for the second* 2415 $\frac{9}{13}$, *and for the third* 1811 $\frac{10}{13}$: *and these three shares added together, will make the totall summe of the whole goods, as you may easily see in this example.*

3623	$\frac{7}{13}$
2415	$\frac{9}{13}$
1811	$\frac{10}{13}$
7851	

Another Question is there proponed thus.

Another like question.

There are 450 *crownes to bee divided betweene three men, so that the first man must have* $\frac{1}{2}$ *and* $\frac{1}{4}$, *the second man* $\frac{1}{3}$ *and* $\frac{1}{4}$; *the third man shall have* $\frac{1}{4}$ *and* $\frac{1}{5}$.

Scholar. I marvell that any man should be so overseen, to propone that question as a thing possible, fifty $\frac{12}{23}, \frac{12}{34}, \frac{12}{45}$, doe make 1 $\frac{12}{13}$, that is almost double the whole summe.

☞ But I perceive it might be thus proponed: that as often as the first man did receive 50 Crownes, so often the second man should receive 35, and the third man 27: for $\frac{1}{2}$ and $\frac{1}{4}$ is equall to $\frac{30}{60}$: and so is $\frac{1}{3}$ and $\frac{1}{4}$ equall to $\frac{35}{60}$, and $\frac{1}{4}$ and $\frac{1}{5}$ is $\frac{27}{60}$: and so working the question the three figures will appear in this forme: whereby the first mans portion is found to be 200 $\frac{12}{56}$: the second mans part is

112	Z 450	
50	Z 200	$\frac{12}{56}$
112	Z 450	
35	Z 140	$\frac{14}{56}$
112	Z 450	
27	Z 108	$\frac{22}{56}$
	140	$\frac{15}{56}$

140 $\frac{11}{36}$ the third mans ſhare 108 $\frac{25}{36}$: which in the whole doth make 450 Crownes to be divided between them.

Maſter. And thus you are (I thinke) ſufficiently inſtructed in the Rule of Fellowſhip.

The Rule of Alligation.

NOw will I goe in hand with the Rule of Alligation ; *which hath his name, for that by it there are divers parcells of ſundry prices, and ſundry quantities alligate, bound or mixed together: whereby alſo it might bee well called the* Rule of Mixture ; *and it hath great uſe in compoſition of Medicines, and alſo in mixtures of Metalls, and ſome uſe it hath in mixtures of Wines : but I wiſh it were leſſe uſed therein then it is now a dayes. The order of this rule is this.*

The rule of Mixture

When any ſummes are propoſed to bee mixed, ſet them in order one over another, and the common number (whereunto you will reduce them) ſet on the left hand ; then marke what ſummes bee leſſer then that common number, and which bee greater, and with a draught of your penne evermore linke two numbers together, ſo that one bee leſſer then the

The reaſon of this Rule.

the common number, and the other greater then hee: (for two greater or two ſmaller cannot well be linked together) and the reaſon is this: that one greater and one ſmaller, may be ſo mixed, that they will make the meane, or common number very well: but two leſſe can never make ſo many as the common number, being taken orderly: no more can two ſummes greater then the mean, ever make the mean in due order, as it ſhall appeare better to you hereafter. And as it is of neceſſity to linke every ſmaller (once at the leaſt) with one greater, and every greater with one ſmaller, ſo it is at liberty to linke them oftner then once, and ſo may there be to one queſtion, many ſolutions. When you have ſo linked them, then marke how much each of the leſſer numbers is ſmaller then the mean or common number, and that difference ſet againſt the greater numbers, which be linked with thoſe ſmaller, each with his match ſtill on the right hand, and likewiſe the exceſſe of the greater numbers above the mean, you ſhall ſet before the leſſer numbers, which be combined with them.

☞ Then ſhall you (by Addition) bring all theſe differences into one ſumme, which ſhall be the firſt number in the golden Rule, and the ſecond number ſhall be the whole maſſe that you will have of all thoſe particulars: the third ſumme ſhall be each difference by it ſelfe, and then by them ſhall be found the fourth number, declaring the juſt portion of every particular in

that

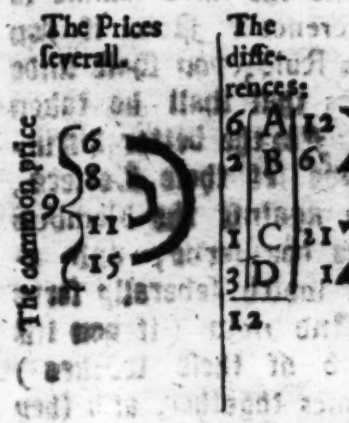
that mixture: As now by these Examples I will make it plaine.

A question of mixing of Wines.

There are foure sorts of VVine, of seuerall prices, one of 6 pence *a gallon, another of* 8 pence, *the third of* 11 pence, *and the fourth of* 15 pence *the gallon. Of all these VVines would I haue a mixture made to the summe of fifty gallons, and so the price of each gallon may be* 9 pence. *Now demand I, how much must be taken of euery sort of VVine?*

Scholar. If it shall please you to worke the first example, that I may marke the applying of it to the Rule: then I trust I shall be able not onely to doe the like, but also to see the reason in the order of the worke.

Master. Marke then this forme, and the placing of euery kinde of number in it.



Here (you see) I haue set downe the seuerall

all prices, which be 6, 8, 11, 5, and have linked together 6 with 15, and 8 with 11. The common price 9, I have set on the left side, and the difference between it, and every particular price, I have set on the right hand, not against the summe (whose difference it is) but against the summe that it is linked withall, so the difference of 15 above 9, is 6, which I have set, not against 15 but 6, that is linked with 15, and the difference between 6 and 9 (that is 3) I have set against 15. So likewise the difference between 8 and 9, is but 1, that I have set against 11; and the difference of 11 above 9 (which is 2) I have set against 8. Then adde I all those foure differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of Gallons that I should have, and the third summe is every particular difference. Now if you worke by the Golden Rule, you shall finde the number of Gallons that shall be taken of each sort of wine: For the better distinction whereof, I have set these Letters, A, B, C, D, both against the Numbers for which the workes doe serve, and ouer the worke also, which severally serue for each of them. And now (if you list to examine the truth of these workes) adde these foure summes together, and they will make 50. that is the totall which I would

The proof of this rule

would have, as by this example you may eaſily perceiue. And (for to proue how the prices doe agree) doe this: Multiply the totall ſumme 50, by the common price 9, and it will make 450: then keep that ſumme by it ſelfe, and afterward multiply euery ſeuerall ſumme of Gallons, by the price belonging to the ſame Gallons, and if that ſumme doe agree with this, which you haue kept ſtill, then is your worke well done. As here 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you ſhall ſet downe, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of Gallons, and it will make $66\frac{2}{3}$: ſo again $4\frac{1}{6}$ multiplied by 11, doth make $45\frac{5}{6}$. And laſt of all, $12\frac{1}{2}$ multiplied by 15, maketh $187\frac{1}{2}$: and theſe added together doe make 450, as in the Example annexed you may ſee, wherefore ſeeing it doth agree with the former ſumme of 50, multiplied by 9, I may iuſtly affirme this worke to be good, and well done.

25	
$8\frac{1}{3}$	
$4\frac{1}{6}$	
$12\frac{1}{2}$	
50	
150	
$66\frac{2}{3}$	
$45\frac{5}{6}$	
$187\frac{1}{2}$	
450	

And now to proue how you can doe the like, I propound the ſame Queſtion, onely willing you to vſe ſome other forme of combining or linking the ſummes.

The variation of this queſtion.

Scholar. That ſhall I proue with your fauour, and therefore I combine 8 with 15, and 6 with 11, & then the forme will be my followeth.

9



Whereby amounteth the same summe in totall of the differences as did before: and yet now the differences be altered as the combination is changed, whereof I understand the reason by your former worke. And therefore here appeareth no strange thing, but that now I have $8\frac{1}{2}$ gallons of 6 pence, and 25 gallons of 8 pence, and 12 gallons and $\frac{1}{2}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{2}$ of 15 pence: so that multiplying $8\frac{1}{2}$ by 6, it maketh 50, and then 25 multiplyed by 8, maketh 200: likewise $12\frac{1}{2}$ multiplyed by 11 yeeld $137\frac{1}{2}$, and $4\frac{1}{2}$ multiplyed by 15, maketh $62\frac{1}{2}$, which 4 summes added into one, will yield in the totall 450. which agreeth with the multiplication of 50 (being the totall summe of gallons) bee 9 the common or mean price.

50
200
$137\frac{1}{2}$
$22\frac{1}{2}$
450

Master. Seeing you conceive this worke so well, I will propound another example unto you of more variety in the Alligations or combinings, as thus:

A question of Spices.

A Merchant being minded to make a bargain for Spices, in a mixt masse (that is to say) of Cloves, Nutmegs, Saffron, Pepper, Ginger, and Almonds: the Cloves being at 6 shillings, Saffron at 10 shillings, Pepper at 3 shillings, Ginger at 2 shillings, and Almonds at 1 shilling.

Now would hee have of each sort some, to the value of 300 pound in the whole, and each pound one with another, to bee in price five shillings: How much shall he have of each sort?

Scholar. That will I try thus.

First I set downe these 6 severall prices, and at the left hand I set the common price five shillings. Then I linke them thus, 1 with 10, 2 with 6, and 3 with 8; as in the example following.



			a		b	
5	a	18	300	18	300	
1	h	5	83⅓	3	50⅔	
		b			e	
3	c	18	300	18	300	
3	g	1	16⅔	2	33⅓	
		c			f	
4	f	18	300	18	300	
18		3	50	4	69⅓	

Master. I had minded to have combined them in more variety: but I am content to see your own worke first, and then more varieties in combination may follow anon.

Scholar;

Scholar. Then to continue as I began I seek the difference between 1 and 5, (which is 4) and that I set against 10, then against 1 I set 5, which is the excesse of 10 above 5, so I gather the difference between 2 and 5, which is 3, and that I set against 6, because it is combined with 2: and likewise the difference of 6 above 5, (which is 1) I set against 2. Then take I the difference of 3 from 5, which is 2, and that I set against 8: and before that 3, I set the difference of 8 above 5, which is 3. Then gather I all these differences by Addition, and they make 18, which I set for my first Number in the Golden Rule, and so appeareth by those workes, that of Almonds I must take 83 $\frac{1}{3}$ pound, of Ginger 16 $\frac{2}{3}$ pound, Pepper 50 pounds, of Cloves 50 pounds, of Nutmegs 33 $\frac{1}{3}$ pounds, and of Saffron 66 $\frac{2}{3}$ pounds.

	83 $\frac{1}{3}$
	33 $\frac{1}{3}$
	150
	300
	266 $\frac{2}{3}$
	666 $\frac{2}{3}$
	1500

Then for triall hereof, I multiply every parcell by his severall price, as 83 $\frac{1}{3}$ which is the summe of Almonds, I multiply by one which is their price.

Also 19 $\frac{2}{3}$ the summe of Ginger, I multiply by 2. which is the price of it: and so each other in his kinde, as this Table annexed doth represent, and then adding them all together I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300, by the common price 5, wherefore

ons if appeareth well inough.

Maister. Now I will make the allegation to proue your cunning somewhat better: but because you shall not thinke your selfe greeued so much, I will also note the differences, as by this Example you may see, where I haue



A B C

alligated 1 with 6 and 8, and therefore haue I set against 1 both their differences, that is 1 and 2. Likewise, because 2 is combined with 3 and 10, I set before them their differences 3 and 5. Against 3 I haue set one, which is the difference of 3 with which 3 is combined (being 3 himselfe) to alligate to 2, and therefore is the difference of 3 there set 1. So likewise with 4 and 5, and therefore haue I set 4 against him, both their differences, and because 6 is ioyned with 5, and therefore haue I set their differences, 1 and 6. And because of reason you may see other columne I haue set the differences, vnto which I haue put numbers, for euery seuerall sort,

Yy 3 any

and have also added them together, whereby apparently that they make 33, and so consequently you see the workes of the Golden Rule set forth. For the six Drugges I have added the letters A.B.C. &c. as before.

Note. But I would not wish you to cleave still to these Elementary aids, but accustome Memory to trust her self: so shall occasion of negligence best be avoyded. And as for the proof try it at more leasure, because the time now is short, and you sufficiently instructed in that proof. And there resteth divers things behinde yet; of which I would gladly give you some taste, before your departure.

Scholar. But if it may please you to let me see all the variations of this question, before you goe from it, for me thinketh I could vary it two or three wages more yet.

Maister. I am content to see you make two or three variations: but I would be loth to stay to see all the variations that may be varied above 300 wayes, although many of them would not well serve to this purpose.

Scholar. I thought it impossible to make so many variations.

Note. Maister. Marvell not thereat, for some questions of this Rule may be varied above 1000 wayes, but I would have you forget such fantasies till a time of more leasure. And now goe forward with some variation of this question.

Scholar. For the first variation, I finde the

the first number 1 with 1 and 10, and 2 I combine with 9 and 10: then ioyne I 3 with 6, 8, and 10, as in this forme.



And so doth there appeare the proportion of weight for every kinde of Drugge in this mixture. Now for the triall.

Master. Nay stay there: you shall not need to make triall in one example so often, or if you list to doe it by your selfe, I am content. But now set forth (for declaration that you conceive the Rule) two or three examples of severall Combinations, and then will we passe to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these bee the varieties: in which, as the



combinations are ſeverall, ſo doth it plainly appear, that the differences by which the proportion of each ſeverall kinde is taken are alſo ſeverall. Marget I ſee in the three firſt of theſe five varieties, and in the one other before, the totall ſumme of the differences to be one, that is to ſay, 18, whereby I perceive that the variety of their mixture doth depend on the variety of their differences ſeverall and not of the variety of their totall ſumme.

Maſter, So is it. And ſeeing you conceive it ſo well, I will make an end of this Rule, onely exhibiting unto you one Queſtion or two of the mixture of Metals, that by it you may deviſe others like, and exerciſe your ſelfe therein alſo, becauſe the uſe of it ſerveth often

in

in busines of charge, not so much for Goldsmiths, as of coynage in Mints. First, I demand of you this question: *If a Mint-master have Gold of 22 Karects, and some of 23 Karects, some of 24: Againe, some 15, some 16, and some of 18 Karects, and would mix them, so that he might have 100 Ounces of 20 Karects: How much must he take of each sort?*

Scholar. To know that, I answer in order thus:



Master. You have wrought the question well: but how chanced you made no doubt of that new name Karect?

Scholar. Because I thought it out of time to demand such questions now, seeing you make so much haste to end: and again in this case the proportion of the number is sufficient for my purpose in this worke, trusting that another time you will instruct me as well of this, as of sundry other things, which as I have heard you talke of, so I have a great desire to them.

Master. Your answer is reasonable, and your request and trust (with Gods helpe)

I intend to ſatisfie. And now to goe forward with this matter, let me ſee your examination of this laſt work.

Scholar. Firſt for the one part I adde together all the particular ſummes, as they appeare in the worke, and they make 100, as here by their Addition doth appear.

And ſo it ſeemeth that the ſums are well gathered: but for the further triall of them, I multiply

120 firſt 20 which is the common or
240 meane ſumme of the Karects by 100,
360 which is the ſum of the whole Maſſe,
550 which I would have, and it maketh
460 2000. Then I multiply every par-
240 ticular ſumme by the Karects that it
2000 doth containe, as 10 by 15, and that maketh 150.

Likewiſe I multiply 15 by 16, and it yeildeth 240: ſo 20 by 18, maketh 360. And 25 by 22, yeildeth 550: likewiſe 20 by 23, bringeth forth 460: and laſt of all, 10 multiplied by 24, yeildeth 240: which ſummes all joined together make 2000, that doth agree with the like ſumme before, wherefore I may well ſay, that the worke is good. And now (if it pleaſe you) I would ſet forth ſome varieties of this queſtion) to prove my wit.

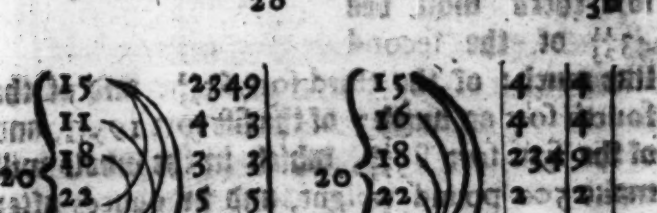
Maſter. Goe to, let me ſee.

Scholar. There be foure varieties,

And



15	3	4	7	15	2	3	5
16	3	3		16	3	4	7
18	2	2		18	4	4	
20	52	2		20	22	5	5
23	4	9		23	5	4	9
25	5	5		24	4	2	6
	28				36		



15	2	3	4	9	15	4	4
11	4	3			16	4	4
18	3	3			18	2	3	4	9
22	5	5			22	2	2
23	52	7			23	2	2
24	54	9			24	5	4	2	11
	37					32			

And more yet could I make, but not like to the number that you speak of in the variation of the other question.

Master, That will I teach you at more leisure, seeing it is a thing rather of pleasure then of any necessitie.

But now for your exercise in this Rule, one other question I will propose. A Mint-master hath sixe Ingots of silver, of sundry fineness, some of foure Ounces fine, and some of five Ounces, some of six, and other of eight, some of 11, *and other of* 12, *and his desire is to mixe* 500 *pounds weight, so that in the whole masse every pound weight should beare* 9 *Ounces of fine silver: How much shall he take (say you) of every sort of silver?*

A question of mixing of silver.

Scholar.

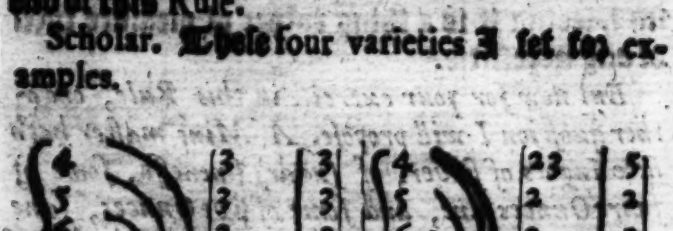
Scholar. To finde out that, I ſet the numbers thus in order.



And gathering the differences it will appear, that of the firſt ſort there muſt be $43\frac{11}{23}$ of the ſecond like much: of the third ſort $65\frac{2}{23}$: and of the fourth ſort as much: of the fift ſort $195\frac{11}{12}$, and of the fixt ſort $86\frac{22}{23}$, which in the whole will make 500 pound weight, and in ounces after 9 ounces ſtir 4500, that is of the firſt ſort $173\frac{21}{23}$, and of the ſecond ſort $217\frac{2}{23}$, of the third ſort $391\frac{7}{23}$, of the fourth ſort $521\frac{12}{23}$, of the fift ſort $2152\frac{14}{23}$, and of the fixt ſort $1045\frac{11}{23}$, which all together do make 4500 ounces, agreeable to the multiplication of 9 by 500.

Maſter. This is well done of you, therefore now make three or four varieties, and ſo an end of this Rule.

Scholar. Theſe four varieties I ſet for examples.



Maſter.



Master. And by these it appeareth, that you can finde out more, with which I will not now meddle, save only (for to shew you the easie help in working the lines of Combination) I will set forth two varieties here.



And this shall suffice now for the Rule of Alligation or mixture: for by these examples may you easily conjecture such other as do appertain to it, as well for the due working, as for variety of framing the lines of Combination.

Scholar. Sir, albeit it pleased you erewhile to put me from my minding of the many varieties that may fall in these Combinations, and

and formed them phantasies, yet my phantasie giveth me, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable: Wherefore ye may well think, that at another time convenient I will require you to aid me herein.

Master. Truth it is, that this consideration may fall in practice as well Politick as Philosophicall, and sundry waies in them be applied: Wherefore when time shall fall fit, for the discussing of this consideration, you shall not want my helping hand.

The Rule of Falshood.

The occasion of the name.

NOw will I briefly also teach you somewhat of the Rule of Falsehood, *which beareth his name, not for that it teacheth any fraud or falshood, but for that by false numbers taken at all adventures, it teacheth how to finde the true numbers you seek for.*

Scholar. So might any other Rule be called the Rule of Falshood, for they work by wrong numbers, and by them finde out the right numbers: so both the Rule of Alligation, the Rule of Fellowship, and the Golden Rule partly.

Master. In the Golden Rule, the Rule of Fellow-

Fellowship, and the Rule of Alligation, although the numbers that you worke by, be not the true numbers that you seek for, yet are they numbers in just proportion, and are found by orderly worke, whereas in this Rule the numbers are not taken in any proportion, nor found by orderly worke, but taken at all adventures.

And therefore I sometimes being merry with my friends, and talking of such questions, do call unto them such Children or fooles, as happned to be in the place, and so take their answer, declaring that I would make them solve those questions, that seemed so doubtfull.

And indeed I did answer to the question, and worke the Triall thereof also by those answers which they happened at all adventures to make: which Numbers seeing they be taken as manifest false, therefore is this Rule called the Rule of false Positions, and for briefnesse, The Rule of Falshood; which Rule for readinesse of remembrance, I have comprised in the few verses following, in form of an obscure Riddle.

Ghesse at this worke as hap doth lead,
By chance to truth you may proceed,
And first work by the question,
Although no truth therein be done.
Such falshood is so good a ground,
That truth by it will soon be found.

From many take too many more,
From too few take too few also:
With too much joyne too few again,
To too few adde too many plain:
In crosse wise multiply contrary kinde,
And all truth by falshood for to finde.

The sense of these Verses, and the summe of this Rule is this.

The exposition of the Rule.

When any *question is proposed* appertaining to this *Rule*, first imagine any number that you list, which you shall name the *first position*, and put it in stead of the true number, and then worke with it as the *question* importeth: and if you have missed, then is the last number of that worke either too great or too *little*: that shall you note as hereafter shall be taught you, and you shall call it the *first errour*.

Then begin againe, and take another number, which shall bee called the *second position*, and worke by the *question*: if you have missed againe, note the excesse or default as it is, and call that the *second errour*. Then multiply crosse-wise the *first position* by the *second errour*, and againe, the *second position* by the *first errour*, and note their totalls severally by the names of totalls: Then marke whether the two errours were *both alike*, that is to say, both *too much*, or both too little: or whether they be *unlike*, that is, the one too much, and the other too little:

little: for if they bee like, then shall you *subtract* the one totall from the other (I mean the lesser from the greater) and the remainder shall be your *Dividend*: so must you abate the lesser errour out of the greater, and the residue shall be the *Divisor*. Now divide the *Dividend* by that *Divisor*, and the *Quotient* will shew you the true number that you seek for. But, and if the errours be unlike, then must you adde both those totalls (which you noted) together, and take that whole number for the *Dividend*, so shall you adde both errours together, and that whole number shall be the *Divisor*, and the *Quotient* of that Division shall give you the true number that the *Question* seeketh for, and this is the whole Rule.

Scholar. This Rule seemeth so unlike any other, that without some example I shall not easily understand it.

Master. Then a good will: propose unto a few lumpy questions and examples of vulgarity, for the better understanding of the words hereof: and for the first, take this example.

A Mason was bound to build a wall in 40 daies, and it was covenanted with him, that every day that he wrought, he should have for his wages 2 shillings 1 penny, *and every day that he wrought not, he should be amerced* 2 shillings 6 pence, *so that when the wall was made, and the reckoning taken of the daies that he wrought, and of the other that he wrought not, the Mason had clearly but* five shillings five pence *for the worke. How*

A question of Masonry the first example.

doe

doe I demand, how many dayes did hee worke of those 40, and how many did he not worke?

Scholar. I pray you expresse the order of the worke, that I may partly by imitation, and partly by comparing it with the Rule, be able again to doe the like.

Master. This order shall you keep in the worke of this Rule: First take some number (as you list) at adventure; as for example, I say he played 12 dayes, and wrought 28 dayes. Note cast you the wages of every day, and see whether it will agree with the summe of 5 shillings 5 pence.

Scholar. The 28 dayes that he wrought after 25 pence the day, yieldeth 700 pence: Then 12 dayes that he wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Master. He had but 65 pence, and by this supposition he should have had 340: therefore is this summe too much by 275, which summe I must set below after this sort as you see here, where first I have made a crosse (commonly called S. Andrews crosse) and at the upper corner on the left hand I have set the first position 12: and at the other corner under it I have set 275, which is the first errour, with this figure † which betokeneth too much, as this line ——— plaine without a crosse line betokeneth too little.

A question of the first example.

12
X
975 †

On

On the right hand of the crosse I have left two like rooms for the second position and his error. Therefore to prosecute the worke, I suppose he played 16 dayes, and wrought 24.

Scholar. I was a while in doubt why you named the dayes of his working, seeing they bee not set in the figure: and I doubted how you knew them, or else whether that you did suppose them at all adventures; as you did the dayes that he played: but now I gather, that seeing 40 dayes is the whole time limited, then the dayes that he played being supposed, the rest of 40 must needs be the dayes that hee wrought, and therefore 28 followed 12 of necessity, and 24 followeth 16 also of necessity, but yet I scarce perceive why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take so that in the second position I take the numbers of the same nature that is here both of working dayes, or both of idle, but now ermine you this second position.

Scholar. If he played 16 dayes, then abating 16 times 30 pence, the sum will bee 480 pence, and for 24 dayes that he wrought, every day yeelding 25 pence, the totall is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therefore it is too much by 55: that must bee set on the right hand of the figure, at the neather part, and over it on the same side 16, which is the second position, thus:

22 16 And as I gather by your words, if were all one if I had set 28 in stead of 22, and 24 in stead of 16.



275 55 Master. So were it. But this shall you marke, that, of what nature soever the two positions be, of the same nature is the quotient. Wherefore when the positions in this question are 22 and 16, which both being numbers of the playing dayes, the quotient shall declare the true number of playing dayes: whereas if the positions had been 28 and 24, which are supposed to be the working dayes, then would the quotient declare the true number of the working dayes, and not of playing dayes, as it will do now. And therefore to continue the work of this question, and to finde the true number of playing dayes, I must multiply crosse-wise the first position by 55, that is the second errour, and the totall will bee 660. Then I multiply 275 and 16, and it yeeldeth 4400. Now because the errors are alike, that is to say, both too much, I must subtract 660 out of 4400, and so remaineth 3740. which is the dividend. Againe, I must subtract the lesse errour 55 out of 275, that is the greater errour, and there will remaine 220, which will be the divisor: then dividing 3740 by 220 the quotient will be 17. Wherefore I say now constantly, that 17 is the true number of dayes that the Mason played: and then it followeth that he wrought 23 dayes: and so is the question answered.

Now for the order of triall of this worke, there needeth none other triall but only this, to worke with this number according to the question, and if it agree, then appeareth the number to be it that you would have. The proof of this rule.

And here now seeing he wrought 23 dayes, and must have for every day 25 pence, the whole summe commeth to 757. Then again, seeing he played 17 dayes, and must abate 30 pence for every day, the whole summe of the abatement will be 510. Therefore I subtract 510 out of 575, and there will remaine 65, which maketh 5 shillings, 5 pence, the cleere wages of the Mason, for his worke, according to the question.

Scholar. Now I trust I understand the worke and the Rule so well (and the better by this proof) that I can be able to do the like: And for a proofe, I take the same question, all save the last number, where I will suppose that he had 10 shillings for his wages cleere. And now to ghesse at the number of the dayes hee wrought, I suppose, first, that he wrought 20 dayes: then say I, if he wrought 20 dayes, his wages must be 500 d. then did he play other 20 dayes, for which must be abated 600 d. and then hee loseth 100 d. And so am I at a stay, for this not like to your former worke.

Master. You should have required of me some question, and not have taken a question of your owne phantasying, untill you were more expert in this Art, for so might you as well

happen to an impossible question, as on a possible: but now to go forward, consider that this number is too little by 220, seeing he should gain by your supposition 120 pence, and in this position hee loseth 100, those both make 220, which you shall set downe for the first errour, with this signe —, betokening too little, as here in this forme following both appeare.

	20
	X
	200

And now for the rest goe forward your selfe once againe.

Scholar. As my errour hath utterred my folly, so it hath procured me better understanding.

Now therefore considering this position not to solve the question, I take another, supposing that he wrought 30 dayes. Then for his wages he must be allowed 750 pence, and for the 10 dayes which he wrought not, he must abate 300 pence, and so remaineth cleer 450 pence, but it should bee onely 120 pence, therefore it is too much by 330, which I set downe in the figure with the former position and his errour, and the figure appeareth thus:

Now first, I multiply in	20	30
crosse wayes 20 by 330, and		X
it will be 6600.		
Then againe I multiply		
30 by 220, and it will be al-	220	3304
so 6600. Wherefore if I		

shall subtract the one out of the other, then will

will remain nothing to be the Dividend.

Maſter. In this you forget your ſelfe again; for in as much as the ſignes in the errours bee unlike, therefore muſt you worke by Addition, adding together thoſe two totalls to make the Dividend, and alſo adding the two errours to make the Diviſor. And becauſe you ſhall no more forget this part of that Rule, take this briefe remembrance.

Unlike require Addition.
And like deſire Subtraction.

Scholar. You mean, that if the errours have like ſignes, then muſt the Dividend, and the Diviſor bee made by Subtraction, as is taught before: And if thoſe ſignes bee unlike (as in this laſt example they bee) then muſt I by Addition gather the Dividend and the Diviſor. Therefore muſt I adde 6600 to 6600, and it will bee 13200, which will be the Dividend. Then againe I adde 220 to 330, and it will be 550, which muſt be the Diviſor: wherefore dividing 13200 by 550, the quotique will be 24, whereby I know that the Maſon wrought 24 dayes, and then it followed, that he played 16 dayes.

Maſter. Examine your worke, whether it be agreeable to the queſtion or no.

Scholar. For 24 dayes worke, the wages muſt be 600 pence, and for 16 dayes which the Maſon wrought not, there muſt be abated 480 pence, and then remaineth cleare to the

Maſon 110, as the queſtion importeth: wherefore it is evident that 24 is the true number of days that he wrought.

Maſter. Although you ſeem vnto to vnderſtand this worke, yet to augment your minde the better with the new Vſage of this Rule, I think it good to propoſe to you 5 or 6 examples more before I make an end of it.

Scholar. Sir, I thanke you that you doe conſider my commodity and profit in knowledge, for vndoubtedly it is practice and exerciſe that maketh men prompt and expert in every kinde of knowledge.

Maſter. You ſay well, ſo that they follow ſome certaine precepts to governe and rule their practice by, elſe may practiſe procure vnto vs error, and a repugnance to exactneſſe of knowledge: namely, as long as the error is not plainly known to the vulgar ſort. But to return to your worke.

A queſtion of wares, the ſecond example.

There is a ſeruant that hath bought of Velvet and Damaſk for his Maſter 46 yards, the Velvet at 20 ſhillings a yard, and the Damaſk at 12 ſhillings, and when he commeth home, his Maſter demandeth of him, how much he hath bought of each ſort: I cannot tell (ſaith he) exactly: but this I know, that I paid for Damaſk 48 ſhillings more then I paid for Velvet: now muſt you ſhewe how many yards there is of each ſort.

Scholar. Although the ſhewe ſeemeth difficult, yet I will proue what I can doe: for I remember your ſaying, that is forceth not how

be lond or false the ghesse bee, & to be found what to the question, and not an answer of a contrary matter.

Therefore first I imagine that he bought 20 yards of Damask, for which hee should pay after the former price 240 shillings, then must he needs have of Velvet other 20 yards, (to make up the 40 yards) and that would cost 400 shillings, so that the totall of the price of the Damask is lesse then the summe paid for Velvet 160 shillings, and should bee more by 48. Wherefore the first stour is 208 too little. Then begin I againe, and suppose he bought of Damask 30 yards, that cost 360 shillings, then had he but 10 yards of Velvet, which cost 200 shillings: and now the price of the Damask is greater then the price of the Velvet by 160 shillings, and should be but 48, therefore is the second errour 112 too much, which I set in forme of figure, as here both appeare. Then doe I multiply in crosse wayes 208 by 30, and the summe will be 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much as the signes of the errours be unlike, I know I must worke by Addition, therefore adde I these two totalls together, and they make 8480, which is the Dividend: then adde I also the two errours together, 208, and 112, and they make 320, which is the Divisor: wherefore

30 20

208 112

dividing 1480 by 320. the quotient will be 26½ which is the true summe of yards of Damask that he bought, and in Velvet 13 yards: and that appeareth by examining thus: 26½ yards of Damask at 12 shillings the yard maketh 318 shillings: then in Velvet he had but 13 yards and ½, and cost 279 shillings, at 20 shillings the yard. Now subtract 279 out of 318, and there will remaine 39 which is the number of shillings that the Damask did cost more then the Velvet.

Master. Now shall you have a question of another kinde.

A question of debt, the third example.

There are three men that do owe money to me, and I have forgotten what the totall summe is, and what the particulers be:

Scholar. Then it is impossible to know the debt.

Master. Nay, you are too hasty, there is more help in it then you perceive. I have three severall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the first and the second to amount to 47 pound, the debt of the first man and the third man did make 71 pound, and the second man his debt with the third, did rise to 88 pound. Now can you tell what every man did owe, and what was the whole summe?

Scholar. Nay, in good faith, but as I perceive that it must be found by conjecture, so will I ghesse at it, supposing that the first man did owe 20 pound, and the second 30, and the third.

Master.

Maſter. Nay ſtay there, you are too farre gone already: you may not ſuppoſe a ſeverall ſumme for every man, for it is enough to ſuppoſe one ſumme for the firſt man, and let the other riſe as the queſtion importeth. Therefore ſeeing you ſet the firſt man his debt to be 20 pound, the ſecond man cannot owe 30 pound, for the declaration is, that their debts added together do make 47 pound, ſo muſt the ſecond man his debt be but 27 pound. Now the ſecond debt with the third muſt make 88, therefore ſubtract 27 out of 88, and there will remain 61, for the third mans debt. Then ſaith the declaration, that the firſt and third mans debts do make 71, but by this ſuppoſition they make 81, that is 10 too much, which I muſt ſet for the firſt errour. Now make you the ſecond poſition.

Scholar. I ſuppoſe the firſt mans debts to be 24 pound: then muſt the ſecond mans debt (by your declaration) be but 23 pound, ſeeing both they make but 47 pound. And the ſecond man his debt with the third, doe make 88 pound, and the ſecond man oweth but 23: therefore the third man muſt owe 65 pound. Now the third mans debt with the firſt, ſhould make by the declaration 71 pound, and they doe make 89 pound, that is 18 pound too much, and that is the ſecond errour, which I ſet down with the firſt, and their poſitions in this forme, and then I doe multiply in croſſe wayes 20 by 18, and it is 360.

And

And 10 by 24 maketh 240. 10 24
And because the signes of the errours be like, I must worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the Dividend: then do I subtract 10 out of 18 by the same reason, and so is the Divisor 8, which is found 15 times in 120. Therefore I say that the first man did owe 15 li. and then the second man must owe 32 li. for those two do make 47 li. and the third man ought to owe 56. for so much remaineth if I abate 15 out of 71, or if I take 72 out of 88.

The fourth example.

Master. *For the fourth example, take this easie question for the variety in work. Two men having several summes, which I know not, doe thus talke together: the first saith to the second, if you give me 2 shillings of your money, then shall I have three times so much money as you. The second man answereth: It were more reason that our summes were made equall, and so will it be if you give mee 3 shillings of your money. Now shew what each of them had.*

Scholar. I imagine that the first had 9 s.

Note.

Master. Consider evermore in your imagination that you take a likely summe, as in this question, take such a summe, that having 2 added unto it, may be divided into three parts even.

Scholar. Why? I remember you said before, it forced not, how rudely soever I guessed.

Master.

Maſter. To the poſſibility of the ſolution, it is truth: but for eaſineſs in working, the aptest numbers are moſt convenient.

Scholar. I thought no leſſe, and therefore I took 9. as an apt number to be parted into three: but I perceive I ſhould have conſidered the aptneſſe of that partition after the addition of two unto it, and then I had been more neer.

Maſter. That is truth, and then ſhould the ſecond man his ſhare be 5; for although he have no what the third part of 9, that is 3, yet you muſt remember, that he lent the firſt man 2, and ſo had he 5.

Scholar. Then to go forward: if the ſecond man had three of the firſt man, then ſhould he have 8 and the firſt man but 4, ſo hath he double to the firſt man, yet he ſaid in the queſtion they ſhou'd have equall: wherefore it appeareth that he hath 4 too much.

Therefore I note that error with his ſuppoſition, and ſhall again that he hath 10 ſhillings: whereunto I adde 2 ſhillings borrowed of the ſecond man, and then he hath 12 s. ſo the ſecond man hath remaining but 4, whereunto if I adde the 2 that he lent to the firſt man, ſo had he but 6 at the beginning.

Then take 3 s. from the firſt man, and give to the ſecond, then hath the firſt man but 7, and the ſecond hath 9, which are not equall, but there are 2 too many.

The like example: 2 queſtion of Lamps.

many, wherefore I set down both the positions with their errours, as before you see, and multiply a croſſe, ſo commeth there 40 and 14: and becauſe the ſignes be like, I take 14 out of 40, and ſo reſteth 26 to be divided: then likewiſe I take 2 out of 84, and there reſteth 2, by which I divide 26, and the quotient will be 13, which is the ſumme that the firſt man had. And ſo appeareth that 2 being added thereto, the ſumme will be 15; ſo hath the ſecond man but 5, and before he had 7: then take 3 from the firſt, and put to his 7, and ſo have each of them 10, and that is equall as the queſtion would.

The fifth example: a queſtion of Lambs.

Maſter. *For the fifth reaſon, take this queſtion. One ſaid to another, I think you had this year two hundred Lambs in ſuit, I ſaid then, but what with paying the tythe of them, and then the ſtracall toſſet, they are much abated: for at one time I loſt halfe as many as I have now left, and at another time the third part of ſo many, and then hit a time ¼ ſaith he. Now gueſſe you how many are left.*

Scholar. Becauſe here is mention made of certaine parts, I muſt take a number that may have all thoſe parts, that is to ſay, ½, ⅓, and ¼ which will be 24, becauſe it hath the ſame parts. Therefore I take firſt 10 to bee the number that doth remaine, ſo hath he loſt 6, 4, and 3, that is 13, and the whole 23, but it ſhould be 200.

Maſter. We are deceived yet ſtill, you have for-

forgotten the 10 part, which must be detained, that is 200, so there remaineth but 1800, and now go on again.

Scholar. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 too few, which I set for the errour. Then for the second position I take 24, whose halfe is 12, the third part 8, and the quarter 6, whereby ariseth 50, which is too little by 1750, therefore I set down both the positions, with their errours, thus:

And multiply in crosse wayes 1775 by 24, whereof commeth 42600. Also I multiply 1750 by 12, and there ariseth 21000. And because the signes are like, I doe subtract the one from the other, and so remaineth the Dividend 616000. Then doe I subtract 1750 out of 1775, and there resteth 25, by which I divide 21600, and the quotient is 864, whereof the halfe is 432, and the third part is 288 the quarter is 216; which all being added together, will make 1800. And if you adde thereto the tenth which was abated before, then will the whole summe be 2000.

12	24
1775	1750

And now doth there come a question to my memory which was demanded of me, but I was not able to answer to it: and now me thinketh I could solve it.

Master. Propone your question.

Scholar.

A question of sheep and tillage, the fifth example.

Scholar. *There is supposed a Law made, that for furthering of tillage every man that doth keep sheep, shall for every ten sheep eat and sow one Acre of ground: and for his allowance in sheep pasture, there is appointed for every foure sheep one Acre of pasture. Now is there a rich Sheep-master which hath 7000 Acres of ground, and would gladly keep as many as he might: by this Statute I demand how many sheep shall hee keep?*

Master. Answer to the question your self.

Scholar. First, I suppose hee may keep 500 sheep, and for them hee shall have in Pasture after the rate of foure sheep to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this errour is 500 little by 6825. Therefore I ghesse again that he may keep 1000 sheep, that is in pasture 250 Acres, and in tillage 100 Acres, which make 350, that is 100 little by 6650. Both these errours with their positions, I set down as you see, and multiply them crosse 6825 by 1000, and it maketh 6825000, also I multiply 6650 by 500, and there commeth 3325000, which summe I subtract out of the former, and there remaineth 3500000 for the Dividend: likewise I subtract the lesser errour out of the greater, and there resteth 175, by which I divide 3500000 (the Dividend aforesaid) and

500	1000
6825	6650

the

the quotient will be 200, so that by this rate he that hath 700 Acres of ground, may keep 20000 sheep.

Another way of working.

Master. You have done well, notwithstanding both this last question, and the next before might bee wrought without the second position by the Rule of Proportion, as this: When in this question you found in the first errour that for 500 sheep there must be 175 Acres, then might you reduce it to 7000 ∠ 20000 by the Golden Rule, thus:

If 170 Acres will admit in allowance 500 sheep, then 7000 will have 2000. And so by one position, with the help of the Golden Rule may you answer that question.

Likewise for the question of Lambs, when you had found that 12 came of 25, you might have set the figure as followeth, and have said:

25 ∠ 12
1800 ∠ 864

If 25 do leave but 12, what shall 1800 leave? and it would appear to be 864.

Scholar. Sir, I thanke you for this art, for it doth much shorten the worke of this Rule.

Another way yet.

Master. Yet againe, I will shew you an other way to answer to this last question without the Rule of false position, and that by the Rule of Fellowship, for it appeareth in the proponing of the question, that ten sheep must

must have in pasture two Acres and $\frac{2}{7}$, and for them must there be eared but one Acre; so it followeth, that for 2 Acres eared, there must bee 5 set to pasture: and if you put them both into one summe, they will make 7. Therefore look what proportion 7 being this totall both bear to 5 and to 2, such proportion shall any totall in this question beare to the pasture ground, and the eared ground.

Scholar. This surbeth wondrous aptly. Therefore to prove it, I demand this by the former supposition: If a man have 300 Acres, how much shall be leave in pasture, and how much shall be turne to tillage? You say, that as 7 is to 5, so shall 300 be to the Acres of pasture: and as 7 is to 2, so is 300 to the Acres of tillage, whereof for both I have set examples here following, whereby appeareth that of Pasture, there shall bee 214 $\frac{2}{7}$ Acres, and of Tillage 85$\frac{5}{7}$, which both summes added together, doe make 300.

7 Z 5 : 300 Z 214$\frac{2}{7}$

7 Z 2 : 300 Z 85$\frac{5}{7}$

A nother question, the seventh example.

Master. *Now take another Example: A man hath three silver Cups with one Cover, the Cover weigheth 18 ounces, the second Cup weigheth even halfe the weight of the first and the third. Now if the Cover be put to the first Cup, they weigh just as much as all the three Cups do weigh: and if the Cover be joyned with the second Cup, they weigh as much as the second twice, and the third: and if the*

Cover

Cover be put to the Cup they will make twice as much as the first and second Cup. Now try you what was the just weight of every Cup.

Scholar. I doe set the weight of the first Cup to be nine Ounces, then in as much as these two (that is to say, the cover and the first Cup) doe weigh the weight of the three Cups, I see that the three Cuppes must weigh 27 ounces, for so much is 18 and 9. Also because the first and the third doe weigh double so much as the second, therefore it is the third part of that weight, that is 9, and then would it follow, that the third Cup also would weigh 9 ounces; but then the question saith that the Cover being joyned to the second Cup, they weigh as much as the second twice, and the third once, that should be 27, and so it doth; that being joyned with the third Cup, they should weigh twice as much as the first and the second, that should be 36, and they weigh but 27, so is that errour 9 too little. Then beginne I again, and say, that the first Cup doth weigh twelve ounces which I joyne with the Cover, and they make thirty Ounces: then seeing the second is of that weight, it must needs weigh ten ounces, and the third must weigh 8 ounces, seeing the first and the third must weigh 20 ounces. Now put I the Cover to the second Cup, and they weigh 28 ounces, which should be even so: then joyne I the cover with the third Cup, and so should it wey twice the first and the second,

that is 44 ounces, and they weigh but 26, that is 18 too little: thoſe errours with their poſitions I ſet down, and multiply in croſſe wayes 9 by 12, whereof commeth 108: Alſo 9 by 18, and that yeeldeth 162: and in as much as the ſignes be like, I abate the leſſer out of the greater, and there doth remaine 54. Then doo I alſo abate the leſſer errour from the greater, and ſo remaineth 9, by which I divide 54, and the quotient is 6, which I take for the true weight of the firſt Cup, which being ſoyned with the Cover, muſt weigh as much as the three Cups, ſo doe they weigh but 24 ounces. Then ſeeing the ſecond Cup is the third part of that weight, for the other two Cups (you ſay) muſt weigh double his weight, the weight of the ſecond Cup is 8 ounces, and ſo the weight of the third Cup muſt be 10 ounces. Now put the Cover to the ſecond Cup, and it will make 26 ounces: that muſt be the weight of the ſecond twice, and the third once, that is, twice 8, and once 10, and ſo is it. Again put the Cover to the third Cup of 10 ounces, and they muſt weigh twice as much as the firſt and the ſecond, that is, 28, and ſo is all agreeable.

9 12
9- 18-

Maſter. Then anſwer to this Queſtion.

A queſtion of water: the eighth example.

There is a Ciſterne with four Cocks; containing 72 barrels of water: and if the greateſt Cocke be opened, the water will avoid clean in ſix houres; at the ſecond Cocke it will aske eight houres:

houres: at the third Cock it will avoid in no leſſe then nine houres: and at the ſmalleſt it will require twelve houres. Now I demand in what ſpace will it avoid, all the Cocker being ſet open?

Scholar. Firſt, I imagine it will avoid in two houres.

Maſter. Then muſt there avoid by the firſt Cock $\frac{1}{3}$ of the water, that is 24 Barrels, and by the ſecond Cock $\frac{1}{4}$, that is 18, and by the third Cock $\frac{2}{9}$, that is 16 Barrels, and by the ſmalleſt Cock $\frac{1}{6}$, that is 12 Barrels, all which ſummes put together, do make 70, as by their Addition it doth appeare, but it ſhould be $72\frac{1}{2}$, therefore the errour is too few.

Scholar. Then will I begin a-gain by your favour, becauſe I thinke I underſtand the worke, and put three houres for the due time: ſo ſhall there run out at the greateſt Cock $\frac{1}{3}$, that is, 36 Barrels, and at the ſecond hole $\frac{1}{8}$, that is 27, and at the third Cocke $\frac{1}{4}$, that is 24, and at the ſmalleſt hole $\frac{1}{4}$, that is 18 Barrels, which all together doe make 105, and ſhould be but 72, ſo is it too much by 33: therefore do I ſet the errours in order of the figure with their poſitions, and worke by Multiplication, in croſſe, ſaying, two times 3 is 6, and two times 33 maketh 66, and becauſe the ſignes are unlike,

24	
18	
16	
12	
70	

2 3

2- 33 +

I muſt adde theſe two totalls together, which make 72: alſo I adde the two errors, and they make 35, by which I divide 72, and the Quotient riſeth $\frac{2}{35}$, whereby I ſee that all the Cocks being ſet open, the water will avoid in two houres, and $\frac{1}{35}$ of an hour.

Maſter. This exerciſe maketh you to grow expert in the Rule. Therefore I will inure you ſomewhat more with a queſtion or two:

A queſtion of partners.

The ninth example.

There were two men that had been partners, and had in account between them 300 Duckets; whereof the one ſhould have for his part 180, and the other 120: but in the parting of them, they fell at variance; ſo that each of them catched as many as he could: yet afterward being reconciled they agreed that he which had gotten moſt part of them, ſhould lay down $\frac{1}{4}$ of them again, and hee that had gotten leaſt, ſhould lay down $\frac{1}{3}$ of thoſe which he had taken, and then parting them into two equall parts, each man to have halfe thereof, and ſo had they their juſt portions, as they ought: now I demand of you what each of them had gotten by the ſcambling?

Scholar. I ſuppoſe he that had leaſt, got 108 Duckets, then the other had 192: wherefore in laying down again of the 192, there was put down $\frac{1}{4}$, that is, 144, and ſo had he left but 48. Alſo of the 108, there was laid down 36, that is $\frac{1}{3}$, and ſo he had left 72. Then I put together 144 and 36, and it maketh 180, which I part into two parts even, and ſo commeth 90 to be given to each of them:

them: which ſumme put to 72 maketh 12 and ſopned to 148, it maketh 238: and now I doubt how I ſhall go forward.

Maſter. You need not to take but one of them, which you liſt, the greater or the ſmaller, for all commeth to one purpoſe: and ſo may you compare it that you take to any of the other ſummes, remembring that you make comparilon to the ſame in the ſecond worke: as for example of the firſt part, If you compare 138 with the leſſer ſumme due, that is, 120, ſo is it 18 too much, and if you compare it with the greater ſumme, then is it 42 too little. Again, if you compare 162 to the greater ſumme, the errour will be 18, as it was in the other: but it will have a contrary ſigne: and if you compare it with the leſſer ſumme, it will be 42 too much: ſo that the errour both wayes is either 18 or 42: and as for the ſignes it little forceth, for in them is nothing conſidered here, but likeneſſe and unlikeneſſe, which in this caſe doth neither further or hinder: But now go on with the work. Note.

Scholar. If it be ſo, then am I out of my greateſt doubt. When I ſopne that 90 (which I found as the half of the latter partition) unto 48, which is left with the one man, and ſo hath he, 138, which (I may ſay) is 18 too many, for the leaſt ſhould be but 120, that errour do I note, and then make a new poſition, ſuppoſing the one man to have 204, and the other to have 96, wherefore of the 204, there

must be laid down 153, and so remaineth with him 51. Also of the 96, there must be laid down $\frac{1}{3}$, that is 32, and so resteth with that man 64 †. Now of the 153 and 32, I make one summe, as 185, which I must divide into two equall parts, and so each man shall have $92\frac{1}{2}$, whereunto if I adde their former positions reserved, then the one shall have $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Now therefore take the lesser summe now again, as I did before, that is, $143\frac{1}{2}$ and finde that he hath too many by $23\frac{1}{2}$, for he should have but 120, and so have I for my two positions two errors, which I set down as here may be seen, each error under his position, and then by the Rule I doe multiply in cross-wayes 108 by $23\frac{1}{2}$, and there riseth 2538, which I note, then again I multiply 96 by 18, and thereof amounteth 1728.

198	96
18 †	$23\frac{1}{2}$ †

Now because the signes are both like, that is, both too many, I must worke by Subtraction, and so abating 1728 out of 2538, there will rest for the Dividend 810: then for the Divisor I subtract 18 out of $23\frac{1}{2}$, and there remaineth $5\frac{1}{2}$, by which I divide 810, and the quotient will be $147\frac{1}{11}$, which is the just portion of him that had the least summe. And if I doe subtract it out of 300, being the totall summe, then will there remain $152\frac{10}{11}$, as the portion that the other did get.

Master

Master. For the proofe of this worke, you may chuse whether you will examine those numbers according to the forme of the questions, or else worke by other two positions for to finde the second number: and if those positions bring the same numbers that did amount by the first two positions, then doth each worke confirme other.

Scholar. By your patience, I will prove both wayes, not onely to seeke their agreement but also to accustome my minde to those workes, for I perceive it is exercise that must be the chiefe engraver of these Rules in my memory.

Master. You consider it well: then go to.

Scholar. First, I will by two other positions try to finde the portion of him which had most.

Master. Although you may doe it with any positions, yet to see the agreement of your worke the better, take the same positions that you did before, comparing them now to the greater, as you did before unto the lesser.

Scholar. Then I suppose that he that had most, had 192, so had the other 108. Now if I take $\frac{1}{4}$ out of 192, that will be 144 and there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is 36, there will remain to him 72: then joyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to each of those

two mens portions remaining with them, the one shall have 138, and the other 162, of which two I take the greater (that is 162) and see it to be 18 too few; for it should be 180, that errour I note under this position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other; then take I $\frac{1}{4}$ of 204, and it will be 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is, 32, and there remaineth to him 64; now put I that 32 to 153, and it yeeldeth 185: which being parted in equall halves, maketh $92\frac{1}{2}$ to be added to each mans remainder, and so the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greatest summe, and it is $23\frac{1}{2}$ too little, that do I note also, and set both these errours under their positions, as in this Example following doth appeare.

And then multiplying 102 by $23\frac{1}{2}$, there doth arise, 4512.

Again, I multiply 204 by 18, and it maketh 3672, which I doe subtract out of 4512, because the signes be like, and there resteth 840 for the dividend, then subtracting 18 out of $28\frac{1}{2}$ there will remain $5\frac{1}{2}$, which I must take for the Divisor. And so dividing 840 by $5\frac{1}{2}$ the quotient will be $152\frac{8}{11}$, whereby I have found an agreeable summe to that which I found by the former positions, for him that had

102	204
18	$23\frac{1}{2}$

most

moſt which I doe ſubtract out of 300, that is the totall, there will reſt $147\frac{3}{11}$, which was the portion of him that had the leaſt part.

Maſter. So by divers poſitions, you ſee, that one doth confirme the worke of the other. Now examine theſe two numbers by the forme of the queſtion, and ſo ſhall you prove your worke good alſo.

Scholar. If that be which gat moſt, had $152\frac{8}{11}$, then muſt he lay down $\frac{3}{4}$ of this ſum. That is $114\frac{6}{11}$, and ſo ſhall remain with him but onely $38\frac{5}{11}$. The other which had leaſt, that is $147\frac{3}{11}$, muſt put down of his ſumme $\frac{1}{3}$, that is $49\frac{1}{11}$, and ſo doth there remain with him yet $98\frac{2}{11}$. Then do I add together $114\frac{6}{11}$, and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$ which I muſt part into equall parts, and that will be $81\frac{6}{11}$, to be given to each of them: putting $81\frac{2}{11}$ unto $38\frac{5}{11}$, there doth amount 120 juſt, which is the true portion of him that ſhould have the leſſer ſum: and adding $81\frac{6}{11}$, $98\frac{2}{11}$, the totall will be 180, the true portion of the other. And ſo is the worke by this proofe alſo tryed to be good. And this I marke by the way, that in their ſcrambling, he got moſt (as it chanceth often) that ought to have had leaſt by juſt partition.

Maſter. Let your ſtudy be to learn truth and juſt Art of proportion, and to diſtribute each part according thereunto, as often as occaſion ſhall be miniſtred. And here would I make an end of this Rule, ſave that I remember one pleaſant queſtion which I cannot overpaſſe,

paſſe which I will declare ſomewhat largely, becauſe you ſhall as well underſtand ſome reaſon in the pleaſant invention, as apt proceeding in the witty working thereof.

The tenth example of gold and ſilver.

Hiero King of the Syracuſans in Sicilia had cauſed to be made a Crown of Gold of a wonderfull weight, to be offered for his good ſucceſſe in wars: in making whereof the Goldſmith fraudulently took out a certain portion of Gold, and put in Silver for it, ſo that there was nothing abated of the full weight, although there was much of the value diminiſhed.

Which thing at length being uttered (as no evill can alwayes lie hid) the King was ſore moved: and being deſirous to know the truth without breaking of the Crown, propoſed the doubt to Archimedes, unto whoſe wit nothing ſeemed unpoſſible, which although preſently he could not anſwer unto, yet he had good hope to deviſe ſome policy for that invention, and ſo muſing thereon, as he chanced to enter into a Bain full of water to waſh him, he obſerved, that as his body entred into the Bain, the water did runne over the Tub, whereby his ready wit of ſuch ſmall effects conjecturing greater works, conceived by and by a reaſon of ſolution to the Kings queſtion, and therefore rejoycing exceedingly, more then if he had gotten the Crown it ſelfe, forgat that he was naked, and ſo ranne home, crying, as hee ranne, εὕρηκα, εὕρηκα, I have found, I have found. And thereupon cauſed two maſſie pieces,

pieces, one of Gold, and another of Silver, to be prepared, of the same weight that the said Crown was of: and considering that Gold is heavier of nature then Silver, and therefore Gold of like weight with Silver, must needs occupie lesse rooms by reason it is more compact and sound in substance, he was assured that putting the masse of Gold into a vessell brim full of water, there would not so much water runne out, as when he should put in the silver masse of the like weight. Wherefore he tried both, and noted not onely the quantities of the water at each time, but also the difference or excesse of the one above the other, whereby hee learned what proportion in quantity is betweene Gold and Silver of equall weight. And then putting the Crown it self into the vessell of water brim full (as before) marked how much water did run out then, and comparing it with the water that runne out when the Gold was put in, noted how much it did exceed that: and likewise comparing it to the water that runne out of the Silver, marking how much it was lesse then that, and by those proportions found out the just quantity of Gold that was taken out of the Crown, and how much Silver was put in stead of it: but seeing Vitruvius which writeth this History, doth not declared the particular worke of this tryall, it shall be no inconvenience to suppose an example for declarations sake, wherein although the true and just pro-

proportion be not expressed, yet the forme of triall shall be truly set forth. And for an example, I suppose the weight of the Crown to be 8 pound, and so of each the other two Masses. And when the Masse of Gold was put into the water, I imagine that there ranne out two pound of water: and when the masse of silver was put in, I suppose there ranne out three pound $\frac{1}{2}$. Againe, when the Crowne was put in, there ran out two pound $\frac{1}{4}$: Now to know what quantity of silver was in the Crown, work by the Rule of false position, and imagine that there was two pound of silver, then must there be sixe pound of Gold, then say thus by the Rule of Proportion. If eight pound of gold do expell two pound of water, what shall sir pound expell? and it will be 1 pound $\frac{1}{2}$. Againe, for the silver: if eight pound of silver expell three pounds $\frac{1}{2}$ of water, what shall two pound of silver put out? it will be $\frac{7}{8}$, now adde those two weights of water together, and they will make two pound $\frac{1}{8}$, and it should be by the supposition two pound $\frac{1}{4}$, so is it too much by $\frac{1}{8}$.

Scholar. Now do I understand the worke as I thinke, therefore I pray you let me worke the rest of the question. And because this first supposition did erre, I note that position and his errour, and take a new position, esteeming the silver to be but one pound, so must there be in Gold seven pound. Then say I: if eight pound of Gold doe yeeld two

pound

pound of water, what ſhall ſeven pound yeeld? and it will be 1 pound $\frac{1}{4}$. Again, if 8 pound of ſilver erpell 3 pound $\frac{1}{2}$ of water, what ſhall 1 pound erpell? and it will be $\frac{1}{28}$. Now muſt I adde theſe two ſummes together, and they make two pound $\frac{1}{16}$, and they ſhould make 2 pound $\frac{1}{4}$, ſo is it too little by $\frac{1}{16}$. Therefore I ſet the poſitions, with their errors in order as here followeth: And then I multiply in croſſe wayes 2 by $\frac{1}{16}$ and it maketh $\frac{1}{8}$: Likewiſe 1 multiply by $\frac{1}{8}$ maketh $\frac{1}{8}$. And becauſe the ſignes be unlike, I muſt adde theſe two ſums which make $\frac{1}{4}$: and that is the dividend.

2 1

$\frac{1}{8}$ † $\frac{1}{16}$

Againe, I muſt adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{1}{16}$, that is the Diviſor. Now I ſhall divide $\frac{1}{4}$ by $\frac{1}{16}$ and the quotient will be $\frac{16}{12}$, that is, $1\frac{1}{3}$: whereby I know that there was put 1 pound and $\frac{1}{3}$ of ſilver into the Crown, and ſo much Gold taken out of it.

Maſter. Prove it now by examination, according to the queſtion.

Scholar. If there were 1 pound $\frac{1}{3}$ of ſilver, then was there of Gold 6 pound $\frac{2}{3}$. Now ſay I by the Rule of Proportion:

If 8 pound of Gold erpell two pound of water, what ſhall 6 pound $\frac{2}{3}$ erpell 7.

8 6 $\frac{2}{3}$ 2 15 7

It

8 Z $2\frac{1}{3}$ $1\frac{1}{3}$ $1\frac{1}{12}$ $1\frac{1}{2}$

It will be 1 pound $\frac{2}{3}$. Againe, if 8 pound of silver expell three pound $\frac{1}{2}$ of water, what shall expell? It will bee $\frac{1}{12}$. Now mine I adde together 1 pound $\frac{2}{3}$ and $\frac{1}{12}$, and they will make 2 pound $\frac{3}{4}$, that is, 2 pound $\frac{1}{4}$, according to the supposition of the question: whereby I perceive the work to be well done. And I cannot but much respect of this excellent invention, to my better is kindled behementlie to be perfectly informed in every part thereof, and namelie in this point, whether the proportion between Water and Gold be such that for 8 pound of Gold put into a vessell full of water, there shall runne out two pound of water, and so as much silver, whether 3 pound $\frac{2}{3}$ of water would avail.

Maister. I perceive your meaning, and conjecture your imagination to be thus, that if you knew the grad proportion between Gold and Silver, and water, both in their weight and quantities, then could you easily finde out the mixtures of them, which thing I have reserved for another worke that intreateth of such matters especially. And at this time you must consider that you learn Arithmeticke, which intreateth of the manner to solve doubtfull questions touching number, without regard what matter is signified by that number: else were it necessary in Arithmeticke to teach all Arts, seeing in it may be moved questions of all Arts.

But

But ſeeing you are ſo deſirous to know theſe things, I will tell you in ſuch a ſort, that you ſhall practiſe your Art in finding it, and propound it in forme of a queſtion. Gold beareth a greater proportion to water, then ſilver doth, and their two proportions be in proportion together, as 48. to 25. But to help you ſomewhat in this Riddle, you ſhall note that the proportion of Quick-ſilver unto water, is the juſt middle number proportionall in progreſſion Geometricall between the proportion of gold and ſilver unto water.

A queſtiod of the proportion of gold ſilver, and quickſilver, unto water.

And this proportion is $\frac{222}{21}$. Now if you will know the juſt numbers of theſe three proportions, then muſt you finde out three numbers in Progreſſion Geometricall, whereof the middlemoſt muſt be $\frac{222}{21}$, and the firſt muſt be unto the laſt, as 25 to 48. And thus I will leave you to finde thoſe numbers when you be at leiſure.

Scholar. Yet Sir, I thank you heartily for thus much, for now I ſee the poſſibility to finde them out. Howbeit, becauſe this queſtion ſeemeth ſtrange, if it might pleaſe you to inſtruct me ſomewhat in the order of working for it, I ſhould the more eaſily finde the true working.

Maſter. You deſire too much if you will ſtudie for nothing: Therefore to occaſion you to ſtudy the better, I will leave this doubt wholly to your own ſearch: But as touching the generality of the Rule, Archimedes needed not to take two Maſſes of Gold

and

and ſilver equall in weight with the Crown, for the proportion might as well be found in any other weight, yea, although the Maſſe of Gold were of one weight, and the Maſſe of Silver of another. As for example: if the Crown were of 18 Pound weight as I did ſuppoſe, and I have not ſo much other fine Gold, but onely one pound, and trying that by water, and finding that it doth expell but ½ of an ounce of water, yet then by it I may inferre, that 8 pound of Gold would expell 6 ounces of water. And likewiſe of ſilver, whereof if I had but two pound, and finde that it doth expell three ounces of water, then might I affirme that 8 pound would expell 12 ounces, that is, one pound weight: and ſo is it good as if the three Maſſes were all of one weight. And thus for this time I will make an end of this other part of Arithmeticke.

Scholar. Although I cannot ſufficiently thank you for this, yet your promiſe made me to looke for the Art of Extraction of Roots, whereof hitherto I have learned nothing.

Maſter. I will not breake my promiſe, but intend (God willing) to performe it within this three or foure moneths, if I perceive this my paines to be well taken in the mean ſeaſon. And you ſhall not repent the tarrying for it: for it ſhall be increaſed by the tarrying. And in the mean time you ſhall take this Addition, not for the ſecond part of Arithmeticke which I promiſed, but for an aug-

augmentation of the first Part, unto which I would have annexed the extraction of Rootes square and cubike, namely for examples of the Statute of Assise of wood, but that in the second part I must write of divers other Roots, and thought it best to reserve those Rules also with their Examples unto the same second Part.

Scholar. Sir, although I cannot recompence your goodnesse, yet I shall alwayes doe mine endeavour to occasion you not to repent your benefit on me thus imployed.

Master. That recompence is sufficient for your part.

FINIS.

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The third Part,

OR,

Addition to this Booke,

Entreateth of briefe Rules, called Rules of Practiſe, of Rare, Pleaſant, and commodious effects abridged into a briefer method then hitherto *hath been publiſhed.*

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The firſt Chapter of this Addition, entreateth of brief Rules, called Rules of Practiſe, with divers neceſſary queſtions, profitable not onely *for Merchants, but alſo for all other Occupiers whatſoever.*

THe working of Multiplication in Practiſe, is no other thing then a certain manner of Multiplying of one kinde by another: whereupon is brought forth, the Product of the proponed number, which is accompliſhed by the means of Diviſion in taking the *halfe*, the *third*, the *fourth*, the *fifth*, or ſuch other parts of the ſumme, which is to be multiplied. Rule 1.

And for the better underſtanding of ſuch converſions, you ſhall underſtand that in the manner and uſe of theſe Rules of Practiſe, you ought firſt to know the even or aliquot parts of a ſhilling, which in this Table following doth appear.

Item { 6 4 3 2 1 } *pence is the* { 1/2 1/3 1/4 1/6 1/12 } *of a ſhilling.*

Wherein as you ſee according to the order

of theſe rules of Practiſe: at 6 pence the yard of any thing, you muſt take ½ of your number which is to be multiplied, and the product that commeth thereof ſhall be ſhillings, if any unite doe remain it is 6 pence.

For 4 d, take the ⅓ of the number that is to be multiplied, and the product alſo produceth ſhillings, if any unites doth remain, each one ſhall be worth in value 4 pence. The like is to be underſtood of the other 3, &c.

I Example.

At 6 d the yard, what 379 yards?

189 ſ —— 6 d

II

At 4 d the yard, what 104 yards?

34 —— 8 d

III

At 3 d the yard, what 5014 yards?

1253 ſ —— 6 d

IV

At 2 d the yard, what 532 yards?

88 ſ —— 8 d

V.

At 1 d the yard, what 409?

341 —— 1 d

Here you may ſee in the firſt example, that 379 yards at 6 d the yard, are worth 189 ſ, 6 d in taking the ½ of 379. And, in the ſecond example the 104 yards at 4 pence the yard, are worth 34 ſ, 8 d, in taking the ⅓ of 104. Likewiſe in the third example, 5014 yards at

3 d

3 d the yard, bringeth forth 1253 s. 6 d in taking the $\frac{1}{4}$ of 5014. Alſo in the fourth Example at 2 d the yard, maketh 88 s. 8 d.

And laſtly, in the fifth Example: 409 yards at 1 d the yard amounteth to 34 s. 1 d. in taking the $\frac{1}{12}$ of 409. and ſo is to be done alſo of all other queſtions the like, when the number of the pence is any of the even or aliquot parts of 12 d.

Item, to bring the Products of theſe ſhillings, and all other the like into pounds is very eaſie in dividing of it in your minde by 20. for it is to be underſtood, that as often as 20 is found in that Product, ſo many pounds doth it contain: which with facility to performe, alwayes ſtrike off the figure towards your right hand, with a right down daſh of your pen, for the 0 that appertaineth to the 20. And then begin at the left hand, in taking the half of the reſt. And if that at the laſt any unite doe remain, the ſame ſhall be joyned with the figure that is cut off, which ſhall repreſent the odde ſhillings contained in that work.

As for example, in your third queſtion at 3 d the yard, which amounteth to 1253 s. 6 d. the Product whereof maketh, 62 li. 13 s. 6 d. as here you may ſee is eaſily performed by this Example.

1	
125	3
02	13
	0

Alſo for the working of one peny the yard, it is ſomething harſh and hard to take the $\frac{1}{12}$ of ſome Products: therefore to eaſe that hard worke, you ſhall firſt bring your delivered

ſumme into groats, by taking $\frac{1}{3}$ part of the product, and if any unites remain of that $\frac{1}{3}$ part, as ſometimes there may, they are pence; and muſt be ſignified with a line from the *groats* with their title of *pence*; and becauſe that 60 *groats* maketh a *Pound* or twenty ſhillings, *ſtrike* off the firſt figure toward your right hand, for the 0 that appertaineth to 60 (as you did even now for the 0 that belongeth to 20:) Then in taking the $\frac{1}{20}$ of that product, if there doe remain any unites, the ſame ſhall you joyne with the figure that you cut off, eſteeming them as *groats*, which keep in your minde, and by taking the $\frac{1}{3}$ part of them, you ſhall turn them into *ſhillings*, and ſo have you done: *As for Example*, by a Queſtion or two hereafter proponed, ſhall more plainely by the worke appear.

At 1 d. the yard, what 54368 yards?

13592 *groats*.

$\frac{1}{6}$ li —— 226-10 s.-8 d.

Here in taking the $\frac{1}{3}$ part of 1359, in coming to the laſt worke, the $\frac{1}{3}$ part of 39 being taken, the *remainder* is 3 2, which joyned with the two that was cut off, maketh 32 *groats*, which converted into *ſhillings*, by taking the $\frac{1}{3}$ part, maketh as appeareth 10 *ſhillings* 8 d. Many other wayes there are, but none more apt for a *young learner* to underſtand then this: wherefore, this one way well impreſſed in memory, is better then 20 wayes doubtfully underſtood.

As

At 1 peny the yard, what 4533 yards?

1133 groats-1 d

½li. 18——17-9 d

At 1 peny the yard, what 64768 yards?

16192 groats

li 269--174 d

NOw followeth alſo to be underſtood that if the number of pence be not an aliquot part of 12, you muſt reduce them into ſome aliquot part of 12: and after the aforeſaid manner, you ſhall make of them two or three Products, as need ſhall require, and adde them together into one ſumme. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken.

for pence	take	and	or	&
5	3	2	4	1
7	4	3	6	1
8	4	4	6	2
9	6	3	4·4	1
10	6	4	4·4	and 2
11	6	& 1	4·4	3

Here in the firſt note of this *Table* at 5 d. you ſhall firſt take for 3 d the ¼ of the number that is to be multiplied: and likewiſe for 2 d. the ⅙ of the ſame number, adding together both the Products: But if you will worke by 4 and 1, you muſt for 4 d, firſt take the ¼ of the number that is to be multiplied: and for 1 d, take the $\frac{1}{12}$ of the whole ſumme, or rather, which is more better, for 1 peny you may take the ¼ of the product which did come of the 4 pence:

pence: becauſe that 1 d, is the $\frac{1}{4}$ of 4 pence. The totall ſummes of theſe two numbers ſhall be the ſolution to the *Queſtion*. And in like manner it is to be done of all others, as by theſe Examples following ſhall appear.

I

At 5 d the yard, what 758 yards?

3 d	189	6 d
2 d	126	4 d
ſhillings,	325	10 d

Otherwiſe,

At 5 d the yard, what 758 yards?

4 d	252	8 d
1 d	63	2 d
ſhillings,	315	10 d

II

At 7 pence the Ell, what 563 Elles?

4 d	187	8 d
3 d	140	9 d
ſhillings,	328	5 d

III

At 8 d the pound, what 112 pound?

4 d	37	4 d
4 d	37	4 d
ſhillings,	74	8 d

Otherwiſe,

At 8 pence the pound, what 112 pounds?

6 d	56	0
2 d	18	8
ſhillings,	74	8 d

IIII

IIII.

At 9 pence the Elle, what	356	Elles?
6 d	178 —	0
3 d	89 —	0
ſhillings	267 —	0 d

V.

At 10 pence the peece, what	795	peeces?
6 d	397 —	6
4 d	265 —	0
ſhillings	662 —	6

VI.

At 11 pence the pound, what	7576	pounds?
6 d	2788 —	0
4 d	2525 —	4
1 d	631 —	4
	6944 —	8 d

Pounds. 347 — 4 ſ — 8 d

1 Here in this firſt example, where it is demanded (at 5 d the yard) what will 758 coſt? Firſt, for 3 d, I take the $\frac{1}{3}$ of 758: and thereof commeth 189 ſ. 6 d: Then for 2 d I take the $\frac{1}{6}$ of the ſame 758, which amounteth to 126 ſ. 4 d, theſe two ſummes added together, doe make 315 *ſhillings* 10 *pence*: and ſo much are the 758 *yards* worth at 5 d the *yard*.

Item alſo for the ſame again: Firſt for 4 d, I take the $\frac{1}{3}$ of 758; and thereof commeth 252 ſ. 8 d: then for 1 peny I take the $\frac{1}{4}$ of the ſame 758, that is to ſay, of 252 ſ, 8 d, and it yeeldeth me 63 ſ. 2 d: which both added together maketh 315 ſ — 10 d, as before.

2 *Item, for* 7 d *there is taken the* $\frac{1}{3}$ *and the* $\frac{1}{4}$ of the whole ſumme which is to be multiplyed,

and adde them together, that is to ſay, firſt, for 4 pence there is taken $\frac{1}{3}$ of 563: which comes to 187 s.-8 d, as appeareth by the worke, and for 3 d there is taken the $\frac{1}{4}$ of the whole ſumme, which amounteth to 140 s. — 9 d. Both which products added together, doe make 328 s. — 5 d. and ſo much comes 563 *Elles* to at 7 d. the *Elle*.

3 *Item, for the firſt* 8 d. *there is taken for* 4 d. the $\frac{1}{3}$ of the whole ſumme, and another $\frac{1}{4}$ for the other 4 d. which added together, as in the example doth evidently appear, amounteth to 74 s.-8 d.

Again, for the ſecond work of 112 li. there is taken firſt the $\frac{1}{2}$ of the whole ſumme for 6 d. which comes to 56 s. then for that 2 d. you have to take $\frac{1}{6}$ of the whole ſumme, or if you will, the $\frac{1}{3}$ of the product that came of 6 d. either of which maketh 18 s.-8 d. Theſe two ſummes being added together, doe make 74 s. 8 d. as in the third example appeareth.

4 *Item, for* 9 d. *there is taken for* 6 *pence* the $\frac{1}{2}$ of the whole ſumme: and the $\frac{1}{4}$ of the whole ſumme for 3 d. or otherwiſe for the 3 d. you may take the $\frac{1}{3}$ of the product that came of 6 d. becauſe 3 pence is the $\frac{1}{2}$ of 6 d. which added together, as plainly appeareth in the *fourth example*, amounteth to 267 s. 0 d.

Item, for 10 d. *firſt there is taken for* 6 d. the $\frac{1}{2}$ of the whole ſumme, which amounteth to 397 s.-6 d. Then for 4 d. there is found 265 s. both which added together, make 662

ſhillings,

ſhillings, 6 d, as appeareth in the *fifth example*: It may alſo be wrought, *as appeareth by the ſecond note in the Table*, by 4 d. twice taken, and the $\frac{1}{2}$ of the product of 4 d. or elſe by the $\frac{1}{6}$ of the whole ſumme, *&c.*

Item, for 11 d. *there is firſt taken the* $\frac{1}{2}$ *for* 6 d. then the $\frac{1}{3}$ of the whole ſumme for 4 d. Laſtly, the $\frac{1}{4}$ of the laſt product for 1 d. All which 3 ſummes added together, maketh in ſhillings 6944 s-8 p and in pounds 347-4 s- 8 d.

Item, likewiſe by the ſame reaſon, when you will multiply (*by ſhillings*) *any number that is under* 20. *you ſhall have in the Product pounds, if you know the even or aliquot parts of* 20. *which are here in this little Table ſet down to ſight.*

Item, s. { 10, 5, 4, 2, 1 } is the { 1/2, 1/4, 1/5, 1/10, 1/20 } of one pound.

So that for 10 s. *which is the* $\frac{1}{2}$ *of a pound*, you may take the $\frac{1}{2}$ of the number which is to be multiplied, and you ſhall have in your product pounds, if an unite doe remain, it ſhall be worth ten *ſhillings*.

Likewiſe for 5 *ſhillings you muſt take the* $\frac{1}{4}$ of the number which is to be multiplied, and if there do remain any *unites*, they ſhall be fourth parts of a *Pound*, every unite being in value five *ſhillings*.

For 4 *ſhillings take the* $\frac{1}{5}$ *of the number* which

is

is to be multiplied: and if there doe remain any *unites*, they shall be fifte parts of a *Pound*, each unite being in value 4 *shillings*.

For 2 *shillings you must take the* $\frac{1}{10}$ *of the* number to be multiplied, wherefore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same number (which is neareſt your right hand) from all the other figures with a small right down line or dash with a Pen, and so have you done for all the other figures which doe remain toward your left hand from the same figure that you doe separate, shall be the said $\frac{1}{10}$ of a *Pound*: and that figure so separated towards your right hand, shall be so many pieces of 2 s. the which figure you must double to make thereof the true number of shillings, as by the Example shall appear.

Finally, for 1 *shilling needeth small work*, for it is so many shillings as be proponed in the summe, which to bring into pounds, hath been already taught in the *first rule*.

Example.

At 10 s. the piece, What		6543 pieces?	
$\frac{1}{2}$	li.	3271	10 s.
At 5 s. the Elle, What		4373 Elles?	
$\frac{1}{2}$	li.	1093	5 s.
At 4 s. the yard, What		7820 yards?	
$\frac{1}{2}$	li.	1567	16 s.
At 2 s. the pound waight, What		7527 pound	
$\frac{1}{10}$	li.	752.	14 s.

At

At 1 s the piece, What $\frac{1}{20}$ li. 757 | 3 pieces?
387 13 l.

NExt, followeth in order to be understood, that if the number of shillings be not some even or aliquot part of 20, you must then convert the same number of shillings into the aliquot parts of twenty, and thereof make two or three products as need shall require: which done, adde them together, and bring them into Pounds. And here for thy furtherance, I have set down a note of the order of their parts, as they are to be taken.

s		s	
3	2 ℔ 1	13	10. 2 ℔ 1
6	4 ℔ 2	14	10. ℔ 4
7	5 ℔ 2	15	10. ℔ 5
8 of	4 ℔ 4 of 10	16 of	10. 5. 1
9	5 ℔ 4	17	10. 5. 2
11	10 ℔ 1	18	10. 4. 4
12	10 ℔ 2	19	10. 5. 4

5 ℔ 1	5. 2. 1	4. 4. 1

For 3 s. according to the tenure that you see is expressed in the Table, you must first take for 2 *shillings* the $\frac{1}{10}$ of the number that is to be multiplied. Then for one *shilling* you must take the $\frac{1}{2}$ of the Product which did come of the same $\frac{1}{10}$ part: which two summes added together produceth the effect desired.

Item, for 6 shillings according to the note set forth in the Table first for 4 s. I take the $\frac{1}{5}$ of the number

number that is to be multiplied: Then for 2 s. the $\frac{1}{2}$ of the product that came of 4 s. and adde them together.

Or elſe as appeareth alſo in the Table, for 5 *ſhillings* you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ part of the product that came of 5 *ſhillings*, and adde them together.

Item, for 7 s. *firſt for* 5 s. *take* $\frac{1}{4}$ *of the Product* that is to be multiplied, then for 2 s. take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, *&c.*

Item, for 8 s. *according to reaſon, and the* intent of the *Table*, for the firſt 4 s. take the $\frac{1}{2}$ of the product, and the ſame number again for the other 4 s. and adde them together.

Item, for 9 *ſhillings: firſt for* 5 *ſhillings*, take *the* $\frac{1}{4}$, then for 4 ſhillings take the $\frac{1}{5}$: and adde them together.

Otherwiſe, as you ſee by the intent of the Table, work twice for 4 ſhillings as was taught even now for 8: and then take the $\frac{1}{4}$ of the laſt product for the 1 ſhilling: but 5 and 4 is the ſhorter.

Item, for 11 s. *firſt diſpatch* 10 s. for which you muſt take the $\frac{1}{2}$ of the product: then laſtly, for 1 ſhilling take the $\frac{1}{10}$ part of the ſumme produced of the $\frac{1}{2}$ of the product, and adde them together.

Item, for 12 *ſhillings, where I will end with the firſt part of my Table.* For take the $\frac{1}{2}$ for 10 *ſhillings*. And then for 2 *ſhillings*, take the $\frac{1}{5}$ of the ſumme that came of 10 *ſhillings*, take and

adde

adde them together, or elſe if you pleaſe for 2 ſhillings, you may take the $\frac{1}{10}$ of the whole given number.

To write more of the manner of taking the true parts, I omit. The deſirous practitioners will (no doubt) conceive it. Alſo the *Table* is ſome aid to help the unperfect, whereupon by and by I will ſet down three or four of theſe notes in *Examples*, and the reſt I will leave to thine own induſtry and practice, to labour upon.

This is the order moſt commonly uſed in practice, when the number of ſhillings is not an *aliquot* part of a pound. But (*loving Reader*) after I have touched the even or *aliquot* parts of a pound that falleth out in pence and ſhillings, I will deliver two new Rules that ſhall drown this common order quite and clean: wherein ſhall be comprehended in one line, or working both of even and odde parts of ſhillings under 20 without regard whether it be an *aliquot*, or not an *aliquot* part; which two Rules (when they come in place) I commit to thy friendly judgement in working.

Now follow the examples upon the notes aforeſaid.

At 6 ſhillings the yard, what	321½ yards?
4 ſhillings	643
2 ſhillings	321 — 10
li.	964 — 16 s.

Otherwise by Multiplicatton of 6.

3215

6 *ſhillings*

1929|0

li —— 964 —— 10 *ſhillings.*

At 7 ſhillings the Ell, what 4563 Ells?

5 ſhillings 1140 —— 15

2 ſhillings 456 —— 6

li 1597 —— 1 ſhilling.

Otherwiſe by Multiplication of 7.

4563

7 s 3194|1

1597 —— 1

At 8 s. the piece, what 7563 pieces?

4 s 1512 —— 12

4 s 1512 —— 12

pounds 3025 —— 4 s.

Otherwiſe by Multiplication.

7563

8 s 6050|4

pounds 3025 —— 4 ſhillings.

At 13 s. the piece, what 401 pieces?

10 s 200 —— 10

2 s 40 —— 2

1 s 20 —— 1

pounds 260 —— 13

Other-

Otherwiſe by Multiplication.

	401	
13 s.	1203	
	401	
	5213	
pounds	260 ——	13 s.

Theſe and ſuch like queſtions of compound number, which I have here in this fourth rule for orders ſake ſet down, for that it hath been heretofore a common courſe of work, I account but ſuperfluous. For in the eight and ninth Rules of this my ſimple Addition ſhall appear, that the given price of any even or odde number of ſhillings, either under or above 20 ſhall be wrought at one or two workings at the moſt, how difficult ſoever the queſtion be.

3 Rule. Item, *there reſteth yet a kinde of practice, how to bring pence into pounds at the firſt working, whereupon you muſt underſtand that* 240 *pence maketh one pound, or* 20s. *In conſideration whereof I cut off the laſt figure or* 0, *and there remaineth but* 24 (*of which* 24) 8 d. *is the* $\frac{1}{3}$ *part thereof,* 6 d. *is the* $\frac{1}{4}$ *part,* 4 d. *is the* $\frac{1}{6}$ *part, and* 2 *pence is the* $\frac{1}{12}$ *part thereof.*

To reduce pence into pounds at one operation.

Whereupon if it were demanded what 1486 yards or pounds of any thing commeth to, at 8 pence the yard, in pricking or cutting off the firſt figure towards your right hand, for

the 0 that appertaineth to 240. There is remaining of the ſaid ſumme 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 li. and there reſteth 1, which 1 I put to the 6, that I prick or cut off, and it maketh 16 pieces of 8 pence, which I double to make into groats, and they make 32, whereof the $\frac{1}{3}$ part maketh 10 s. and there remaineth $\frac{2}{3}$ s. which is 8d, whereby it followeth, that the 1489 yards at 8 pence the yard maketh 49 li. 10 s. 8 d. as by the example ſhall appear.

Item, for 6d. take $\frac{1}{4}$ part of the number from the prickt figure; and if any unites remain, they are ſo many ſix pences, whereof taking the $\frac{1}{3}$ they are ſhillings, if there doe remain yet one, it is in value ſix pence.

Item, for 4d. take the $\frac{1}{3}$ part of the number from the prickt figure; if any unites doe remain, they are ſo many groats, which to convert into ſhillings, take the $\frac{1}{3}$ part. And if any yet remain, they are thirds of ſhillings, each one in value being worth 4 pence.

Item, for 3 pence, take the $\frac{1}{3}$ part from the prickt figure, if any unites remain, they are ſo many pieces of 3 pence, whereof in taking the $\frac{1}{4}$ part, maketh ſhillings: if any thing yet remain, they are the fourth parts of ſhillings, each one being in value 3 pence.

Item, for 2 pence, as appeareth alſo by the Table, take the $\frac{1}{12}$ part of the number from the prickt figure: if any thing remain, they are ſo many pieces of 2 pence, which by taking

the

the $\frac{1}{6}$ part, you ſhal turne into ſhillings, and if any unites remain, they are ſo many ſix parts of ſhillings, or pieces of two pence, whether you will.

If one coſt 8 pence, what 1486?
maketh pounds 49--10--8d.

If one coſt 6 pence, what 7865?
maketh pounds 196—12-6d

At 4 pence the yard, what 8736 yards?
maketh pounds. 145—12--0d.

If one coſt 3 pence, what 9874 worth?
maketh pounds 123—8—6d.

At 2 d. the Ell, what 7894 Ells to?
maketh pounds 65—15—8d.

But if your number of pence be not an aliquot or even part of 24, then muſt you bring them into the aliquot parts of 24. and make thereof diverſe products which muſt be added together, as by the queſtion hereafter following ſhall appear. 6 Rule.

Item, for 5 d. firſt take for 3 d. then for 2 d. and adde them together, according to the inſtruction of the *ſecond Rule*: or elſe firſt take for 4 d. then 1 d.

Item, for 7d. firſt take for 4 d. then for 3 d. and adde them together.

Item, for 9 d. firſt take for 6 d. then for 3 d. and adde them together.

Item, for 10 d. firſt take for 6 d. then for 4d. and adde them together.

Item, for 11 d. firſt take for 8 d then for 3d. and adde them together: as by theſe Examples.

Examples.

1 If one yard coſt 5 d. what 749|6

4 pence 126 — 12

1 peny 31 — 13

maketh pounds 158 — 5

Otherwiſe.

1 — 5 — 7591|6

3 pence 94 — 19

2 pence 63 — 6

maketh pounds 158 — 5 s.

2 If one coſt 7 d. what 98|7 worth?

4 pence 16 — 9

3 pence 12 — 6 — 9

maketh pounds 28 — 15-9 d.

Otherwiſe.

1 — 7 — 98|7

6 pence 24 — 13 — 6

1 peny 4 — 2 — 3

maketh pounds 28 — 15 — 9d.

3 If one coſt 9 d. what 98|7 worth?

6 pence 24 — 13 — 6

3 pence 12 — 6 — 9

maketh pounds 37 — 0 — 3 d.

Other-

Otherwiſe.

	1 — 9 — 98|7 ?
6 pence	24 — 13 — 6
3 pence	12 — 6 — 9
maketh pounds	37 — 00 — 3

4 If one coſt 10 pence, what 98\|7 ?

6 pence	24 — 13 — 6
4 pence	16 — 9 — 0
maketh pounds	41 — 2 — 6

5 If one coſt 11 pence, what 98\|7 ?

8 pence	32 — 18 — 0
3 pence	12 — 6 — 9
maketh pounds	45 — 6 — 9

But if you have any ſhillings and pence to be multiplyed together, then are you to take for the ſhillings according to the inſtruction of the third Rule: and for the pence according to the firſt Rule before mentioned: unleſſe you can ſpie the advantage thereof, and thereby help your ſelfe; as appeareth in this ſecond example, where firſt I work for 6 d. which is to be rebated out of the given number, and I have 719 li. 11 s. my deſire.

At 19s. 9d. the yard, what 738 yards?

	738	
10 s.	369 — 0	
5 s.	184 — 10	
4 s.	14 — 12	
6 d.	18 — 9	
pounds	719 — 11 s.	

Otherwise by Rebating.

738	
6 d.	18 — 9 s.
li.	7 — 19 — 11 s.

The like again is done by rebating, as by these two examples appeareth.

At 18 s. the Ell, what 418 Ells?

20 s.	418 — 16
pounds	376 — 4 s.

At 16 s. the Ell, what 517 Ells?

4 s.	10 — 8
pounds	413 — 12 s.

7 Rule. *And now I will touch a little the even part of a pound, that falleth out in pence and shillings, whereof for these parts you shall take such like part out of the given number that is to be multiplied, as the price of that given number beareth in proportion to a pound which also for their better aid is here set down.*

1 s. 8 d.	is the	part of a pound.
2 . 6		
3 . 4		
6 . 8		

Item,

Item, firſt for 1 ſhilling 8 pence take the $\frac{1}{12}$ part of the given number, and if any thing do remain they are twelve parts of a pound, each one being in value 1 ſhillings 8 pence.

Item, for 2 ſhillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplied; and if any thing doe remaine, they are eight parts of 1 pound, each one being in value 2 ſhillings ſix pence.

Item, for 3 ſhillings 4 pence, as appeareth by the Table, you muſt take the $\frac{1}{6}$ part of the given number, and if any thing do remain, they are 6 parts of a pound, each one being in value 3 ſhillings 4 pence.

Item, for 6 ſhillings 8 pence take the $\frac{1}{3}$ part of the number that is to be multiplied: And if any unites doe remaine, they are thirds of a pound, every one being worth 6 ſhillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreviation into the proportionate parts of a *pound*, as 16 ſhillings 8 pence maketh $\frac{1}{6}$: which 16 ſhillings 8 pence is eaſily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats.

Then ſet 60 the groats of a pound under 50: cutting off the two Ciphers as is here performed.

And then have you brought 16 *ſhillings* 8 pence into the knowne parts of a *pound*, which maketh

16--8
3
50
60

But

But yet gentle Reader, for thy further instruction, I have hereunto annexed in a *Table*, how pence and shillings bear proportion to a pound, which I commit to thy friendly benevolence; it will be some aid unto the ungrounded Practitioner; but I count him the best workman that can presently reduce his given price into the known and proportionate parts of a pound.

A Table of the Aliquot parts of a pound, or 20 shillings.

s.	d.	l.
0	2	1/10
0	3	1/80
0	4	1/60
0	6	1/40
0	8	1/30
1	0	1/20
1	3	1/16
1	8	1/12
2	0	1/10
2	6	1/8
3	0	1/6
3	4	1/6
3	9	1/16
4	0	1/5
5	0	1/4
6	0	3/10
6	3	1/16
6	8	1/3
7	0	7/20
7	6	1/8
8	0	2/5

s.	d.	l.
8	4	1/12
8	9	9/20
9	0	9/20
10	0	1/2
11	0	11/20
11	3	9/20
12	8	3/5
12	0	3/10
13	0	13/20
13	4	2/3
13	9	13/20
14	0	7/10
15	0	3/4
16	0	4/5
16	8	5/6
17	0	17/20
17	6	7/8
18	0	9/10
18	4	11/12
18	9	19/20
19	0	19/20

Here follow four examples upon the four Notes delivered.

At 1 s. 8 d. the yard, what	3884 yards?
maketh pounds	323—13—4d.
At 2 s. 6 d. the yard, what	4563 yards?
maketh pounds	570—7—6 d.
At 6 s. 8 d. the Ell, what	7562 Ells?
maketh pounds	2520—13-4d

Now by cuſtome you are able to work by all ſorts of ſummes being delivered in ſhillings and pence, as one ſhilling one peny, two ſhillings two pence, three ſhillings three pence, and ſo of all other: wiſhing you to have ſome conſideration of your queſtions, when they are ſet down, for there are many ſubtile abbreviations, and great advantages to be gotten, and eaſily to be perceived.

An of 3 s.———8 d. of 2 s. and 1s. 8d.
Of 4 s.———2d. of 3s.———4d. & 10 d. which 10 d. is ½ of 3 s.———4d.
Of 5 s.———8 d. of 4 s. 1 s.——8d.
Of 5 s. 10 d. of 5 s, and 10 d. which 10 d. is ⅛ of 5 s.

And by this mean when you have taken one product, you may oftentimes upon the ſame take

take another more briefly then upon the ſum which is to be multiplied, &c.

4 Rule.

NNow (Gentle Reader) that you have ſeen the vertue of the even or aliquot parts of a pound in ſhillings alone, and alſo in the aliquot parts of ſhillings and pence: according to my promiſe hereafter followeth a briefe and eaſier method for any even number of ſhillings, either under or above 20, then ever yet hath been publiſhed; Notwithſtanding M. Humphrey Baker, *whoſe travell is worthy commendation, and whom for knowledge ſake I reverence, hath in ſome part touched this firſt part, though not in this method. The work of the Rule both pleaſant, ready, and brief, as by the variety of the examples delivered thereupon ſhall appear. And firſt I will ſet forth a queſtion, thereby the better to expreſſe or teach you the order thereof: which is this.*

If one coſt 6 s. what 8574?

1	6.	8574
maketh pounds		2572——4 s.

Mr. Iohn Mellis his firſt Rule.

To the underſtanding of this example, after you have ſet down your given number in form of the Rule of 3, with a line drawn under it, you ſhall preſently ſet a prick under your firſt figure 4. toward your right hand drawing from the prick, as heretofore hath been practiſed, a little ſhort line, thereto ſet down the

the ſhillings anon, which done, multiply the firſt figure 4 by 6, the value of your price; (which here you ſee ſtandeth in ſight above the line) it maketh 24, which is one pound foure ſhillings. The one pound keep to carry to the next place, and the foure ſhillings ſet down at the end of the preſcribed line towards your right hand. Thus have you done now with 6 above the line, and alſo with 4 in the firſt place (for the prick under 4 doth ſignifie that 4 hath done his office.) Then ſecondarily for a generall Rule take but the ½ of the given price, which here is 3, which 3 is the number that ſhall now continue the reſt of the multiplication, and end the work, whereupon I multiply 3 into 7, ſtanding in the ſecond place it maketh 21, and with the 1 pound I kept in 22; ſet down 2, and keep 2 in mind, working according to the Rule of multiplication, delivering the tenths in minde in their due place, which done, the product from the prick to your left hand repreſenteth the pounds, and the other at the end of the ſhillings, as appeareth by the examples.

Note a generall rule.

If one yard coſt 2 s. what		7536?
1	2	7536
maketh pounds		753 — 12 s.
If one yard coſt 4 s. what		8792?
1	4	8792
maketh pounds		1758 — 8 s.

If

If one piece coſt 6 s. what 9537 ?
1 6 9537
maketh pounds 2861——2 s.

If one piece coſt 8s. what 7509 ?
1 8 7509
maketh pounds 300——12s.

If one coſt 12 s. what 5794?
1 12 5794
maketh pounds 3476——8 s.

If one coſt 14 s. what 3705 ?
1 14 3705
maketh pounds 2593——10 s.

If one coſt 18 s. what 5703 ?
1 18 5703
maketh pounds 5132——14s.

If one coſt 22 s. what 953 ?
1 22 953
maketh pounds 1048——6 s.

Let theſe ſuffice (gentle Reader) for an entrance into even numbers. And now I will ſhew the like rule for any odde or uneven part of a pound.

TO help you to the underſtanding of theſe other queſtions that hereafter follow: where in my firſt Example the given number is 6487

at

at 3 s. the yard: I multiply 3 above the dint into 7, it maketh 21. The one shilling is set down, and the 1 pound I keep. Now am I to take the ½ of three, which because it is an odde number I cannot.

Mr. Iohn Mellis his second Rule.

Therefore I shall keep and continue my multiplication by three still, and work by the ½ of the rest of the given figures or number, to wit, 648. And first the ½ of 8 which is 4 multiplied into 3, maketh 12, there to joyn the 1li. in minde, it maketh 13 set down 3, keep one. Then again multiply by two the ½ of four it maketh six, and with one in minde it maketh 7. Then lastly, take the ½ of six, which is 3. saying, 3 times 3 is 9, which 9 set down, and so is the question answered, as appeareth by the practice, and examples following.

At 3 s. the yard, what 6487 ?

1 3 6487

maketh pounds 973 — 1 s.

If one yard cost 5 s. what 4269 ?

1 5 4269

maketh pounds 167 — 5 s.

At 7 s. the Ell, what 6489 ?

1 7 6489

maketh pounds 2271 — 3 s.

If one Ell cost 9 s. what 2807 ?

1 9 2807

maketh pounds 1263 — 3 s.

At

At 11s. the Piſtolet, what		8263?
1	11	8263
maketh pounds		4544——13s.

If one piece coſt 13s. what		4629?
1	13	4629
maketh pounds		3008——17s.

But now note (gentle Reader) when the given price falleth upon an odde number, as 3, 5, 7, 11. 13, &c. then it is to be preſuppoſed that the given ſumme to be multiplyed, muſt be a ſumme made of even numbers, 2, 4, 6, 8, 10, &c. elſe cannot that queſtion be wrought at one line or working.

Providing alwayes that it may bear an odde figure in the firſt place towards your right hand, as appeareth in theſe ſix examples, which laſt were wrought, and ſuch like, &c. which may bear an odde number for the price, and be done at one line, or working very well.

But if the given price be an odde number, and the ſumme to be multiplied, odde numbers alſo: then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithſtanding to bring to paſſe, take this for a generall Rule. Firſt, worke for the even number, contained in that queſtion, or given price, according as you have learned, and then afterwards for the one odde ſhilling, take the ½ of

A gentrall Rule.

the ſumme given to be multiplyed, omitting the firſt pricke place, as was taught for the working of one ſhilling in my firſt Rule of Practice, and adde thoſe two together, and you ſhall haye your deſire.

Examples.

At 3s. the yard, what 7539 yards?

2s.	7539	18
1s.	3769	19
maketh pounds	1130	17s.

At 7s. the Ell, what 7539?

5s.	7539	
2s.	2261	14
1s.	3769	19
maketh pounds	6238	13

At 13s. the yard, what 7534?

10s.	3767	0
2s.	753	8
1s.	376	2
maketh pounds	4897	10

Note this well.

And thus have I abridged into theſe two rules how to bring any number of ſhillings, whatſoever they be, into pounds, with a briefer Method, then never yet hath been publiſhed, which I commend unto thy friendly cenſure and judgement in the uſe and practice thereof.

If

If one coſt 6s. 5d. what 1231?

6s.	369		6
4d.	20	10	4
1d.	5	2	7
maketh pounds	394	18	11

At 14s. 2 d. what 2825?

14s.	1977	10	
2d.	23	10	10
maketh pounds	2001	0	10

At 16s. 4d. what 2531?

16s.	2024	16	
4d.	24	3	8
maketh pounds	2066	19	8

At 3s. the Piſtolet, what 8325?
maketh pounds 1248——15s.

At 5s. the Crown, what 6529?

2285——35.

At 9s. the piece, what 6567?
maketh pounds. 2955——3s.

Theſe three laſt queſtions may ſeeme ſomething harder, yet they are eaſie enough, if you mark them well, If I ſhould explain them, then are they too eaſie. Therefore I leave them to whet the minds of the deſirous.

10 Rule. ITem *when any one of the ſummes, which is to be multiplyed, is compoſed of many Denominations, and the given number but of one figure alone; then ſhall you multiply all the Denominations of the other ſumme by the ſame one figure, beginning firſt with that ſumme which is leaſt in value toward your right hand, and bring the Product of thoſe pence into ſhillings, and the Product of the ſhillings into pounds, as by this example appeareth.*

At 3 li. 7s. 4d. a yard, what are 9 worth? maketh pounds. 30—6s—0d.

11 Rule. BUt *if any of the ſummes that are to be multiplyed, there be a broken number; Firſt worke for the whole according to the inſtructions that you have learned, and then take ſuch part of the given price, as that broken number beareth in proportion to the price, as in the examples following. After you have wrought for 3s. and for 6d. then are you to take the* ½ *of 3s. 6d. for the* ½ *yard, and adde that to the ſumme: So adding all the 3 Products together, which make 43 li. 2s. 9d. the juſt price of 245* ½ *Ells, and thus muſt you doe of all other.*

At 3s. 6d. the Ell, what 245 ½?

3s.	36——18
6d.	6——3
½	1—9
maketh	43—2—9

At

At 16s. 4d. the piece, what $14\frac{1}{4}$?

16s.	11		4
4d.	0	4	8
$\frac{1}{4}$		12	3
maketh pounds	12	0	11

If one piece coſt 4 li. 3s. $6\frac{1}{2}$ d. what 12 pieces?

4 li.	48		
3 s.	1	16	
6 d.		6	6
$\frac{1}{2}$			6
maketh pounds	50	2	7

The Proofe.

If 12 pieces coſt 50li. 2s. 6d. what one piece?
maketh pounds 4 — 3 — $6\frac{1}{2}$

ITem, *touching the manner how to underſtand the order of this queſtion, and others the like, firſt ſeeke how many times* 12 *is contained in* 50, *which is* 4 *times, and ſo reſteth* 2 *Pound, which* 2 *Pound converted into ſhillings, and joyned with the other* 2 *ſhillings, maketh* 42 *ſhillings: wherein is found* 12, *three times reſteth* 6s. *which turned into pence, putting thereto the* 6 *pence in the firſt place, it maketh* 78, *wherein* 12 *is found ſixe times, reſteth* 6d. *which containeth* 12, *but* $\frac{1}{2}$ *a time, put that* $\frac{1}{2}$ *to the* 6d. *and then the ſolution is* 4 lib. 3s. $6\frac{1}{2}$ d. *as appeareth by the Practice thereof.* 14 Rule

13 Rule. Item, *The like is to be done of any thing that is bought or sold after five score to the hundred, or the Quintall. As for example.*

If 100 pound coſt 27 li. 13s. 4d. what one pound?

27 li.—13s.—4d.

2

5. 5 | 53
12

1 | 10

53

d. 6 | 4 | 0
10 0. or $\frac{2}{3}$

Maketh 5s. $6\frac{2}{3}$d.

I have wrought this at length for the aid of the young learner, becauſe he ſhould underſtand how all the Multiplication is ſet down.

But to work it more neatly, it is by a little underſtanding, ended thus.

27 li.—13s.—4d.

20

5. 5 | 53
12

d. 6 | 40

100

Maketh 5s. $6\frac{2}{3}$d.

Item, *To the underſtanding of this and ſuch like queſtions, the right down line is all the guide, which is pulled down cloſe by 20. as you ſee in the example, where 27 pound 13 ſhillings is reduced all into ſhillings and maketh 553 ſhillings.*

The 5 towards the left hand being ſeparated with the hanging or right down line, is the juſt num-

number of ſhillings, that anſwereth the queſtion. Nextly, 53 ſhillings is multiplyed by 12, to reduce them to pence, putting to the 4d. it yeeldeth for the multiplication of the firſt figure two 116. the one beyond the line towards the left hand, is 1 peny towards the reſt of the price: then 53 alſo multiplyed by 1 yeeldeth 53: but the 5 behinde the line toward the left hand, is alſo 5 pence more, towards the price, which 1 and 5 I adde together under the line, it maketh 6d. So is there found now as appeareth by the Title, of ſhillings and pence, 5 ſhillings 6 pence.

Finally, I come now on this ſide the line towards the right hand, and under 12 I finde firſt 10, and then 3, which added together, maketh 40. under which 40, you muſt put the 100, and it maketh $\frac{40}{100}$ *which abbreviated, commeth to* $\frac{2}{5}$. *So the juſt price of one pound after 5 ſcore to the hundred, maketh* 5s. $6\frac{2}{5}$d. One example more, and ſo I will leave this Rule.

If 100 coſt $10\frac{1}{4}$d. what 9874 ?

6d	246	17	
4d	164	11	4
$\frac{2}{4}$d	20	11	5
$\frac{2}{4}$d	10	5	$8\frac{1}{2}$

Maketh	li.	4	42	5	$5\frac{1}{2}$
	s.	8	45		parts of a peny,
	d.	5	$45\frac{1}{2}$ 91		
			100 100		

Alſo the like may bee done of the uſuall weights here in *England*, (which is 112, for every hundred weight) in caſe you know the aliquot parts of a hundred weight, which are theſe, 56 li. 28 li. 14 li. and 7 li. For 56 li. is the $\frac{1}{2}$ of 112 li. 28 li. is the $\frac{1}{4}$ of 112 li. 14 li. is the $\frac{1}{8}$ and 7 li. is $\frac{1}{16}$ part.

Therefore for 56 li. take the $\frac{1}{2}$ of the ſumme of the money that 112 li. weight is worth.

For 28 li. take the $\frac{1}{4}$ of the ſumme of money that 112 li. weight is worth.

For 14 li. take the $\frac{1}{8}$ of the ſumme that 112 li. is worth.

And for 7 li. the $\frac{1}{16}$ of the ſumme of money that 112 li. is worth.

As for example; at 17 li. 19s. the hundreth pounds weight, that is to ſay, the 112 li. what ſhall 3 quarters and 7 pound coſt:

1. C. —— 17 li. —— 19s.	3 q. —— 7 li.
2 quarterns 8 ——	19 —— 6
1 quartern 4 ——	9 —— 9
7 pounds 1 ——	2 —— $5\frac{1}{4}$
Maketh pounds 14 ——	11 —— $8\frac{1}{4}$

The

The ſecond Chapter intreateth of the Reduction of divers meaſures to others value by Rules of Practice.

NOw will I ſhew a few examples of *Practice in reducing of Meaſures, as* Ells, Yards, Braces, Pawnes of Genee, &c. *Much more would I have touched, but that I feare the Booke will riſe to too great a Volume.* 18 Rule.

In 864 Ells of *Antwerpe*, how many yards of *London*?

864	864
432	216
216	648

maketh 648 yards of *London*.

ITem, *in theſe and ſuch like queſtions of* Flemmiſh *meaſure, to be brought into yards* Engliſh, *firſt take the* $\frac{1}{2}$ *of the given number as appeareth in the firſt example towards your left hand. Then take halfe of that Product, or the* $\frac{1}{4}$ *of the given number, and adde thoſe two Products together, as they ſhall be yards* Engliſh; *as by the example you may perceive.*

The

The second example toward your right hand is yet briefer then the first, whose worke is this; Take the $\frac{1}{4}$ of the delivered number, and that Product subtract out of the given number, and the rest sheweth your desire. Of these two wayes use which you thinke best.

The Proof.

In 648 yards *London*,
How many Ells of *Antwerpe*?

648
216
maketh 864 Ells of *Antwerpe*.

15 Rule. ITem, *for the understanding of this worke, first take the* $\frac{1}{3}$ *part of the yards of* London, *which found, adde that* $\frac{1}{3}$ *part and the yards together, as appeareth by the Practice, and the Product sheweth the Ells of* Antwerpe.

Item, in 20 yards of *London*,
How many Ells of *Antwerpe*?
maketh 426 $\frac{2}{3}$ Ells.

320 yards
106 $\frac{2}{3}$
426 $\frac{2}{3}$ Ells $\frac{1}{3}$

Proof.
426 $\frac{2}{3}$ Ells.
106 $\frac{2}{3}$
320 yards.

Other Reductions.

16 Rule ITem, *you shall understand, that forasmuch as six braces of* Millain, *make five Ells of* Antwerpe; *whereupon according to the* Rules of Practice, *you may reduce the one into the other, by the like reasons aforesaid, in taking the* $\frac{1}{6}$ *part*

and

and then ſubtract the ſame, to make Ells of Antwerpe. And again, by the contrary, taking the ½ part with adding the given number, to turne the Ells to Braces. As for example.

In 876 Braces, how many Ells of *Antwerpe*?

876

146

Ells 730 *Antwerpe.*

The contrary.

730 Ells *Flemmiſh.*

146 Braces.

816 Braces.

Ells 730 *Antwerpe.*

182 ½.

Yards 547 ½ *Engliſh.*

Thus appeareth, that 876 Braces by Practice, make 730 Ells *Flemmiſh*, which Ells *Flemmiſh* reduce into *Engliſh yards*.

So again upon the ſame firſt queſtion of Braces, I would know how many yards *Engliſh* they make.

After the rate that 100 Braces are worth 62 ½ yards.

876 Braces.

438

209 ½

I anſwer, 547 ½ yards.

ITem, *To the underſtanding of this work, and ſuch like, firſt take the ½ of the given Braces, and after take the ¼ of that halfe, or the ½ of the given number, and adde them together, and the Products are alſo yards* Engliſh.

Item,

ITem, *three Ells of* Rochell *make* 5 *Ells at* Lisbone. *So likewise three Ells at* Lions *make* 5 *Ells at* Antwerpe.

To worke these and such like, double the Ells of Lions, *and the Ells of* Rochell, *and from their Products subtract the* $\frac{1}{21}$ *and the rest shall be the Ells of* Antwerpe, *or the Ells of* Lisbone.

I. *Example.*

In 63 Ells of *Lions*, how many Ells of *Antwerpe?*

63
63
126
$\frac{1}{21}$
21

Ans. 105 Ells *Ant.*

In 100 Ells of *Rochel*, how many Ells of *Lisbone?*

110
200
$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{3}$

Ans. $166\frac{2}{3}$ Ells of *Lisb.*

Touching the proofe or return of these and such like questions for a generall Rule, you shal first take the $\frac{1}{3}$ of the given number; and adde that $\frac{1}{2}$ and the given number together, and the $\frac{1}{2}$ of that Product shall be your desire.

Example.

In 105 Ells of *Antwerpe*, how many Ells of *Lions?*

105
$\frac{1}{3}$ 21
$\frac{1}{3}$ 126

Ans. 63 Ells of *Lions.*

In $166\frac{2}{3}$ Ells of *Lisbone*, how many Ells of *Rochell?*

$166\frac{2}{3}$
$\frac{1}{3}$ $33\frac{1}{3}$
$\frac{1}{2}$ 200

Ans. 160 of *Roch.*

Que-

Queſtions of Factoridge and Intereſt, briefe and truly reſolved by the Rule of Practice without Time.

1 *Queſtion.*

AT 5 ſhillings *per Centum*, what comes 8860 li. 15s. 4 d. unto?

Anſwer. Note that 5s. is the fourth of 20. s. I take the $\frac{1}{4}$ part of 8860 li. 15s. 4d, which makes 2215 li. 3s. 10 d. Now the Root is 100, which you ſhould divide by, ſo cutting the two laſt figures away of the pounds, you have 22 li. then multiply 15 li. by 23s. ſo adde the 3 unto, you ſhall have 303s. cut away the two laſt figures, there reſteth 36s. Laſtly, there remains 3s. which I multiply by 12 to bring into pence, and ſo I finde 0d. and $\frac{44}{100}$ remaining which being abbreviated makes $\frac{22}{50}$ parts of a peny, ſo I find that there is gained 22 li. 3s. 0d $\frac{22}{50}$ parts of a peny.

	li. 22.	15s. 03.	10
		20	
s. 3		03	
		12	
		06	
0 d.		4	
	4	23	
	100	50	

Queſt.

2. *Queſt.* At 10 s. *per centum*, what comes unto? 1448 li. 16s. 8d.

Anſwer. Note that 10 s. is the $\frac{1}{2}$ of 20s. I take the $\frac{1}{2}$ of 1448 li. 16s. 8d. which makes 724 li. 8s. 4d. cut off the two laſt figures, and there reſteth 7 li. then multiply the 24 li. by 20s. and adde the 8 s. and it maketh 488 s. cut the two laſt figures off, and there reſteth 4s. then multiply 88s. by 12 d. and take in 4d. and there reſteth 1060d. cut off the two laſt figures, and there reſteth 10d. and $\frac{60}{100}$ which is $\frac{1}{5}$ of a penny: ſo the whole ſumme is 7li. 4s. 10$\frac{1}{5}$ which is the anſwer to the queſtion.

li. 7. 24. 8. 4.
20.
s. 488
12
180
88
d. 1060 | 10 | 0 | $\frac{3}{5}$

3. *Queſt.* At 15 s. *per centum*, what comes unto? 1008 l. 12s. 0d.

Anſwer. Note 15. that is $\frac{1}{2}$ and $\frac{1}{4}$ of 20. take the $\frac{1}{2}$ of 1008 lib. 12s. there reſteth 504 li. 6s. then take the $\frac{1}{4}$ adde them together, the total will be 756 li. 9s. cut off the two laſt figures, reſteth 7li. then multiply by 20s. and take in your 9s. it maketh 1129s. cut off the two laſt figures, there reſteth 11s. then multiply by 12 d.

504.6
252.3
756.9
20
11 | 29
12
58
29
3 | 48 | 24 | 12
100 | 50 | 25

there

there commeth 348d. cut off the last two figures, there resteth 3 d. and $\frac{48}{100}$ which being abbreviated maketh $\frac{12}{25}$ parts of a peny, so shall you finde 7 li. 11 s. 3d. $\frac{12}{25}$. which is the answer to the question.

li. 868 li. 13s. 4d. unto?

20

s. 13 | 73

12

150

73

d. 8 | 8 | 0 | 4

10 | 0 | 5

4. *Quest.* At 1 li. *per centum*, what comes unto?

Answer. Cut away the two last figures, and multiply by 20, and 12, and take in your shillings and pence. And you shall finde 8 li. 13 s. 8d. $\frac{4}{5}$ as doth appear by this work.

5. *Quest.* At 2 li. *per centum*, what comes unto?

Answer. Multiply the whole summe by 2 lib. thus, then cut off the two last figures of your pounds, as you did before, and you shall finde 112 li. then multiply by 20 and by 12, taking in your shillings & pence, and you shall finde 112 li. 3s. 4 d. which is either for Factor or Broker, &c.

5628 li. 6s. 8d.

2

112 | 16. | 13.4

20

3 | 33

12

70

33

4 | 00

6 *Quest.*

6. *Question.* At 3 li. *per centum*, what comes

Answer. Multiply the summe by 3 l. thus; then cut off the two last figures, and you shall find 24 li. then multiply by 20, and by 12. taking your shillings and pence and you shall finde 0s. 6d. $\frac{12}{50}$ parts of a peny, which is something above a half-peny.

800 li. 18s. 2d. unto?

3

24 | 02. 14. 6.

20

6 | 54

12

114

54

6 | 54 | 27

100 50

7. *Question.* At foure *per centum* what comes

Answer. Multiply by 4 li. thus, cut off the two last figures, Multiply by 20, and by 12. taking in your shillings and pence, and you shall finde 11 li. 19 s. 0d. $\frac{2}{25}$ parts of a peny, which is something above a farthing.

298 li. 15s. 9d. unto?

4

11 | 95. 3. 0.

20

19 | 03

12

06

03

036 | 18 | 9

100. 50. 25

8. Quest.

8 *Queſt.* At 5 li. $\frac{1}{4}$ *per centum*, what comes 3658 li. 16 s. 8 d. unto 5

Anſwer, Multiply by 5 li. thus, then take the $\frac{1}{2}$ of the whole ſumme and place the figures even, then take the $\frac{1}{2}$ of that $\frac{1}{2}$, and add all three ſums together, cut off the two laſt figures, then multiply by 20. and by 12. taking in your ſhillings, and pence, and you ſhall find 210 l. 7 s. 7 d. $\frac{2}{10}$ parts of a peny, which is the anſwer to the queſtion.

1829	04	4	
914	14	2	
210	38	05	10
	20		
	7	65	
	12		
	130		
	66		
	7	19	10
		10	8

9 *Queſt.* At 6 l. $\frac{1}{2}$ *per centum*, what comes 5684 li. 12 s. 6 d. unto 6

Anſwer, Multiply by 6 li. and then take $\frac{1}{2}$ of the whole ſumme, adde them both together, then multiply by 20, and by 12, taking in your odd ſhillings and pence, and you ſhall find 369 li. 10 s. 10 d. $\frac{2}{10}$ parts of a peny, which is the anſwer to your queſtion.

3410	7	15	0
284	2	06	3
369	50	01	3
1	20		
	05		
	01		
	15	3	
	100	20	

10 *Queſt.* At 7 li. $\frac{1}{2}$ *per Centum*, what comes 3868 li. 13s. 4d. unto?

Anſwer. Multiply by 7 li. then take the $\frac{1}{2}$ add them together, cut off the two laſt figures, then multiply by 20, you ſhall finde 290 li. 3s. The anſwer to the queſtion.

7

3708o. 13. 4
1934. 06. 8

290 1500. 0
20
3 00
284

11 *Queſt.* At 8 li. *per Centum*, what comes 2560 l. 17s. 6d. unto?

Anſwer. Multiply 8 li. cut off the two laſt figures, multiply by 20, and by 12, and you ſhall finde 204 li. 17s. 5d. $\frac{1}{25}$ parts of a peny.

8

204 87. 02. 0
20
17 42
12
84
42
5 04 | 2 | 2
100 | 50 | 25

11 *Queſt.*

Questions of Interest with Time, wrought by Practice.

1 *Question.*

AT 6 *per Centum* what comes unto for 1 month 368 li. 18 s. 8 d.

Answer; Multiply by 6 lib. there commeth 2813 li. 00 s. 0 d. then take for 1 moneth the $\frac{1}{12}$ of the Totall, and you shall find 234 li. 8 ? 4 d. of the two luft figures of the pounds, Multiply by 20 and by 12, taking in your odde money, and you shall finde 2 li. 6 s. 10 d. $\frac{1}{2}$ parts of a peny, which is the answer to the question.

2813. 00 0

li: 234. 08. 4

20

s. 6 88

12

180

88

d. 10 6 10 1/3

100 5

2 *Queſt.* At 7 li. ½ *per centum*, what comes unto for 2 months.

Anſwer, Multiply by 7 li. then take ½, adde them two together, then for your two months take the ⅙ of the Total, multiply by 20 and 12, taking in your odde ſhillings and pence, and you ſhall find 47 l. 10 s. 1 d. $\frac{2}{10}$ parts of a peny, which is the anſwer to the queſtion.

3 *Queſtion*, At 8 li. *per centum*, what comes unto for 3 months.

Anſwer, Multiply by 8 li. then take for your 3 monthes ¼ of the Totall, multiply by 20, and by 12 adding in your odde ſhillings and pence, and you ſhall finde 197 li. 5 s. 11 d. $\frac{2}{25}$ parts of a peny, your demand.

3800 li. 12 s. 8 d.

7

26604. 08 8

1900. 06 4

28504. 15 0

47 50 15 10

20

10 15

12

30

1 6

1 9 0

10 0

9864 li. 16 s. 4 d

8

78918. 10 8

197 29. 12 8

20

5 92

12

192

93

11 12 6 3

100 50 25

A

4 *Queſt.*

4 *Queſt.* At 6 li. $\frac{1}{2}$ *per centum*, what comes unto for 4 months.

Anſwer, Multiply by 6 lib. then take $\frac{1}{2}$ adde both together, then for your 4 months take $\frac{1}{3}$ part of the whole, cut away your two laſt figures, multiply by 20, and by 12: adde in your odde ſhillings and pence, and you ſhall finde 131 li. 14s. 10d. $\frac{42}{55}$ parts of a peny, your demand.

5 *Queſtion*, At 8 *per centum*, what comes unto for 5 months.

Anſwer, Multiply by 8li. then for 5 months take $\frac{1}{4}$ of the Totall, cut off the two laſt figures of your pounds, Multiply by 20 and by 12, add in your odde ſhillings and pence, and you ſhall finde 100. li. 13s. 4d. your demand.

6080 li. 13s. 0d.

8

36483 18 0
2040 06 6
39524 04

13174 11 2
20

14 91
12

184
91

10 94 47

200 50

3020 li. 00s. 00d.

8

24160 00 00.

6040 00 00
4026 13 04

100 66 13 04
20

13 33
12

70
33

4 00

6 *Question*, At 8 *per centum*, what comes unto for 6 months.

Answer, Multiply by 8 li. then for your 6 moneths take the ½ of the Totall, cut off the two last figures of your pounds. Multiply by 12. taking in your odde shillings and pence, and you shall finde 22 lib. 8s. 5d. $\frac{12}{15}$ parts of a penny, your desire.

7 *Quest*. At 8 li. *per centum*, what comes unto for 4 moneths.

Answer, Multiply by 8 li. then for your 7 months take ½ and ¼ of the Totall, cut off the two last figures of your pounds, then multiply by 20 and 12, taking in your odde mony, and you shall finde 275 li. 2s. 11 d. ½ your desire.

8 *Quest*.

8 *Question*, At 8 *per centum*, what comes unto for 8 months.

Answer, Multiply by 8 li. then for 8 months take $\frac{1}{3}$ of the totall, cut off the two last figures of your pounds, then multiply by 20, and by 12 adde in your odde money, and you shall finde 116 li. 5s. 9d. $\frac{2}{25}$ your desire.

3680 li. 08s. 03d.

8

29443. 04 0

9813. 08 0

9814. 08 0

116 28 16 0

20

5 76

12

152 1

76

912 322 160

100 50 25

9 *Quest.* At 8 li. *per centum*, what comes unto for 9 months.

Answer, Multiply by 8 li. then for your 9 moneths take $\frac{1}{2}$ and $\frac{1}{4}$ of the whole summe, cut off the two last figures of the pounds, then multiply by 29, and by 12. taking in your odd shillings and pence, and you shall finde 221 li. 1s. 11d. $\frac{2}{25}$ which is something above a farthing.

3684 li. 19s. 0d.

8

29479. 12 0

14739. 16 9

7369. 18 0

221 09. 14 0

20

1 94

12

188

94

11 28 14 1

100 50 25

10 *Queſt.* At 6. $\frac{1}{4}$. *per centum*, what comes unto for 10 months.

Anſwer, Multiply by 6 li. then take the $\frac{1}{2}$ and $\frac{1}{4}$ of 100 li. adde all 3 ſummes together, then for the 10 months take $\frac{1}{2}$ and $\frac{1}{3}$ of the Totall, add them together, cut off the two laſt figures of the pounds, multiply by 20, and 12, adding in your ſhillings and pence, cutting off the laſt figures of your ſhillings and pence, you ſhall finde 5 li. 12 s. 6 d. your deſire.

100. li. 05. 0d.
6
600. 10. 0
50. 0. 0
25. 0. 0
675. 0. 0
337. 10. 0
225. 00. 0
562. 10. 0
20
12 50
12
1 0
50
6 00

11 *Queſt.*

11 *Queſt.* At 8 li. *per centum*, what comes unto for 11 months.

Anſwer, Multiply by 8 li. then for 11 months take $\frac{1}{2}$ and $\frac{1}{4}$, from the Totall adde all three ſummes together; cut off the two laſt figures of your pounds. Multiply by 20 and by 12, adding in of your ſhillings and pence, cutting off the two laſt figures of your ſhillings and pence, and you ſhall finde 65 li. 0s. 7d. $\frac{17}{25}$ parts of a peny, your deſire.

886 li.	16s.	0d.	
		8	
7094	08	0	
2364	16	0	
2364	16	0	
1773	12	0	
65	03	04	8
20			
0 64			
12			
128			
64			
768	34	17	
100	50	25	

12 *Queſt.* At 8 li. *per centum*, what comes unto for 12 months.

Anſwer, Multiply by 8l, cut off the two laſt figures of the pounds, Multiply by 20, and by 12, adding in your ſhillings and pence, cut off the two laſt figures of your ſhillings, and the two laſt of your pence, and you ſhall find 726l. 8s. 11d. $\frac{17}{25}$ parts of a peny, your deſire.

9080 li.	12s.	2d.	
		8	
726	44.	17	4
	20		
8 97			
12			
11	68	34	17
	100	50	25

FINIS. The

The third Chapter teacheth of the Order and work of the Rule of *Three* in broken numbers after the Trade of *Merchants*, digreſſing ſomething from Maſter *Records*, which is comprehended in three Rules.

NOw, *that I have ſomewhat intreated of the* Rules of Practiſe, *I will give a few inſtructions, after my ſimple order, for the working of the* Rule of Three *in broken numbers, wherein I ſhall need to ſay the leſſe, becauſe I hope the ſtudious Learner, that hath travelled any thing in the* Grounds of Arts, *is not unfurniſhed of knowledge capable to underſtand me.*

But before I deliver any Inſtructions for broken Numbers, I will propone a queſtion which ſhall be wrought three ſundry wayes, thereby to ſhew as it were, three Degrees of Compariſon: how farre the Ruſe of Three in broken, for more ſpeed of worke, differeth from the whole, which I rather ſet downe for a view, that the ſtudious herein may be more deſirous to attaine broken, leaving any more to diſcourſe in Dialogue forme, but onely to give inſtructions where neede is: and in the reſt to put forth the queſtions, with their anſwers.

My

My first question is thus.

If one yard cost 6s. 8 d, what are 789 worth at that rate?

1 ——— 6s. ——— 8d. ——— 789 The first way.
12 80

80 63120 d.

Here the Product of the summe are pence, according to the nature of the middle number.

xx
63120 (5260 (263
400 320
220
200
answer ——— 263 li.

1 ——— 6⅔s ——— 789 The second way.
20
3 20
15780 s.

Here the Product of the summe are s. according to the nature of the middle number.

x
15786 (5260 (263
300 220
200
li.

1 ——— ⅓ ——— 789 The third way.
3 1 1
789

Here

Here the Product is pounds, according to the title of the ſecond number.

689 (263

I anſwer, 263 li.

Now that you have ſeen the three former vertues of the Rule of three, whoſe Products have firſt brought forth d. next s. and laſtly li. I will deliver three notes in order following: and with them a dozen queſtions, that ſhall ſhew the work of the Rule of three in broken Numbers or Fractions.

Note theſe three well.

1 The firſt foure ſhall be ſundry queſtions of a Fraction comming in the ſecond place.

2 The ſecond foure ſhall be of two Fractions comming in the ſecond or third place.

3 The third foure of Fractions in all three places.

Notes upon the firſt Rule for a Fraction comming in the ſecond place.

My firſt Queſtion is this.

1 Rule. *If one yard coſt me* 3s. 4d. *what are* 756 *worth at that price?*

The firſt variety. In ſetting down the queſtion to perform the work, I turn four pence into the part of a ſhilling, which is $\frac{1}{3}$ and then the queſtion ſtandeth thus:

1 —— $63\frac{1}{3}$ —— 756.

To

To the ready working of this queſtion, and all ſuch other like: my firſt note is this, which take for a generall Rule: That when any one Fraction ſhall come, either in the ſecond or third place, that the Denominator of that Fraction or Fractions, muſt alwayes be brought unto the Number, or Numerator of the firſt place; and thereby multiply the one into the other. A generall Rule.

And this benefit is alwayes gotten by the vertue of bringing the Denominator of the ſecond Numbers Fractions unto the firſt place: For the Fraction in the middle number is now releaſed: and the Product that commeth of the Multiplication, is of the nature and like the denomination of the whole number in the ſecond place which here are ſhillings. Note this,

Whereupon now to work the Queſtion, I bring 3, the Denominator of the Fraction in the ſecond place, unto the firſt Number 1, with a line ſet under thus $\frac{1}{}$ and the third under it thus $\frac{1}{3}$, ſaying, once 3, is 3 my Diviſor: that done, reduce $3\frac{1}{3}$ ſaying, 3 times 3 is 9, and the other 1 over 3 make 10, my ſecond number in the *Rule of Three*, by which 10 I doe multiply my laſt number 756, as appeareth by the worke thereof, and it yeeldeth 7560 ſhillings, my Dividend.

Then dividing 7560 by 3. my Diviſor, it yieldeth in quotient 2520 ſhillings, which maketh 126 pounds, as appeareth here moſt plainly, both by the Example, and the worke.

At

At 3s. 4d. the yard, what 756 yards?

1 — 3 1/3 — 756

3 — 10 — 18

75608.

2	2	
7560	25 20	(126.
3333	8220	

Answer 126 li.

Yet otherwise upon the same question, altering the price now into the proportion it beareth to a Pound, for the 3s. 4d. is ⅓ part of a Pound: which Example first standeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

1 — ⅓ — 756

The second variety.

1 — ⅓ — 756 6 — 1 — 1

756 pounds.

As soon as I have carried 6 the Denominator of my middle number unto my first place, as before hath been taught, I pull down 1, the numerator of 6, with a line under 6, thus. $\frac{1}{1}$, and that one in custome I pull down in sight; being the figure that I will multiply my third or last number by, according to the tenour of the *Rule of Three*. And because one can neither Multiply nor yet Divide (though here it is set down in form of Multiplication, the rather for your understanding) the Product of the Multiplication according to the Declaration

of

of this my first Rule or Note, is converted into the Title of my second number, which here are pounds. Now followeth the division performed in my Divisor 6, to make an end of that question.

23
756 (126. *which maketh* 126 li. *as before.*
666

And thus much for the variety in working that question.

And now followeth another.

My second Question.

If one yard of Cotton cost 8 $\frac{1}{4}$ d. what 859?

1 — 8$\frac{1}{4}$ — 859
4 — 33 — 33
2577
2577
28347

x			li.	s.	d.
2|322	2|1				
28347 (7886	(990	(29	10	6$\frac{3}{4}$
4444 1222	220				

This Question was also wrought like the first, and bringeth forth 29 li. 10 s. 6$\frac{3}{4}$ d. *the price of* 859 *yards.*

My

My third Question.

If seven pounds of any thing cost 3 li. —10s. what comes 987 pounds to?

li.

7 —— 3½ —— 987

2 —— 7 —— 7

14 6909

08

142

2347

6909 (493 11/14

1444

12

Answer, 493 li. —10s.

Notes upon my second Rule for two Fractions coming in the second and third place.

My first Question is this.

IF one Ell cost 13s. —— 4d. what halfe a quarter or ⅛ of an Ell?

Answer, *First bring* 13s. —4d. *into the parts of a pound, which is* ⅔, *and then will the question stand thus.*

1 —— ⅔ li. —— ⅛

ITem, for the performance of this work, doe as before was taught in the first Rule: first

bring

bring 3 *the Denominator of the ſecond Fraction unto your firſt number* 1, ſetting a line under it thus: $\frac{1}{}$, Saying once 3 is 3, that done, bring 8 the *Denominator of the third Fraction*, ſetting it under 3, and multiply them together, ſaying, 3 times 8 maketh 24, which 24 is your *Diviſor*. (Now have you done with the *Denominator* 8) therefore you ſhall put a line under thus, $\frac{3}{}$. And the like line alſo under $\frac{8}{}$, ſetting or pulling down under them their own *numerators*, that is, 2 under 3, and alſo 1 under 8, as appeareth in the Example, which *numerators* for a generall rule evermore to be pulled downe of cuſtome in ſight, to multiply the one by the other, according to the tenour of the *Rule of Three*. Then I multiply the one by the other, ſaying, once 2 is two, which ſignifieth 2 li. being of the nature and like denomination of the middle number, which 2 li. is to be reduced into ſhillings, otherwiſe it cannot be divided by my firſt number 24.

Then dividing 40 by 24, the quotient bringeth forth 1 $\frac{2}{3}$. So much is $\frac{1}{8}$ of an Ell worth after that rate. Otherwiſe, although 2 pound could not be divided by 24, yet it might have beene abbreviated to $\frac{1}{12}$ of a pound: which is worth 1s. 8d. as before.

li.

1 —— 2/3 —— 2/8 (1

3 2 1 2 (6

8 20 40 (1 2/3 s.

24 40 24

Second Queſtion.

IF *one pound of any weight coſt* 13 *ſhillings* 4 *pence, what are* 2/8 *of the pound worth after that rate?*

Anſwer, Reduce the 13 ſhillings 4 pence into the parts of a pound: which is 2/3. and then will the queſtion ſtand thus.

li.

1 —— 2/3 —— 2/8

THen, *for the underſtanding of this, if you mark well the laſt Example, this and the reſt lieth open, and needs ſmall inſtruction. For as you did laſt, ſo again, bring the* Denominator *of the ſecond and third* Fraction, *unto the firſt figure* 1, *multiplying the one into the other* 3, *which maketh alſo* 24, *your* Diviſor.

Note. Then making a line under 3 thus, 3 and a line under 8 thus, 8 and pulling downe their *Numerators* under each figure, that is 2 under 5, and 7 under 8, which as I ſaid before for a generall rule I pull down of cuſtome in ſight,

ſight, to be the two Numbers, that of duty ought to be multiplied together; which done, I bring 2, being the leſſer figure under 7, multiplying them together, it maketh 14, which are of the nature of the middle number: that is to wit, pounds, which 14 cannot aptly be divided among 24. therefore are reduced into *ſhillings*, as is plainly to be ſeen in the example: then 280 *ſhillings* parted among 24 yieldeth for his *quotient* 11s. 8d. your deſire, and the juſt price of $\frac{2}{3}$ of an *Ell*. Otherwiſe 14. though it could not be divided by 24. might by *Mediation* or *Diviſion* in broken *Numbers* have been divided or abbreviated to $\frac{7}{12}$, which in effect being reduced to his known parts, maketh 11s. 8d. as before. But my good will and meaning is to aid young beginners: therefore have I reduced the 14 *pound* into *ſhillings*, which is the eaſier way.

Now followeth the example.

1	2	7	1
3	3	8	2
8	2	7	4 | 6
24		3	280 (11 $\frac{2}{3}$ s.
		14	214
		20	2

280 s. I anſwer, 11 $\frac{2}{3}$ s.

The third example.

If one yard coſt me 2s.——6d. what $345\frac{1}{4}$ yards?

Anſwer, Firſt put 6d. into the parts of a ſhilling, and then the queſtion ſtandeth thus:

$$1 \text{——} 2\tfrac{1}{2} \text{——} 345\tfrac{1}{4}.$$

Item, to the ready underſtanding of this, and all ſuch like, according as before hath been declared, bring the *Denominators* of the ſecond and third *Fractions* unto the firſt place, multiplying them the one into the other, all which make 8 for the common *Diviſor*. Then next reduce your ſecond number: ſaying, two times 2 is 4. and 1 is 5. as was taught in the example aforeſaid. Laſtly, reduce your third number $345\frac{1}{4}$ all into fourths, and they make 1381. which 1381 is to be multiplied by 4. according to the tenour of the *Rule of Three*: which done, maketh 6905s. and divided by 8. your *Diviſor* yieldeth in Quotient $863\frac{1}{4}$s. which maketh in pounds 42li.3s. $1\frac{1}{2}$: and ſo much are the $345\frac{1}{4}$ yards worth at that price.

The ſame queſtion wrought again by two ſhillings 6 pence, is now converted into the parts of a pound, and ſtandeth thus:

$$1 \text{——} \tfrac{1}{3} \text{——} 345\tfrac{1}{4}.$$

Item, After I have brought here my ſecond and

and third *denominator* unto my firſt place, and found 32 to be my *diviſor*; having thus finiſhed my firſt place with all things unto him belonging (which is meant of bringing and Multiplying the *denominators* of the ſecond and third *Fractions* into him) I then goe in hand to ſee what is to do in my ſecond place, where preſently of cuſtome I pull down my Numerator 1 under 8. being the figure in ſight that ſhall multiply my third number.

Then laſtly, I reduce $345\frac{1}{4}$ all into fourths as afore was practiſed, which maketh 1381 the which 1381 I am to multiply by 1 my ſecond number, they are nothing increaſed, but by the *Metamorphoſis* of my work they are now 1381. pound, being of the nature of the middle number, as I have often ſhewed you, which divided by 32 my Diviſor yieldeth 43 pound, and $\frac{5}{32}$, which $\frac{1}{32}$ of a pound reduced into known numbers, make 3 ſhillings $1\frac{1}{2}$d. as before.

Example.

						2		
1	——	$\frac{1}{8}$	——	$345\frac{1}{4}$		105		
32		1		1381	(43	5		
			1381	522		32		
				3				

NOw follow foure other queſtions, vvhich are in all three places broken numbers; or vvhole and broken together.

Item, Firſt, for the finding out of your *Diviſor*, you ſhall take this for a moſt certaine, and generall Rule: That you muſt multiply the *Numerator* of the firſt *number* in the queſtion, by the *Denominator* of the ſecond: And that *Product* againe, by the *Denominator* of the third: And the totall thereof ſhall be your *Diviſor*.

Secondly, for a generall rule to find out your *Dividend*, *multiply* the *Denominator* of the firſt number by the *Numerator* of the ſecond, and the whole thereof by the *Numerator* of the third. And the totall thereof ſhall evermore be your *Dividend*.

Now for an example, I propone this queſtion, thereby to make my meaning more plaine, and to ſhew you, as I have done in the reſt, the manner and order of the work.

If $\frac{2}{3}$ *of any weight or meaſure coſt* $\frac{3}{8}$ *of a pound, or* 20s. *what are* $\frac{1}{8}$ *of the like weight or meaſure worth after that rate?*

Example.

$\frac{2}{3}$ —— $\frac{3}{6}$ —— $\frac{1}{8}$

ITem, for the more plainer underſtanding hereof, and all other the like in broken Numbers: Firſt, you ſhall pull down two, the *Numerator* of the firſt Number or Fraction, with a line under, thus, $3\frac{1}{2}$: that done, according as you have learned before, bring 6, the *Denominator* of the ſecond Fraction, and ſet it under

under two, multiplying the one into the other, which maketh 12. Then laſtly, bring 8. the *denominator* of the third *Fraction*, and ſet it under 12. multiplying that 12 by 8. which amounteth to 96. or elſe for more brief, multiply 6 by 8 ſaying, ſix times 8 makes 48. which 48 ſet under 2. and multiply the one into the other, it maketh 96. as before. And this 96 is the firſt number in the *Rule of Three*. That ſhall alwayes for a moſt generall Rule be your *diviſor*.

Secondly, to work for your *dividend*, you ſhall, (as it hath beene ſufficiently declared before) pull down 5. the *numerator* of your ſecond *Fraction*, and ſet it under 6, with a line under, thus $\frac{5}{6}$.

That done (as you know) you are to pull down 3, the *numerator* of the third *Fraction*, and ſet it under 8, with a line under it, thus, $\frac{3}{8}$, multiplying the one into the other, according to the Tenour of the *Rule of Three*; which maketh 15. Then according to my Note, forget not to bring the *denominator* of the firſt *Fraction*, which is 3. under 15. and multiply them together, which maketh 45. which 45 is your *dividend*, and are of the nature of *denomination* of the middle number, as I have taught you before: And therefore are 44 li. which aptly cannot be divided by 96. Therefore you ſhall reduce the 45 li. into s. as you ſee performed in the Example, which amounteth to 90s. which divided by 96 your *di-*

viſor, it yieldeth 9s. and $\frac{16}{20}$ of a ſhilling, vvhich in leſſer termes is $\frac{4}{5}$: vvhich $\frac{4}{5}$ in mony maketh $4\frac{1}{2}$d. and ſo much vvill the aforeſaid $\frac{4}{5}$ coſt, as by the vvork followving ſhall appear.

The Example.

2 5 3
3 6 8
2 5 3
6 5 3
12 15 9 6
8 3 9 6 (9 $\frac{16}{20}$ 4
96 45 96
20
900

Otherviſe though 45 could not be divided by 96. yet by Diviſion in broken numbers it might have been abbreviated to $\frac{15}{31}$ of a pound, vvhich reduced into known parts, will make 9s. $4\frac{1}{2}$d. as before.

Now my ſecond example ſhall be the proof of this queſtion.

If $\frac{1}{18}$ yards coſt $\frac{12}{32}$ of a pound, or 20 ſhillings, what ſhall $\frac{2}{3}$ coſt?

Anſwer, Work as was taught you before, and you ſhall have your deſire.

Here

$$\frac{\frac{2}{3}}{\frac{3}{32}} \quad \frac{\frac{11}{32}}{\frac{15}{2}} \quad \frac{3}{2}$$

$$\frac{32}{96} \quad \frac{2}{30}$$

$$\frac{3}{288} \quad \frac{8}{240}$$

Here as appeareth by the vvork, the Multiplication being ended, 240 is to be divided by 288, vvhich to some perchance may seem hard, yet notvvithstanding is the vvork good. Therefore abbreviate 240. by 288. as you see here is practised: and the end of your abbreviation shall come to $\frac{1}{6}$ your desire, $\frac{240}{288}\ \frac{120}{144}\ \frac{60}{72}\ \frac{30}{36}\ \frac{5}{6}$.

Otherwise, 240 | 120 | 60 | 30 | 5
288 | 144 | 72 | 36 | 6

Otherwise, 340 | 40 | 5
288 | 48 | 6

The third Question.

If $\frac{3}{4}$ elle coste 13s 4d. what 156 $\frac{1}{2}$ elles?

Answer, To vvork this question the shortest vvay reduce 13s 4d. into the parts of a pound, vvhich is $\frac{2}{3}$.

Then as you did afore, after you have set dovvn the question, the *Numerator* of the first *Fraction* 3 is pulled dovvn under 4. and *Denominators* of the other tvvo *Fractions* multiplyed into

into him; which maketh 18. your *Diviſor*.

Then the *Numerators* of the ſecond *Fraction* is pulled downe, under 3 in cuſtome now in ſight, ready to *multiply* my third number, by which is performed as ſoon as the laſt numbers $156\frac{1}{2}$ is reduced into halfs.

Then laſtly, I multiply that Product by 4. the *Denominator* of the *Fraction*: it yieldeth 1504. which I divide by 18. and my quotient is 139. l. and $\frac{8}{9}$ of a pound remaining, which is worth 2s. $2\frac{2}{3}$d. And ſo much will $156\frac{1}{2}$ Elle coſt, as by the work following doth appear.

3	2	$156\frac{1}{2}$		
4	3	313	47	li.
3	2	2	1762	
6	626	2504	(139$\frac{8}{9}$	
18	4	1888		
	2504	16		

The fourth queſtion.

If $2\frac{1}{2}$ *Elle coſt* $1\frac{1}{3}$ *Pound, what commeth* $26\frac{1}{4}$ *Ells to?*

Item, to the workmanſhip of this queſtion, firſt conclude your ſecond *number* in ſaying, three times 1 is 3, and 1 is 50. Then bring the *multiplication* of the *Denominators* of the ſecond and third *Fractions* which maketh 20. and *multiply* that 20. by 5. your firſt *numerator*, and it maketh 60. which is your *diviſor*.

Then the *Reduction* of the ſecond Number, which

which is 5, *multiplyed* by 117 the Product of the laſt *numbers* reduction, make 585, which 585 yet reſteth to be *multiplied* by 2, the *Denominator* of the Fraction in the firſt place, yieldeth 170, which divided by your *Diviſor*, 160. yieldeth 19. pound, 10s. as appeareth by the work thereof.

Thus having now touched the 12 queſtions whereof I firſt pretended, which with diligence and oft practice, I truſt are ſufficient to aid the deſirous unto the working of any broken numbers, I will now intreat of divers neceſſary rules incident unto traffick, as hereafter followeth.

The fourth Chapter teacheth of loſſe, and gaine, in the Trade of Merchandiſe.

IF one yard coſt 6s. —— 8d. and the ſame is ſold againe for 8s. —— 6d. the queſtion is, what is gained in one hundred pounds laying out on ſuch commodities?

Anſwer, The *Rule of Three* direct, applyed two manner of wayes to doe the ſame: the one is to ſay, If $6\frac{3}{4}$ give $8\frac{1}{2}$, what giveth 100? *Multiply* and Divide, and looke what your quotient bringeth forth, above your laying out, is the neat gaines and ſolution to your queſtion:

queſtion: If you follow the worke, your ſolution will bring forth 127li. ——10s. which is 27li. ——10. more then your principall, and ſo much is gained in the 100 pounds laying out.

Item, to work it the other way, which I take the neareſt, ſeek the difference betwixt the juſt price and the other price, which is one ſhilling ten pence, then ſay by the *Rule of Three*.

If $6\frac{3}{8}$s. gain $1\frac{5}{8}$s. what ſhall 100 pound gain? *Multiply* and divide, and you ſhall find 27li. 10s. and ſo much is gained in 100li. laying out.

You may uſe which of theſe two wayes you think good.

The proofe.

If a yard of cloth be delivered for 8s. 6d. whereupon was gained after the rate of 27li. 10s. in 100. pounds laying out: The queſtion is, what the yard coſt at the firſt hand?

Anſwer, Put your gain 27li. ——10s. to 100 pounds, all maketh 127li.--10s. Then ſay, If 127li. 10s. give but 100. pounds, what giveth 8½s. Work, and you ſhall finde 6s. 8d. the true ſolution to your queſtion.

Yet another example or proofe upon the firſt Queſtion.

If one yard coſt 6s.--8d. the queſtion is, at what price the ſame is to be ſold again, for to gain

gain 27 lib. 10s. in 100. pounds laying out?

Anſwer, *Say by the* Rule of Three, *if* 100li. *gain* 127li. 10s. *what giveth* 6 $\frac{2}{3}$s? *Multiply and divide, and you ſhall finde* 8s. 6d. *your true ſolution.*

If one Ell coſt 7s. 8d. and be ſold again for 8s. 6d. *The queſtion is,* What is gained in 20. pound laying out in ſuch commodities?

Anſwer, *Seeke the difference betwixt the juſt price, and the other price which is ten pence, and then apply the* Rule of Three, *as before is taught, ſaying, If* 7 $\frac{2}{3}$s. *give* $\frac{1}{6}$ *ſhillings, what giveth* 20li? *Multiply and divide, and you ſhall finde* 2li. 3 $\frac{11}{23}$s. *and ſo much is gained in* 20li. *laying out.*

The proof alſo by an example of Loſs.

A Merchant hath bought Holland cloth at 8s. 6d. *the ell, which proveth not to his expectation, whereupon he is content to loſe* 2li. 3 $\frac{11}{23}$ *in* 20. *pounds laying out. The queſtion is, what price ought to be made of the Cloth, abating this loſſe?*

Anſwer, Doe as before in Gains hath been taught, putting 2li. 3 $\frac{11}{23}$s. to your 20. pound, all together, maketh 22li.-3 $\frac{11}{23}$. Then ſay by the *Rule of Three.* If 22li 3 $\frac{11}{23}$s. give but 20ſi. what ſhall come of 8 $\frac{1}{2}$s? work, and you ſhall finde 7s.-8d. the juſt price that the Ell ought to be ſold for after the rate of this loſſe.

Thus

Thus it appeareth evidently, as in company the *Rule* is appliable as well to gain as loſſe.

If 20 $\frac{1}{4}$ *yards coſt* 36 li. $\frac{1}{2}$ 10s. *how ſhall I ſell the ſame again* $\frac{2}{3}$ *of the Principall, or to make of* 3,4. *which is all one.*

Anſwer, By the *Rule of Three*, If 3 doe give 4, what will 36 $\frac{1}{2}$ give? Multiply and divide, and you ſhall find 48 $\frac{2}{3}$ pounds, Then ſay again, if 20 $\frac{1}{4}$ yards do give 48 $\frac{2}{3}$ pounds, as well principall as gain, what will one yard be worth at that price? Multiply and divide, and you ſhall find 2 li. $\frac{16}{243}$.

If one Ell of Cloth coſt me 8s. 8d. *and afterwards I ſell* 10 $\frac{1}{2}$ *Ells thereof for* 5 li. 13s. 4d. *I would know, whether I winne or loſe: and how much upon the* 100 *pounds of mony.*

Anſwer, See firſt at 8s. 8d. the Ell, what 10 $\frac{1}{2}$ Ells comes to, and you ſhall finde 4 li. 11s. and I ſold the ſame for 5 li.—13s.—4d. ſo that I did gain upon the 10 $\frac{1}{2}$ Ells 22 ſhillings 4d. Then if you would know how much is gained in 100 pounds, I ſay by the *Rule of Three*, if 4 li.—11s. did gain 22s.—4d. what will 100 pounds gain? Multiply and divide, and you ſhall find 24 li.—10s.—10d. $\frac{40}{91}$ and ſo much is gained in the 100. pound of mony.

If 18 $\frac{1}{2}$ *yards coſt me* 11 *Pound five ſhillings, and I ſell the yard again for* 16 *ſhillings, the queſtion is whether I doe winne or loſe, and how much in or upon the pound of mony?*

An-

Anſwer, Look what the 12 ½ yards come to at 16s the yard, and you ſhall finde ten pound. But they coſt 11 pound 5 ſhillings. So there is loſt upon the whole 1 pound 5s. Then to know how much is loſt in the pound, ſay by the *Rule of Three*, if 11 ¼ pound doe loſe 1 ¼ pound, what will 1 pound loſe? Multiply and divide, and you ſhall finde 2s. 2 d. ⅔. and ſo much is loſt in the pound of mony.

If I ſell the 100 weight of any commodity for 4 Pound, whereupon I doe loſe after ten Pound in the 100 Pound, I demand how much I ſhall leſe or gain in the 100 li. if in caſe I had ſold the ſame for 4 Pound ten ſhillings.

Anſwer, Say, if 90 pound yield 100. how much will 4 give? Multiply and divide, and you ſhall finde 4 4/9. Then ſay again, if 4 4/9 give me 4 ⅓ what will 100 come to? Multiply and divide, and you ſhall find 101 pound ¼ which is more then 100 pound by 1 pound 5 ſhillings: and ſo much is gained in the 100. pound.

A Merchant hath ſold Currants for the ſum of 430 Pound, and he hath gained therein after ten Pound in the 100. Pound. The queſtion is, to know how much he gained in all.

Anſwer, Say by the *Rule of Three*. If 100 pound doe gain ten pound, what will 430 pound gain? Multiply and divide, and you ſhall finde 43, and ſo much hath hee gained in all.

If

If one yard be worth 28 $\frac{1}{3}$s. *for how much ſhall* 10 *yards be ſold to gain after* 8li. 6s. 8d. *in the* 100 *pound?*

Anſwer, Firſt, adde 8li.—6s.—8d. to 100. Then ſay, if 100li. do give 108 $\frac{1}{3}$s. for principall and gain, what will 28 $\frac{1}{3}$s. principall yield? Multiply and divide, and you ſhall finde 30$\frac{1}{9}$s. Then ſay, again, by the *Rule of Three*, if 1 yard do give 30$\frac{1}{9}$s. (which is as well the principall as the gain) what ſhall ten yards give? Multiply and divide, and you ſhall finde 15l. 8s. 9d. And for the ſame price ſhall the ten yards be ſold for to gain after the rate of 8li—6s.—8d. upon the 100.

A branch or proof out of this Queſtion.

A Merchant hath ſold clothes for 15li.-8s.-9d. *and he hath gained in the whole the ſumme of* 1li.—3s.—9d. *The queſtion is to know how much he hath gained in the* 100. *pound?*

Anſwer, To know this, firſt rebate the gains from the price, and there will remain 14li. 5s. 0d. Then ſay by the *Rule of Three* direct. If 14 li. $\frac{1}{5}$ give me 1 li. 3 $\frac{1}{4}$, what will 100li. give? Multiply and divide, and you ſhal find 8li. 6s. 8d. the effect deſired, the proof is apparent in the queſtion before.

Yet

Yet another branch or proofe of the first Queſtion.

If ten yards be delivered for 15 li. 8 s. 9 d. *whereupon was gained after the rate of* 8 li. 6 s. 8 d. *upon the* 100. *Pound. The queſtion is, what the yard did coſt at the firſt hand?*

Anſwer. Firſt, ſay by the *Rule of Three*, if ten with principall and gain yield 15 li. 8 s. $\frac{1}{2}$ ſhillings, what ſhall 1 yield? Multiply and divide, and you ſhall find 30 $\frac{3}{8}$ s. Then ſay again by the *Rule of Three*, if 108 $\frac{1}{2}$ principall and gain give but 100. what ſhall 30 $\frac{3}{8}$ of principall and gain yield? Work, and you ſhall finde 21 $\frac{1}{2}$ s. And ſo much did the yard coſt at the firſt peny.

If one yard coſt 36 s. *how much ſhall* 12 *yards be ſold for, to gain after the rate of* 10 li. *in the* 100?

Anſwer. Firſt, ſay, If 100. give 110 li. principall and gain, what will 36 s. give? Multiply and divide, and you ſhall finde 39 $\frac{3}{5}$ s. Then ſay again by the *Rule of Three*: If one yard of principall and gain yield 39 $\frac{3}{5}$ ſhillings, what ſhall 12 yards gain? Multiply and divide, and you ſhall finde 23 li. 15. $\frac{1}{5}$ s. which $\frac{1}{5}$ s. in knowne number, is 2 $\frac{2}{5}$ d. And for the ſame price ſhall the 12 yards be ſold, to gain after the rate of 10 in the 100.

The Proof.

If 12 *yards be ſold for* 23 li. 15 s. 2 $\frac{2}{5}$ d. *where-*

upon is gained after 10 li. *in the* 100. *The question is, what the yard cost at the first peny?*

Answer, First say, If 12 give 23 li. $15\frac{2}{3}$s. what one yard? Multiply and divide, and you shall finde $39\frac{2}{3}$s. Then say againe by the *Rule of Three*, If 110. pounds give but a 100. what shall $39\frac{2}{3}$s. give? Work, and you shall find 36s. the just price of the yard at the first hand.

Item, *When one Merchant selleth wares to another, and he giveth to the buyer* 1 li. 6s. 8d. *upon the score, or* 20 li. *The question is, How much shall the buyer gain upon the* 100 li. *after that rate?*

Answer, First, adde 1 li. 6s. 8d. unto 20 li. and they are $21\frac{1}{3}$. Then say, If 20 pound give $21\frac{1}{3}$, what shall 100. give? Multiply and divide, and you shall finde $106\frac{2}{3}$. So the buyer getteth after the rate of $6\frac{1}{3}$ li. upon the 100 li.

Gentle Reader, other necessary questions appertaining to Losse and Gain, you shall have in the eighth Chapter of this Treatise.

The

The fifth Chapter entreateth of Loſſe and Gain upon time, wrought by *the double Rule of Three, or by the Rule compoſed:* which is contained in foure ſpeciall ſelected branches, or queſtions of divers formes, *each one of them ſpringing from the firſt Queſtion,* and each one of them alſo being a proof to other, *&c.*

IF one yard coſt me 2ſ. 8d. *ready money, and after I ſell the ſame again for* 2ſ. 10d. *to be paid for it at the end of three months: the queſtion is, what I gain upon the* 100 li. *in* 12. *moneths.*

Anſwer, Firſt ſay, if $2\frac{1}{3}$ gain $\frac{1}{3}$, what ſhall 100 li. gain? Multiply and divide, and you ſhall find $6\frac{1}{4}$ li. Then ſay again, by the *Rule of Three,* if three months gain $6\frac{1}{4}$ pound, what ſhall 12. months gain? Work, and you ſhall find 25 li. and ſo much ſhall I gaine in 12 moneths after that rate.

Item, You may alſo work it all at one working by the firſt part of the *Rule of Three* compoſed, ſaying, if $2\frac{2}{3}$ d. in three months doe gain $\frac{2}{3}$ of a ſhilling, (which is 2 d.) what will 100 li. gain in 12 months? Which for thy further encouragement, the work of this one example I

have here put down, to verifie that I affirme in the firſt part of this *Ground of Arts*, that this Rule, and ſo all others, more rejoyceth in *Broken*, then in *Whole*.

s.	months	s.	li.	mo.
$2\frac{2}{3}$	3	$\frac{1}{6}$	100	12
8		1	20	
3 2			2000	
24 72000		5	3	
6 24444	(5000 (2		6300	
144 144	2220		12	
	1		72000	

Where the Multiplication and the diviſion being ended, maketh 25 li. your deſire.

If a yard be delivered for 2s. 10 d. *to be paid at* 3 *months, whereupon was gained after the rate of* 25 li. *in the* 100. *for* 12 *months, the queſtion is now, what the yard coſt at the firſt hand?*

Anſwer. Firſt ſay, if 12 months gain 25 li. what ſhall 3 months gain? Work, and you ſhall finde $9\frac{1}{4}$ li. Then ſay again the ſecond time, if $106\frac{1}{4}$ li. give but 100. what ſhall $2\frac{1}{6}$ s. give? Work, and you ſhall finde 2s. 8 d. which is the juſt price that the yard coſt at the firſt hand.

If one yard of Cloth coſt me 2 s. 8 d. *ready money, for what term ſhall I ſell the ſame again for* 2 s. 10 d. *ſo that I might gain after the rate*

of

of 25 pound upon the 100. *pound in* 12. *moneths?*

Anſwer. Firſt ſay, if 2 $\frac{2}{3}$ gain $\frac{1}{6}$, what ſhall 100 pound gain? Multiply and divide, and you ſhall finde 6 $\frac{1}{4}$ pound. Then ſay again for the ſecond work. If 25. pound be come of twelve months, what ſhall come of 6 $\frac{1}{4}$? Work, and you ſhall finde 3 moneths, the juſt term of time that the Cloth ought to be delivered at 22 s. 10 d. to gain 25 pound upon the 100. li. in 12 moneths.

If one yard coſt me 2 s. 8 d. *ready money, for what price ſhall I ſell the ſame again to be paid at the end of* 3. *moneths, ſo that I may gain after the rate of* 25. *Pound in the* 100. *pound for* 12. *moneths?*

Anſwer. Firſt ſay, if 12. gain 25 li. what ſhall three moneths gain? Multiply and divide, and you ſhall finde 6 $\frac{1}{4}$ li. Then ſay for the ſecond worke, if 100. lib. give 106 $\frac{1}{4}$, what giveth 3 $\frac{2}{3}$ s? Worke and you ſhall finde 2 s. 10 d. and for that price muſt the yard be ſold to gain after 25. pound in the 100 pound for twelve moneths.

Many other of theſe queſtions I might here have delivered, but for feare the Booke would riſe to too thick a volume, and ſo to make the price ſo much the dearer, whereby it might not be ſo portable to my Countrie-men, as I wiſh it. But theſe 4 I have of purpoſe framed in this order, having relation one to another, aſſuring you that what queſtion ſoever may be propoſed within the compaſſe of this Rule

you ſhall finde by one of theſe 4. to make a ſolution. And moreover, divers others are yet to be delivered where the Creditor giveth divers dayes of payment, which can never be well wrought, nor yet underſtood, unleſſe you can firſt finde by Art the juſt times that all thoſe payments, how different ſoever they be, ought to be paid at once: whereupon firſt I think good here to give ſome inſtructions unto ſuch a Rule, for it is the onely aid for the finiſhing of ſuch queſtions as hereafter ſhall follow.

The ſixth Chapter entreateth of Rules of Payment, which is a right neceſſary Rule, and one of the chiefeſt handmaids that attendeth upon buying and ſelling, *&c.*

Example.

A Merchant doth owe a ſumme of money, whereof the ⅓ is to be paid at ſixe months, and the ½ at eight months, and the reſt at a year. If he would pay all at one payment, the queſtion is, what time ought to be given him?

Anſwer, I have omitted the quantity of the ſumme, for you ſhall underſtand the Rule is appliable,

appliable, and yieldeth a true ſolution to what ſumme ſoever ſhall be propoſed: But now for order ſake in teaching, I do imagine the ſumme to be 60 pounds, whereupon the manner of this worke is to multiply the proportionate part of the money by the time, as in company. Then 20. being the firſt payment, and the $\frac{1}{3}$ of 60 which $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the Example in the Operation, maketh two moneths: next 30. which is the $\frac{1}{2}$ multiplyed by his terme 8. yields 4. moneths, then the reſt which is 10 li. muſt needs be abbreviated into the proportion it beareth to 60. which is $\frac{1}{8}$, which $\frac{1}{6}$ multiplied by his time 12. moneths, produceth $\frac{12}{6}$, maketh two moneths. All which added together, as appeareth in the Operation, maketh 8. moneths, which is the juſt time that all thoſe payments ought to be paid at once.

$\frac{1}{3}$ by $\frac{6}{1}$	$\frac{6}{3}$	2 Moneths.
$\frac{1}{2}$ by $\frac{8}{1}$	$\frac{8}{2}$	4 Moneths.
$\frac{1}{6}$ by $\frac{12}{1}$	$\frac{12}{6}$	2 Moneths.
	$\frac{26}{8}$	

A Merchant hath 800 li. *to pay the* $\frac{1}{4}$ *thereof ready money, the* $\frac{1}{4}$ *at two moneths, the* $\frac{1}{2}$ *at* 4. *moneths, and the reſt at a yeare. The queſtion is, if he would pay all at one payment, what time ought to be given him?*

Answer. The ready money is never multiplied, then $\frac{1}{4}$ multiplied by two moneths, as you did before, maketh $\frac{1}{12}$, then $\frac{1}{3}$ by 4 produceth 2 moneths, as appeareth here in the operation: But now for the rest of the mony you cannot multiply it untill you have sought what proportion it beareth to 800. pounds. Therefore you must subtract the ready mony, the $\frac{1}{4}$ and $\frac{1}{3}$ out of the principall. The rest will be 66 $\frac{1}{3}$ li. which you must look what part it beareth to the principal, which you shall finde to be $\frac{1}{12}$, the same you must also multiply by his time 12 months, and it yieldeth one month, so all make 3 $\frac{1}{2}$ months, as appeareth in the operation.

A Merchant is to pay 1200 li. *in three terms, that is to wit,* 400 li. *at two weeks: and* 600 li. *at foure months: Lastly,* 200 li. *at five months: The question is, in what time they ought to be paid at once.*

Answer. Proportionate the parts, and you shall finde that 400 is $\frac{1}{3}$ part, and for 600. you shall finde $\frac{1}{2}$, and likewise 200. is the $\frac{1}{6}$ part, which multiply by their times as before, and you shall have $\frac{2}{3}$ weeks, more eight weeks: and lastly 3 $\frac{1}{3}$ weeks, which together maketh 12. weeks, or three months, your desire.

A Merchant is to pay 600. *Pound in three termes, whereof* 100. *Pound is paid present, more*

300. Pound at twenty dayes, and the reſt at five months, accounting thirty dayes to a month. The queſtion is, what time ought theſe Payments to be paid at once?

Anſwer. Worke, and you ſhall finde two moneths.

The ſeaventh Chapter intreateth of buying and ſelling in the Trade of Merchandize, wherein is taken part ready money, and divers dayes of Payment given for the reſt, and what is wonne or loſt in the 100 Pound forbearance for twelve moneths more or leſſe, according to the quantity of money, or proportion of time, &c.

A Merchant hath bought ſatins which coſt 8 ſhillings the yard ready money: and he ſelleth the ſame again to another man for 10 s. the yard, but he giveth 2 dayes for the payment, that is to ſay, three months for the one halfe, and five months for the other halfe. The queſtion is to know how much the ſeller doth gain upon the 100. lib. in 12. moneths after that rate.

Anſwer. Seeke firſt, by the Rules of payment, at what time thoſe two payments ought to be paid at once, and you ſhall finde foure months,

moneths, at which time the ſecond Merchant ought to have paid the whole entire payment: And therefore ſay by the firſt part of the *Rule of Three* compoſed:

If 8. ſhillings in 4. moneths doe gaine 2 s. what will 100. lib. gain in 12. moneths?

	s.	m.	s.	li.	m.
	8	4	1	10	12
	4			20	
	32			2000	
x				2	
				4000	
				12	
	48			48000	
	32			1500	(75
	32			220	

Multiply and divide, and you ſhall finde 75. pounds, as appeareth in the example, and ſo much doth the firſt Merchant gain upon the 100. pounds in 12. moneths.

A Merchant hath ſold 50. Clothes, at 9½ li. the Piece, to be paid the one ½ at foure moneths, the ⅓ at five moneths, and the ⅙ at ſeven moneths, and the ſellers minde is to take no more but after eight Pounds in the 100. for 12. moneths. The queſtion is now, what the firſt Merchant gaineth in the ſale of theſe Clothes after that rate.

Anſwer, Firſt, looke what the 40 Clothes come to at that price, and you ſhall finde 475. pounds. Then ſecondly, according to your direction in the Rules of payment, ſeeke at what time all the payments are to be performed at once. And you ſhall finde 4 ⅙ moneths. Then thirdly ſay, by the firſt part of the *Rule of Three* com-

composed. If 100 li. in 12 months gain 8 li. what will 475 li. gain in $4\frac{1}{6}$ months? Work, and you shall finde 15 li. and $\frac{11}{32}$ of a pound, which is the neat gains that the first Merchant hath after the rate aforesaid.

A Merchant hath bought Holland at 7s. 3d. *the Ell ready money, and he selleth the same again, for* 8s. 4d. *the Ell, to be paid* $\frac{1}{4}$ *part in ready money, three parts at* 2 *months, and the rest at* 4 *months. The question is now to know how much the first Merchant doth gain upon the* 100 *pounds in* 12. *months after that rate?*

Answer, According to the direction delivered you in the Rule of payment, the ready mony is not to be multiplied. Then working for the other two payments to finde out the true proportion at what time they ought to be paid at once, you shall find for $\frac{3}{4}$ at two months, $\frac{2}{3}$ of a month. And the rest of the money which is $\frac{1}{16}$ multiplied by his term 4 months, yieldeth $1\frac{1}{4}$ months, both which added together make $2\frac{1}{3}$ months, the just time that both the payments ought to be performed at once. And therefore say by the first part of the *Rule of* 3. composed, If $74\frac{1}{4}$ in $2\frac{1}{3}$ months doe gain $\frac{12}{46}$ of a pound, what shall 102 pounds gain in 12 months after that rate? Work, and you shall find 78 $\frac{12}{603}$ pounds. And so much doth he gain upon 100. pounds in 12 months.

A Merchant hath bought 30. *Clothes at* 6. *pounds the piece for ready money: Afterward he selleth ten of them for* 7 *pound the piece, for three*

months

months term: And the other 20. he ſelleth for 8. Pound the piece, for 4. months Term. The queſtion is now, what he gaineth upon 100. Pounds in 12. months?

Anſwer. Firſt finde the value of the thirty Clothes, which amount to 180. pounds. Secondly, ſeek what the ten pieces come to at 7. pounds, and what the twenty pieces come to at 80. pounds; the one comes to 70. and the other to 60, both which together make 230. which is 50. pounds more then they coſt. Thirdly, as I have taught you in the Rule of Payment, proportionate the firſt and ſecond prices unto the proportion they bear unto 230 the Product of their two prices, and you ſhall finde $\frac{7}{23}$ for the firſt, and $\frac{16}{23}$ for the latter. Then fourthly, multiply thoſe parts by their times, and you ſhall have $\frac{21}{23}$ and $\frac{64}{23}$ both which together maketh three whole months, and $\frac{16}{23}$ of a month, which is the juſt time that both thoſe Payments ought to be paid at once.

Then ſay by the firſt part of the *Rule of Three* compoſed; If 180 pounds in $3\frac{16}{23}$ months doe gain 50. pounds, what ſhall 102. gain in twelve months? Multiply and divide, and you ſhall finde 90 $\frac{10}{11}$ pound, and ſo much doth he gain upon 100. pounds in 12 months.

A Merchant hath bought Cinnamon which coſt him 9 ſhillings the Pound ready money. The queſtion is now, at what price he ought to ſell the 100. weight. To wit, 112 Pounds to be paid the $\frac{1}{4}$ *at two moneths, and the reſidue at the end of*

three

three months, ſo that he may gain after the rate of ten pounds upon 100. *pounds for* 12. *months.*

Anſwer, Seek firſt by the Rules of payment at what term both the payments ought to be paid at once, where the $\frac{3}{4}$ multiplyed by his term two months, maketh $\frac{3}{4}$ months.

Likewiſe the next payment, which is $\frac{1}{4}$ multiplied by his term three months, maketh $2\frac{1}{4}$ moneths, both which added together, maketh $2\frac{3}{4}$ months, which is the time that both the payments ought to be paid at once. Then ſay by the *Rule of Three*, if 12. months do give me ten pounds, what will $2\frac{3}{4}$ months give? Multiply and divide, and you ſhall finde $2\frac{3}{24}$ pounds. Then ſay again by the *Rule of Three*, If one pound coſt me 9s. what will 112. pounds coſt? Multiply and divide, and you ſhall finde 51. li. 15. Then ſay once again: If 100. pound doe give $102\frac{2}{24}$, what will $50\frac{3}{5}$ pounds give? Multiply and divide, and you ſhall find 51 li. 11 s. $1\frac{1}{3}$ d. and for that price ought I to ſell 112. pound of Cinnamon to be paid at the two ſeverall payments aforeſaid, to gain thereby after the rate of tenne pounds upon the hundred pound in twelve month.

Briefe

Brief Rules for our hundred weight here at *London*, which is after 112. pound for the 160.

ITem, *Who that multiplieth the pence that one pound weight is worth by* 7. *and divideth the* Product *by* 15. *ſhall finde how many pounds in money* 112 *pound weight is worth.*

And contrariwiſe, he that multiplieth the pounds that 112. *pounds weight is worth by* 15. *and divideth the* Product *by* 7. *ſhall finde how many pence in money the one pounds weight is worth.*

Example.

At 10 pence the pound weight, what is 112. pounds weight worth?

Anſwer, Multiply 10 by 7. and thereof commeth 70. the which divided by 15. and you finde 4⅔ pounds. And thus the 112 pounds is worth 4 li. 13 s. 4 d. after the rate of 10. pence the pound aforeſaid.

At 6 pounds the 112 pounds weight, what is one pound worth?

Anſwer, Multiply 6 by 15. and thereof commeth 90. the which divide by 7. and you ſhall finde 12 $\frac{6}{7}$ d. So much is one pound worth when the 112. pounds did coſt 6 pounds.

The

The eight Chapter intreateth of Tares, and allowances of Merchandize ſold by weight, and of Loſſes and Gains therein, &c.



AT 16 pound the 100 Suttle, what ſhall 895 pound Suttle be worth, in giving 4 pound weight upon every 100. for Treat?

Anſwer, Adde 4 unto 100, and you ſhall have 104. Then ſay by the Rule of Three, If 104. be worth 16. pounds, what are 895 pounds worth? Multiply and diuide, and you ſhall finde 237li. 13s. 10 $\frac{2}{13}$ d. and ſo much ſhall the 895 pounds weight be worth.

Item, at 3s. 4d. the pound weight what ſhall 754 $\frac{1}{2}$ pound be worth, in giving 4 pounds weight upon every hundred for Treat?

Anſwer, See firſt by the *Rule of Three* what the 100. pound is worth, ſaying, If one coſt 3 $\frac{1}{3}$s. what 100? Multiply and divide, and you ſhall finde 16 $\frac{2}{3}$ pounds. Then adde 4 unto one 100, and they are 104. Then ſay again by the *Rule of Three*, If 104 be ſold for 16 $\frac{2}{3}$ pounds, for how much ſhall 754 $\frac{1}{2}$ be ſold for? Multiply and divide, and you ſhall find 120li. 18s. 3 $\frac{4}{13}$d.

And

And for ſo much ſhall the 754½ pound be ſold for at 3s. 4d. the pound, in giving 4 upon the 100.

Other neceſſary briefe Rules there are for the finding of Treats, or caſting up of Cheſts of Sugar, &c. which for that it is a myſtery, I omit: if any lack inſtruction that way, they ſhall finde me ready to pleaſure them.

Item, If 100 pounds be worth 36s. 8d. what ſhall 860 pounds be worth in rebating 4 pounds upon every hundred for tare and cloſe?

Anſwer, Multiply 860 by 4, and thereof commeth 3440, the which divide by 100, and you ſhall have 34⅖ pounds, abate 84⅖ from 860 and there will remain 825⅗ pounds. Then ſay, by the *Rule of Three*: If 100 li. coſt 36 2/3 s. what will 825⅗ coſt after that rate? Multiply and divide, and you ſhall finde 15 li. 2s. 9 15/25 d. And ſo much ſhall the 860 coſt in rebating foure pounds upon every hundred, for tare and cloſe.

Item, *Whether doth he loſe more that giveth 4. Pounds upon the 100. or he that rebateth 4 Pounds upon the 100?*

Anſwer. Firſt note, that he that giveth 4. pounds on 100. giveth 104. for 100. And he which rebateth 4. pounds upon the 100. giveth the 100. for 96. Therefore ſay by the *Rule of Three*,

three, if 104 be delivered for 100, for how much ſhall the 100 be delivered? Multiply and divide, and you ſhall find 96 $\frac{2}{13}$, and he which rebateth 4 in the 100, maketh but 96 pounds of 100, ſo that he loſeth 4 pounds in the 100, and the other which giveth 4 pounds upon the 100 loſeth but 3 $\frac{11}{13}$ pounds upon the 100. Thus you may ſee, that he which rebateth 4 pounds in the 100, loſeth more by $\frac{11}{13}$ pound in the 100 pounds, then the other which gave 4 pounds upon the 100, for tare and cloffe.

If 100 pounds of any thing coſt me 23 s. 4 d. the queſtion is, how I ſhall ſell the pound, to gain after the rate of ten pounds, upon the 100 pound.

Anſwer. Say by the *Rule of three*, if 100 pounds give 100 pounds, what ſhall 23 $\frac{1}{3}$ s. give? Multiply and divide, and you ſhall finde 1 $\frac{17}{60}$ pounds. Then ſay again, if 100 pound be worth 1 $\frac{17}{60}$ pounds, what is one pound worth? Multiply and divide, and you ſhall find 3 d. $\frac{3}{13}$ And ſo much is the pound worth in gaining ten pounds upon the 100.

Item, *A Grocer hath bought C. weight of commodity for* 6 li. 10 s. *The queſtion is now to know how many pounds thereof he ſhall ſell for* 33 s. 4 d. *to gain* 20 s. *in C. weight.*

Anſwer. Adde 20 s. unto 6 li. 10 s. and they

make 7 li. 10 s. Then ſay, if 7 ½ pound yeeld me 112 pound, what ſhall 1 ⅔ pounds yeeld? Multiply and divide, and you ſhall find 24 4/9 li. And ſo many pound ought he to ſell to gain 20 s. in his C. weight.

If one pound weight coſt 3 s. 4 d. *and I ſell the ſame again for* 4 s. *what is gained in a hundred pound of mony laid out in that commodity?*

Anſwer. You may ſay, If 3 ½ s. give 4, what will 100 pound gain? But then when you have found, you muſt ſubtract 100 pounds out of the Product, the reſt is your neat gain: or elſe to produce the neat gain in your work at the firſt, ſubtract the juſt price out of the overprice, as I taught before in the firſt beginning of Loſſe and Gain, and your concluſion ſhall be all one. Multiply and divide by which of the two wayes you think good, and you ſhall finde that he gaineth 20 pounds in the 100 pound.

Item, *If the pound weight which coſt* 4 s. *be ſold again for* 3 s. 4 d. *I demand what is loſt in the* 100 *pounds of mony.*

Anſwer. Say, If 4 s. loſe ⅔ s. what ſhall 100 loſe? Multiply and divide, and you ſhall finde 16 li. 13 s. 4 d. and ſo much is loſt upon the 100 of mony.

Item, *If C. weight of any commodity coſt* 45 *pounds, and the buyer repenting, would loſe five pounds*

pounds in the 100 of mony, I demand how the pounds may be ſold, his loſſe to be neither more nor leſſe then after the rate aforeſaid of five by the hundred?

Anſwer. By the *Rule of three*, if 100 loſe 5, what ſhall 45 loſe? Work, and you ſhall finde 2 $\frac{1}{4}$ pound, which rebated from the principall 45, reſteth 42 l. 15 s. Laſtly ſay, if 112 yeeldeth but 42 li. 15 s. what one pound? Multiply and divide, and you ſhall find 7 s. 7 d. $\frac{12}{28}$ And ſo much is the pound worth after that loſſe.

A Grocer hath bought three pieces of Raiſins weighing 175 $\frac{1}{2}$ pounds, 182 $\frac{1}{4}$ pounds, 191 pounds: tare for each fraile 2 $\frac{3}{4}$ pounds, at 25 $\frac{1}{2}$ s. the C. weight. The queſtion is, what they amount to in mony.

I anſwer, 6 li. —— 3 s. —— 4 $\frac{21}{28}$ d.

A Grocer hath bought three ſacks of Almonds weighing 267 $\frac{1}{2}$ pound, tare two pound, 257 $\frac{1}{2}$ pounds, tare 2 $\frac{1}{2}$ pound, 252 pound, tare 3 pound, at 2 s. 10 $\frac{1}{2}$ d. the pound, what amount they to in mony?

I anſwer, 110 li. —— 12 s. —— 3 $\frac{1}{4}$ d.

The ninth Chapter intreateth of lengths and breadths of Arras and other Cloths, with other queſtions incident unto length and breadth.

IF a piece of Arras be 7 Elles and $\frac{1}{4}$ long and five Elles and $\frac{2}{3}$ broad, how many Elles ſquare doth the ſame piece containe?

Anſwer. Multiply the length by breadth, that is to ſay, $7\frac{1}{4}$ by $5\frac{2}{3}$ And thereof will come $43\frac{1}{12}$ elles: ſo many elles ſquare doth the ſame piece contain.

Item, *more, a piece of Arras doth contain 22 Elles ſquare, and if the ſame were in length* $3\frac{1}{4}$ *elles, I demand how many elles in breadth the ſame piece doth contain.*

Anſwer. Divide 22 elles by $3\frac{1}{4}$ and thereof commeth $6\frac{12}{13}$ So many elles doth the ſame contain in breadth.

Item, *more, a Merchant hath* $3\frac{1}{4}$ *elles of Arras, at* $1\frac{2}{3}$ *elles broad, which he will change with another man for a piece of Arras, that is* $\frac{7}{8}$ *elles ſquare. The queſtion is, how many elles of that ſquareneſſe ought the firſt Merchant to have?*

Anſwer

Anſwer. Multiply the firſt merchants piece his length by the breadth, and you ſhall finde it containeth 5 $\frac{5}{12}$ ells, which $\frac{5}{12}$ ells you ſhall divide by $\frac{7}{8}$ and you ſhall find 6 $\frac{4}{21}$ ells, and ſo many ells of that ſquareneſſe ought the latter merchant to give the firſt.

Item, *A Student hath bought* 3 $\frac{1}{2}$ *yards of broad cloth, at* 7 *quarters broad, to make a gown, and ſhould line the ſame throughout with Lamb at a foot ſquare each skin, the queſtion is now how many skins he ought to have.*

Anſwer. Seek firſt the number of yards ſquare that his cloth containeth, which to doe, multiply 3 $\frac{1}{2}$ his length, by 1 $\frac{1}{4}$ his breadth, and you ſhall finde 6 $\frac{1}{8}$ yards ſquare: then ſay by the *Rule of three*, if one yard ſquare give 9 foot, what ſhall 6 $\frac{1}{8}$? Work, and you ſhall find 59 $\frac{1}{8}$ skins.

Item, *more, a Lawyer hath a rich peece of ſeeling come home which is* 24 *foot and* 3 *inches long, and* 7 *foot and* 2 $\frac{1}{2}$ *inches high: the Ioyner is to be paid by the yard ſquare: the queſtion is how many yards this containeth.*

Anſwer. Multiply his length by his breadth, that is to wit, 24 $\frac{1}{4}$ foot by 7 $\frac{1}{24}$ foot, and you ſhall find 174 $\frac{17}{96}$ foot ſquare, which 174 you ſhall divide by 9 (for ſo many foot make a yard ſquare) and you ſhall finde 19 yards 3 foot

and $\frac{11}{12}$ of a foot, and ſo many yards doth this piece hold.

Item, *I bought a piece of Holland cloth containing 36 Els $\frac{1}{3}$ Flemmiſh. The queſtion is how many Ells Engliſh it makes.*

Anſwer. You muſt note, that five ells *Flemmiſh* doth make but three ells *Engliſh*.

Therefore ſay by the *Rule of three*, if five ells *Flemmiſh* make but three ells of *Engliſh*, how many ells *Engliſh* will 36 $\frac{1}{3}$ ells *Flemmiſh* make? Multiply and divide, and you ſhall finde 21 $\frac{4}{5}$ and ſo many *Engliſh* doth 36 $\frac{1}{3}$ ells *Flemmiſh* contain. The like is to be done of others.

Item, *more, I have bought 342 Ells Flemmiſh, of Arras work, at two Ells broad Flemmiſh, and I would line the ſame with Ell broad Canvas of Engliſh meaſure. The queſtion is, how many Ells Engliſh will ſerve my turn?*

Anſwer. For as much as three ells engliſh are worth five ells *Flemmiſh*, therefore put three ells *Engliſh* into his ſquare, in multiplying three by himſelfe, which maketh nine. Likewiſe multiply the *Engliſh* ell, which is five quarters, every way into himſelfe ſquarely, and you ſhall finde 25. Then multiply 342 which is the length of the piece, by 2, which is the breadth, and thereof commeth 684, then

ſay

ſay by the *Rule of three*, as before: if 25 ells ſquare of *Flemmiſh* meaſure, be worth nine ells ſquare of *Engliſh* meaſure, what are 684 of *Flemmiſh* meaſure? Multiply and divide, and you ſhall finde 246 $\frac{6}{25}$ ells *Engliſh*.

The ſame is alſo wrought by the Backer *Rule of three*, in ſeeking the ſquares contained in the *Flemmiſh* ell of two ells broad (which are 18) and alſo in ſeeking the ſquares contained in the *Engliſh* ell (which are 25) then ſay by the *Rule of three* backward, If 18 quarters require 342 ells, what ſhall 25 quarters give? Multiply and divide by the *Rule of three* Reverſe, and you ſhall find as before 246 $\frac{6}{25}$ ells *Engliſh*?

Item, *more, at three ſhillings foure pence the Flemmiſh Ell, what is the Engliſh Ell worth after the rate?*

Anſwer. Say, if three quarters give 3 $\frac{1}{3}$ s. what giveth five quarters? Multiply and divide, and you ſhall finde 5 s. 6 $\frac{2}{3}$ d.

Item, *more, at* 8 s. 4 d. *the Flemmiſh Ell ſquare, what is the Engliſh Ell worth after that rate?*

Anſwer. According to the reaſon of the laſt Queſtion, conſider that a *Flemmiſh* ell ſquare is equall to nine quarters of a yard *Engliſh*, and

an *Engliſh* ell ſquare is equall to 25 quarters of a yard. Therefore ſay by the *Rule of three*, if 9 quarters give $1\frac{2}{3}$ s. what 25 quarters? Work and find 23 s. $1\frac{1}{9}$ pence. And ſo is the *Engliſh* ell worth.

Item, *more, at 6 s. 8 d. the ell ſquare: what ſhall a piece of Cloth coſt that is* $7\frac{1}{2}$ *ells long, and* $3\frac{1}{4}$ *ells broad?*

Anſwer. Multiply the breadth by the length, and you ſhall find $24\frac{1}{8}$ ells ſquare coſt $6\frac{2}{3}$ s. what $24\frac{1}{8}$ els? Multiply and divide, and you ſhall find 8 pounds, 2 s. 6 pence, and ſo much the ſame piece of cloth coſt.

Item, *more, a Mercer ſold 3 pieces of Silk. To wit* $24\frac{1}{2}$. $13\frac{2}{3}$ *and 25 yards, at* $9\frac{1}{4}$ s. *the yard, and was glad to receive in part of payment again, a cloth containing* $34\frac{1}{2}$ *yards at* $7\frac{2}{3}$ *ſhillings the yard. The queſtion is now, what the Debtor is in the Creditors debt? Work, and you ſhall finde he oweth the Mercer 22 pounds 3 ſhillings* $2\frac{1}{4}$ *pence.*

The tenth Chapter intreateth of reducing of Pawns of Geanes into Engliſh yards.

Note that 100 *Pawnes doe make* 26 *yards, whereupon three Pawnes* $\frac{11}{13}$, *doe make one yard, and one Pawn after the rate and proportion is* $\frac{11}{50}$ *of a yard.*

In 4563 *Pawnes of Geanes, how many yards engliſh?*

Anſwer. Say by the *Rule of three*, if a hundred Pawns doe make 26 yards, what will 4563 Pawns make? Multiply and divide, and you ſhall find 1186 yards $\frac{19}{50}$ So many yards doe 4563 Pawns make.

Otherwiſe take ſome other number at your pleaſure, as ten pawns, which is the $\frac{1}{10}$ part of 100, then to find his proportion, take the $\frac{1}{10}$ part of 26, which is $2\frac{1}{5}$ and then ſay alſo by the *Rule of three*, if ten Pawns give $2\frac{1}{5}$ yards, what will 4563 Pawns give? Work, and you ſhall find 1186 $\frac{19}{50}$ yards, as before.

More, at 2 s. 6 d. *the Pawns of Geans, what will*

will the Engliſh yard be worth after the rate?

Anſwer. Say by the *Rule of three*, *if* $\frac{11}{10}$ of a yard coſt $\frac{1}{2}$ s. what one yard? Multiply and divide and you ſhall find 9 s. 7 $\frac{2}{13}$ d.

More, if 346 $\frac{1}{2}$ *Pawnes coſt* 30 li. 13 s. 4 d. *ſterling, what is that the Engliſh yard after the rate?*

Anſwer. Say by the *Rule of three*, if 346 $\frac{1}{2}$ Pawns coſt 30 $\frac{2}{3}$ pounds, what are 3 $\frac{11}{13}$ Pawns worth (for ſo many Pawns make a yard?) Multiply and divide, and you ſhall find $\frac{9200}{27827}$ parts of a pound, which in known numbers is worth 6 s. 9 d. $\frac{6271}{9009}$.

The eleventh Chapter intreateth of Rules of Loan and Intereſt, with certain neceſſary queſtions and proofs incident thereunto, &c.

ITem, *I lent my friend* 326 *pounds for* 5 $\frac{1}{2}$ *moneths ſimply without any Intereſt, upon condition, to have the like courteſie againe when I need. But when I came to borrow, he could ſpare me but* 149 li. 8 s. 4 d. *The queſtion is now how*

how long time I ought to have the uſe thereof, to countervaile my friendſhip before time ſhewed him?

Anſwer. Say by the backer *Rule of three*, if 326 pounds give $5\frac{1}{2}$ months, what time will 149 $\frac{2}{12}$ pounds give? Multiply and divide, and you ſhall finde twelve months, and ſo long time ought I to uſe his mony.

The Proofe.

Item, *lent my friend* 149 li. 8 s. 4 d. *for twelve months, The queſtion is now how much mony he ought to lend me again for* $5\frac{1}{2}$ *months to recompence my friendſhip ſhewed him?*

Anſwer. Say by the Backer or Reverſe *Rule of three.* If twelve months give 149 $\frac{2}{12}$ what ſhall $5\frac{1}{2}$ months give? Work, and you ſhall find 326 pounds, and ſo much ought he to lend me to requite my gentleneſſe or good turn.

Two other branches, yet more, for proofe out of the ſame queſtion.

Item, *lent my friend* 149 li. 8s. 4 d. *for 12 months, to have the like friendſhip again when I need. And comming to borrow of him, he very courteouſly took me 326 pounds (for that he could well then ſpare the ſame) The queſtion is now, how long I ought to occupy it, not uſurping friendſhip, but in his due time to reſtore it again.*

Anſwer,

Anſwer. Say (by the *Rule of three* reverſe, if 149 $\frac{7}{12}$ pounds give 12 moneths, what ſhall 326 pounds give ? Multiply and divide, and you ſhall finde, that at 5 $\frac{1}{2}$ moneths terme, I ought to reſtore it again.

Proofe.

Item. *Lent my friend* 326 *pounds for five* $\frac{1}{2}$ *moneths. The queſtion is now, how many pounds he ought to lend me for* 12 *moneths to recompence this pleaſure again.*

Scholar. Work by the *Rule of three* reverſe, as you have done before, and you ſhall finde 149 li —— 8 s —— 4 d.

Again, four other ſelected queſtions, of Loan and Intereſt, all out of one branch, and each one alſo a neceſſary queſtion, and a particular proofe to other.

ITem, *Lent my friend* 430 *pounds at Intereſt for three moneths, to receive after the rate of* 8 *pounds in the* 100 *pounds for* 12 *moneths. The queſtion is what the intereſt commeth to.*

You may, if you pleaſe, work it at two workings by the *Rule of three* direct, in ſaying, if 12 moneths

months give 8 pounds, what giveth three moneths? Multiply and divide; and it giveth 2 pound.

Then for the ſecond work ſay: If a hundred pound yeeld 2 pounds, what yeeldeth 430 li? multiply and divide, and you ſhall find 8 li. 12 s. and ſo much comes the loane of 430 li. to for 3 moneths after the rate of 8 pound in the hundred pounds of 12 moneths.

Otherwiſe wrought thus by the rule of three at twice alſo.

If 100 pounds give 8 pounds, what giveth 430 pounds? multiply and divide, and you ſhall find 34 pounds $\frac{2}{5}$; Then again for the ſecond work ſay: If 12 moneths give 34 pounds $\frac{2}{5}$ what giveth three moneths? Worke and find 8 li. 12 s. as before.

Otherwiſe yet at one working: By the firſt part of the rule of five numbers forward, in ſaying if 100 pounds in twelve moneths, gain 8 pounds, what ſhall 430 pounds gain in three moneths? multiply the firſt by the ſecond for your Diviſor, and the other three, the one into the other for the dividend, and you ſhall finde eight pounds 12 ſhillings, as aforeſaid.

Proofe.

Item, *A friend of mine received of me* 8 *pounds* 12 *ſhillings for the Intereſt and Vſe of* 430 *pounds for three moneths terme: The queſtion is*

now

now, what he tooke in the 100 pound for 12 months after that rate?

Anſwer. For moſt brief, ſay by the firſt part or rule of five numbers forward: If 430 pounds in three moneths did pay 8 li. 12 s. what doth 100 pounds in 12 moneths take after the rate? Work, and you ſhall find 8 pounds, and ſo much he took upon the 100 pounds for 12 moneths.

A third Queſtion and proof alſo by the Backer Rule of five Numbers.

Item, *I lent my friend 430 paunds to receive for the intereſt thereof, after the rate of 8 paunds in the 100 for 12 moneths. The queſtion is now how long time my friend ought to give the uſe thereof, that it may be returned with 8 li. 12 s. gains.*

You may work it, if you pleaſe, by the Rule of three direct at twice, in ſaying: If 100 li. yeeld 8 pounds, what yeeldeth 430 pound? Multiply and divide, and find 34 pound and $\frac{1}{2}$.

Then again for the ſecond work ſay, if 34 $\frac{1}{2}$ pounds, give twelve moneths, what giveth 8 $\frac{1}{2}$ pounds? Multiply and divide, and you ſhall find three moneths, and ſo long time ought my friend to uſe it to return with 8 li. 12 s. gain.

Otherwiſe at one working by the Backer Rule

Rule of 5 numbers, in ſaying, if 100 pounds in 12 moneths doe gain 8 pounds, how much time ſhall 430 pounds be a gaining of 8 pounds 12 s.? Multiply the firſt and the ſecond into the laſt for your Dividend, and the third and fourth multiply together for your Diviſor, and then divide, and you ſhall find three moneths, the juſt time that my friend ought to uſe it to return it with 8 li. 12 s. gain.

A fourth derived queſtion out of this Branch, which is a proof of this laſt, and alſo of the other two going before.

ITem, *How much mony ought a Merchant to deliver after* 8 *pounds in the* 100 *for twelve months, that in three months he may gain* 8 *li. twelve ſhillings.*

Anſwer. You may alſo if you pleaſe, work it by the Golden Rule of three at twice: firſt ſaying, If three months give 8 $\frac{1}{5}$ pound, what 12 months gain? You ſhall finde 34 $\frac{2}{3}$ Then ſay again, If 8 pounds be come of 100 pounds, what ſhall come of 34 li. 8 s? Work, and you ſhall find the anſwer to the queſtion, which is 430 pounds, and ſo much ought the Merchant to deliver.

But moſt briefly it is anſwered by the Backer Rule of 5 numbers, where I argue thus, ſaying:

If

If 100 li. be 12 moneths a gaining of 8 li. then but for three moneths terme onely to take 8 li. 12 s. muſt needs be a good round ſumme to work it, ſet your numbers thus: 100 —— 12 — 8 —— s —— 8 1/5; multiplying the firſt into the ſecond, and alſo by 43 the product of the fifth, for your dividend, and the third and fourth together with 5. the Denominator of your fraction for your Diviſor: then divide, and you ſhall find as before 430 pounds: the true ſolution to your queſtion.

The twelfth Chapter intreateth of the making of Factors, which is taken in two ſorts.

THe firſt is, when the eſtimation of the Factor is taken upon the ſending of the Merchant, as if the eſtimation of his perſon be 1/4 it is underſtood that he ſhall have 1/4 of the gain, the Merchant the other 1/4

The other ſort is, when the eſtimation of his making is out of the ſending of the Merchant, as if the order and agreement between them were ſuch, that the Merchant ſhall put in 800 li. and the Factor for his making ſhall have 1/8: nevertheleſſe he ſhall have but 1/8 of the gain

or

or profit, for the $\frac{1}{4}$ of 800 is 200 (for the eſtimation of his making) which with the 800 pounds in all make 1000 pounds, whereof the 200 pound, is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, under ſuch condition, that the ſaid Factor ſhall have the $\frac{1}{4}$. And after certain time they finde in profit 124 lib. 6 s. 8 d. I demand how much the Merchant ſhall have hereof, and how much ought the Factor to have?

Anſwer. *When the eſtimation of the Factor is out of the ſending of the Merchant, it maketh*

li.	s.	d.		
99	9	4	*for the*	*Merchant.*
24	17	4		*Factor.*

But if that his eſtimation be at the ſending of the Merchant, then it maketh but

li.	s.	d.		
93	5	0	*for the*	*Merchant.*
31	1	8		*Factor.*

For the Merchant is then to have $\frac{1}{4}$, *and the Factor* $\frac{1}{5}$.

A Merchant doth put into the hands of his Factor 800 pounds, and the Factor 400 lib.

to have the $\frac{1}{5}$ part of the profit: I demand now for how much his perſon is eſteemed, when the ſame is counted upon the ſending of the Merchant.

Anſwer. *According to the Tenour and order before preſcribed in the firſt Rule, that is if his eſtimate be $\frac{1}{4}$ he ſhall have the $\frac{1}{4}$ of the gain. Therefore ſay by the* Rule of Three direct: *If $\frac{1}{4}$ taken out in 400 pound, what is the eſtimate, or putting in of $\frac{1}{5}$ taking? Multiply and divide, and you ſhall finde 320 pounds, and ſo much is the perſon of the Factor eſtimated.*

Otherwiſe.

To finde the eſtimation of the perſon of the Factor, you ſhall conſider, that ſeeing it was agreed betweene them, that the Factor ſhould take the $\frac{1}{5}$, then the Merchant ſhall have the reſidue, which are $\frac{4}{5}$: wherefore the gain of the Merchant unto that of the Factor is in ſuch proportion as 5 unto 4. Then if you will know the eſtimation of the perſon of the Factor, ſay, If 5 give 4, what will 400 give? Multiply and divide, and you ſhall finde 320 pound. And ſo much is the perſon of the Factor eſteemed to be worth.

Other conditions then theſe aforeſaid, may alſo be between Merchants and Factors, without reſpect either of ſending, or not ſending of the Merchant, where moſt commonly the eſtimation

mation of the body of the Factor is in ſuch proportion of the ſtock which the Merchant layeth in, as the gain of the ſaid Factor is unto the gain of the Merchant. As thus, if a Merchant doe deliver into the hands of his Factor 400 pound, and he to have half the profit, the perſon of the ſaid Factor ſhall be eſteemed to be worth 400 pound: and if the Factor do take but $\frac{1}{3}$ of the gain, he ſhould have but $\frac{1}{2}$ ſo much of the gain as the Merchant taketh, which muſt have $\frac{2}{3}$, wherefore the perſon of the Factor is eſteemed but the $\frac{1}{2}$ of that which the Merchant layeth in, that is to ſay, two hundred pound.

And if the Factor did take the $\frac{2}{5}$ of the gain, then the Merchant ſhall take the reſidue which are $\frac{3}{5}$, wherefore the gain of the Merchants unto the Factor is then in ſuch proportion as 3 unto 2: whereupon if you will then know the eſtimation of the perſon of the Factor, ſay, If 3 give 2, what ſhall 400 give? Worke, and you ſhall finde 266 $\frac{2}{3}$ pounds. And ſo much is the perſon of the Factor eſteemed to be worth.

And if the Merchant ſhould deliver unto his Factor 400 pound, and the Factor would lay in 80, and his perſon, to the end he might have the $\frac{1}{2}$ of the gain, I demand how much ſhall his perſon be eſteemed?

Anſwer. *Abate* 80 *from* 400, *and there will remain* 320. *And at ſo much ſhall his Perſon be eſteemed.*

A Merchant hath delivered unto his Factor 900 pounds to govern in the Trade of Merchandize, upon condition that he ſhall have the $\frac{1}{3}$ of the gain, if any thing be gained, and alſo to bear the $\frac{1}{3}$ of the loſſe, if any thing be loſt. Now I demand how much his perſon was eſteemed at?

Anſwer. *Seeing that the Factor taketh the* $\frac{1}{3}$ *of the gain, his Perſon ought to be eſteemed as much as* $\frac{1}{2}$ *of the ſtocke which the Merchant layeth in: that is to ſay, the* $\frac{1}{2}$ *of* 900 *Pound, which is* 450. *The reaſon is, becauſe* $\frac{1}{3}$ *of the gain that the Factor taketh is the* $\frac{1}{2}$ *of the* $\frac{1}{3}$ *of the gaine that the Merchant taketh, and ſo the Factor his Perſon is eſteemed to be worth* 450. *Pounds.*

A Merchant hath delivered unto his Factor 600 li. and the factor layeth in 250 li. and his perſon. Now becauſe he layeth in 250 li. and his perſon, it is agreed between them, that he ſhall take the $\frac{2}{5}$ of the gain. I demand for how much his perſon was eſteemed?

Anſwer. *For as much as the Factor taketh* $\frac{1}{3}$ *of the gain, he taketh* $\frac{2}{3}$ *of that which the Merchant taketh, for* $\frac{2}{5}$ *are the* $\frac{3}{3}$ *of* $\frac{2}{5}$. *And therefore the Factors laying in ought to be* 400 li. *which is* $\frac{2}{3}$ *of* 600 *pound that the Merchant laid in. Then ſubtract* 250, *which the Factor did lay in, from* 400 *pound which ſhould have been his whole ſtock, and there remaineth* 150 *pound for the eſtimation of his perſon.*

More, a Merchant hath delivered unto his factor 800 pound, upon condition that the

factor

Factor ſhall have the gain of 160 li. as though he laid in ſo much ready money: I demand what portion of the gain the ſaid Factor ſhall take?

Anſwer, *See what part the* 160 (*which the Factor laid in*) *is of* 960, *which is the whole ſtock of their company, and you ſhall finde* $\frac{1}{6}$: *And ſuch part of the gain ſhall the Factor take.*

But in caſe, that in making their Covenants, it were ſo agreed between them, that the Factor ſhould have the gain of 160 *Pound of the whole ſtock which the Merchant layeth in, that is to ſay, of the* 800 *Pound: then ſhould the Factor take* $\frac{1}{5}$ *of the gains, for* 160 *is* $\frac{1}{5}$ *of* 800 *Pound.*

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandize, which is diſtinct into ſeven Rules, with divers other neceſſary queſtions incident thereunto.

The firſt Rule.

TWo Merchants, willing to change their Merchandize the one with the other: The one hath 24 broad clothes at 10 li. 10 s. the piece: The other hath Mace at 12 ſhillings the pound. The queſtion is, how many pounds of Mace he ought

to give for his Cloth, to save himſelfe harmleſſe, and be no loſer?

Anſwer. *Seek firſt by the* Rule of Three, *what the 24 Clothes coſt at 10 pounds 10 ſhillings the piece, and you ſhall finde 252 pound. Then ſeek the quantity of Mace, ſay again by the Rule of Three, If 12 ſhillings buy one pound, what ſhall 252 pound buy me? worke, and you ſhall finde 420 pound of Mace. And ſo many pounds ought he to give for his Clothes.*

The Proof.

Two barter. The one hath 420 pounds of Mace at 12 s. the pound, to barter or change broad Clothes, at 10 pounds 10 ſhillings the piece. The queſtion is, how many broad Clothes he ought to give for all his Mace?

Anſwer. *Firſt ſay, If one coſt 12 ſhillings, what 420? you ſhall finde 5040 s. Then ſay again, If 10 pounds give one Cloth, what ſhall 5040 ſhillings give? Worke, and you ſhall finde 24 Clothes, your deſire.*

The ſecond Rule.

Two change Merchandize for Merchandize: The one hath Peppet at two ſhillings foure pence the pound, to ſell for ready money. But in barter he will have no leſſe then three ſhillings the pound. And the other hath Hol-

land

ſand at five ſhillings ſix pence the Ell ready money. The queſtion is now, at what price he ought to deliver the Ell in the barter to ſave himſelf harmleſſe.

Anſwer. *Say by the* Rule of Three direct: *If* 2 $\frac{1}{3}$ *ready money give* 3 s. *in barter, what ſhall* 5 $\frac{1}{2}$ *give in barter? You ſhall finde* 7 $\frac{1}{14}$ s. *and at that price ought the ſecond Merchant to ſell his Holland in barter.*

The Proof.

Two barter. The one hath Holland at 5 s. 6d. the Ell to ſell for ready money. And in barter he will have 7 $\frac{1}{14}$ s. The other hath Pepper at 2 s. 4 d. the pound, to ſell for ready money. The queſtion is now, how he ought to ſell in barter?

Anſwer. *Say by the* Rule of Three direct, *If* 5 $\frac{1}{2}$ *ready money give* 7 $\frac{1}{14}$ s. *in barter; what ought* 2 $\frac{1}{3}$ *to take in barter? Multiply and divide, and you ſhall finde* 3 *ſhillings your deſire.*

The third Rule.

Two barter. The one hath cloth of Arras at 30 s. the Ell ready money, but in barter he will have 35 $\frac{1}{2}$ s. And the other hath white Wines which he delivered in barter for 16 pounds the Tun. The queſtion is now, what his Wines coſt the Tun in ready money.

Anſwer. *Say by the* Rule of Three direct, *If* 35 ½ s. *in barter, give but* 30 *ready money, what did* 16 *pound in barter coſt the Worke; and you ſhall finde* 13 li. 10 s. $\frac{12}{71}$. *And ſo much coſt his VVines for a Tun ready money.*

The proofe.

Two barter Merchandize for Merchandize: The one hath white Wines at 13 li. 10 s. $\frac{12}{71}$ s. the Tun to ſell for ready money: But in barter he delivered it for 16 pounds. The other, to make his match good and ſave himſelfe harmleſſe, delivereth Arras at 35 ½ s. the Ell. The queſtion is now, what an Ell of his Arras coſt in ready money?

Anſwer. *Say by the* Rule of Three direct: *If* 16 *pounds in barter give but* 13 li. 10 $\frac{12}{71}$ s. *in ready money, what ſhall* 35 ½ s. *yield in barter? VVorke, and you ſhall finde* 30 s. *your deſire.*

The fourth Rule.

Two barter: The one hath Kerſeyes at 14 *pounds the Piece ready money: But in barter he will have* 18 *pounds: and yet he will have the* ⅓ *part of his over-price in ready money. And the other hath Ginger at eight groats the pound to ſell for ready money. The queſtion is, how he ought to deliver the Ginger by the pound in barter to ſave himſelfe harmleſſe, and make the barter equall.*

Anſwer.

Answer. Item, for the working of this queſtion, and ſuch other the like, you muſt underſtand, if the party over-ſelling his wares, require to have alſo ſome portion in ready money, as $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ &c. Then ſhall you firſt rebate the ſame demanded part, whatſoever it be, from the over-price, and alſo from the juſt price. And thoſe two numbers that ſhall remain after the ſubtraction is made, ſhall be the two firſt numbers in the *Rule of Three*. And the juſt price of the ſame Merchandize ſhall be the third number, which by the operation of the *Rule of Three direct*, ſhall yield you a true ſolution, how, and at what price you ſhall overſell that your Merchandize, to ſave your ſelf harmleſſe, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighteen) which is the over-price of his Cloth, which $\frac{1}{3}$ of eighteen is ſix, which you muſt ſubtract from 18, there reſt 12. And alſo abate it from 14, which is the juſt price of the Cloth, and there remaineth 8, which and 12 are the two firſt numbers in the *Rule of Three*. Then take eight groats, or 2 $\frac{2}{3}$ ſhillings for the third number. Then ſay by the *Rule of Three direct*: If eight pounds give 12 pounds, what ſhall 2 $\frac{2}{3}$ s. give?

18	14
6	6
12	8

Mul-

Multiply and divide, and you shall finde 4s. And for so much shall the second Merchant sell his Ginger, or his commodity in barter, to ballance the same equall.

The Proof.

Two barter; the one hath five Kerseys, at 14 Pounds the piece ready money: But in barter he will have 18 Pounds: and yet he will have the ⅓ part of his over-price in ready money. And the other hath Ginger, which he having enough to make the barter equall, delivered in barter for 4s. the li. The question is now, what his Ginger cost him ready money?

Answer. After you have made the subtraction, abating 6 the ⅓ part of 18, both from 18 and 14 (as before was taught you:) then will there remain 8 and 12 for your two first numbers in the *Rule of Three*. Then say, If 12 give 8, what shall come of 4 the over-price of the pound of Ginger? Multiply and divide, and you shall find 2s. 8d. your desire.

Two Merchants barter Merchandize, for Merchandize. The one hath Devonshire *whites at* 7 li. 13 s. 4 d. *the piece ready money: but in barter he doth them away for* 8 li. 3s. 4d. *and yet he will have the ⅔ part of his price in ready money. And the other hath Cottens at three Pounds the piece ready money. The question is now, at what price he aught to sell or exchange his Cottens in barter*

barter to ſave himſelfe harmleſſe, and make the barter equall?

li.	ſ.	d.	li.	ſ.	d.
7	13	4	3		4
2	14	5⅓	2	14	4⅓
4	88	10⅔	5	8	10⅔

Anſwer. Firſt ſeek the ⅓ part of 8 li. 7s. 4d. which is 2 li. 14 s. 5⅓ d. which rebated from 8 li. 7s. 4d. there reſteth as appeareth by the Example aboveſaid, 5 li. 8s. 10⅔ d. which is ⅔ parts of 8 li. 7s. 4d. alſo rebated from 7 lib. 13s. 4d. there reſteth 4 li. 18s. 10⅔ d. the two firſt numbers in the *Rule of Three*, and the three pounds, which is the neat price of the piece of Cotten, is the third number: Then ſay by the *Rule of Three direct*, as was taught before. If 4 li. 18 s. 10⅔ d. give 5 li. 8s. 10⅔ d. what ſhall three pounds give? Multiply and divide, and you ſhall find three pounds 6s. $\frac{22}{39}$ d. The juſt price that he ought to deliver his Cottens in barter.

The fifth Rule.

Two Merchants will change Merchandize for Merchandize. The one hath Kerſeys at 43 s. the piece to ſell them for ready money. And in barter he will ſell them for 56 s. 8 d. and he will gain after ten Pounds upon the 100 Pounds. And

yet he will have the $\frac{1}{3}$ *of his over-price in ready money. The other hath Flax at* 3 d. *the Pound ready money. The question is now, how he shall sell the Pound of his Flax in barter?*

Answer. See first at 10 pound upon the 100 pounds what the $56\frac{2}{3}$ s. commeth to, in saying, (by the *Rule of Three direct*) If 100 pounds give 110 pounds, what $56\frac{2}{3}$ s? Multiply and divide, and you shall finde 3 pound 2 shillings 4 pence, of which the $\frac{1}{3}$ that he demandeth in ready money, is 1 pound 11 shillings 2 pence; the same 31 s. 2 d. abated from 40 shillings, and also from 56 s. 8 d. there will remaine 8 s. 10 d. and 25 s. 6 d. for the two first numbers in the *Rule of Three*, and 3 pence the price of the pound of Flax for the third number. Then multiply and divide, and you shall find $8\frac{1}{4}$ d. And for so much shall he sell the pound of Flax in barter.

The sixth Rule.

Two are willing to exchange Merchandize: The one hath Norwich *Grograns at* 25 s. *the Piece ready money: and in barter he will have* 30 s. *and he will have the* $\frac{1}{4}$ *part of his over-price in ready money. The other hath* Norwich *Stockings at* 40 s. *the dozen to sell for ready money. But in as much as the first Merchants Grograns are no better, he would deliver them so to ballance the barter, that he may gain* 10 *Pounds in the* 100. *Pounds. The question is now, how he shall sell his*

his hose the dozen in barter, according to his request.

Answer. Say, if 100 give 110 l. what shall 40 s. give, which is the just price of the dozen of stockins? Multiply and divide, and you shall finde 44 s. Then take the $\frac{1}{4}$ of 30 s. which is 7 s. 6 d. and subtract it from 25 s. and also from 30 s. and there will remaine 17 s. 6 d. and 22 s. 6 d. for the two first numbers in the *Rule of Three*, and 44 shillings, which is the just price (with his gain in the dozen of Stockins) for the third number. Then multiply and divide, and you shall finde 56 s. 6 $\frac{4}{7}$ d. and for so much he is to sell his dozen of stockins in barter.

The seventh Rule.

Two Merchants will change their Merchandize one with the other: The one hath 720 Ells of Cambricke at 5 s. the Ell to sell for ready money, but in barter he requireth 6 s. 8 d. And yet notwithstanding, he loseth by it after 10 pounds upon the hundred pounds, whereupon he requireth one halfe of his overprice in ready money: and the other Merchant having skill enough to make the barter equall, delivered English *Saffrons at 30 s. the pound. The question is now, what his Saffrons cost the pound in ready money.*

Answer. You must first seeke what is lost upon the 100 pound, which to do, you may say, (if you please) If 100 pound lose 10, what shall

ſhall 6 ⅔ loſe? Worke, and you ſhall finde 5s. (or 8 d.) which muſt be rebated from 6s. 8 d. ſo reſteth 6 s. ſtill. Or you may ſay, If 100 pound give me but 90 pounds, what ſhall 6 s. 8 d. give? Work this way either, and you ſhall find alſo as before directly in your quotient 6s. your deſire. Then are you next to caſt up what the 720 Ells of Cambrick commeth to at 6s. 8d. the Ell, and you ſhall find 240 pounds: the ½ whereof the Cambrick Merchant will have in ready money (which is 120 pounds:) Nextly, you muſt caſt what the Cambrick commeth to after his loſſe in the 100 pound, which as you found, is but 6 s. an Ell, and you ſhall find 216 pounds: Now muſt you ſubtract his ready money (which is 120 pounds in all) out of 240 pound, and alſo out of 216 pound, and there will remain 120 pounds, and 96 pounds for your two firſt numbers in the *Rule of Three*, and 38 ſhillings is the over-price of your Saffron for the third number: Then multiply and divide, and you ſhall finde 24 ſhillings. And ſo much did his Saffron coſt in ready money.

Two Merchants barter, the one hath fifty Clothes to put away for ready money at 11 *pounds the Cloth, and in barter putteth them away for* 12 *pounds, taking Holland Cloth at* 20 d. *the Flemmiſh Ell, which was worth no more but* 18 d. *The queſtion is now, what Holland payeth for the Cloth, and what he winneth or loſeth by the bargain?*

Anſwer.

Anſwer. Fifty Clothes at 11 pounds the Cloth commeth to 550 pounds, and put away at 12 pounds the piece, maketh 600 pound. Then to finde what Holland payeth for the Cloth, ſay by the *Rule of Three direct*, If 20 d. buy one Ell, what 600 pounds? Worke, and you ſhall finde 7200 Ells. Now to finde the Eſtate of his gain or loſſe, you muſt ſeek what his 7200 Ells commeth to at 18d. the Ell: Work by the Rule of proportion direct, and you ſhall finde 540 pounds, which is not ſo much as his Clothes were worth in ready money by tenne pounds: and ſo much loſt the firſt Merchant by his Exchange.

A Venetian hath in London 100 *pieces of ſilk, to put away for ready money at* 3 li. *the piece. But in barter he delivered them for* 4 li. *the piece, taking wools of a Felmonger at* 7 li. 10 s. *the C. weight, which was worth no more but ſix pounds, the C. ready money. The queſtion is now, what wools payeth for the ſilkes, and which of them winneth or loſeth by the barter?*

Anſwer. A hundred pieces of Silke at 3 li. is in all 300 lib. and at 4 lib. is 400 lib. Then to find what Wools payeth for the Silke, ſay by the *Rule of Three direct*: If $7\frac{1}{2}$ buy me 100 weight, what 400 pound? Work, and find $53\frac{1}{3}$ C. weight of wooll. Now to finde the Eſtate of their gaine and loſſe, caſt up his wooll at 6 lib. the C. (for ſo much they were worth ready money) and you ſhall finde 320 pound, which is 20 pound more then the Silkes were

to

to be sold for ready money, whereby the Venetian gained 20 pounds by the Barter.

A Merchant hath 53 ½ weight of Wooll at 6 pounds the C. to sell for ready money, but in barter he will have 7 pounds 10 s. And another doth barter with him for Silks, which are worth three pounds a piece ready money. The question is now, how he ought to deliver his Silkes the piece in barter, and how many payeth for the Wools.

Answer. Say by the Rule of Proportion, (or by the *Rule of Three direct*) If 6 pounds for C. weight ready money, yield me 7 li. 10 s. what will 3 li. yield, which is the just price of a piece of Silk in barter, to make the Truck equall? Work, and finde 3 li. 15 s. the price of a piece of Silk in barter: then say, If 3 li. 15 s. require one piece of Silk, how many pieces of Silk are bought with 400 pound, which is the value of 53 ½ C. weight of Wooll, at 7 li. 10 s? Worke by the *Rule of Three direct*, and you shall finde 160 pieces of Silk and ½ of a piece, and so many of Silk pay for the Wooll, and neither party hath advantage of other.

Two men will change Merchandize, the one with the other. The one of them hath beer at 6 s. 8 d. the barrell, to sell for ready money, but in barter he will sell the barrell for 8 s. and yet he will gain moreover after 10 pound upon the 100 pounds. And the other hath white Spanish Wooll at 2 s. the Rove, to sell for ready money. The question is now, how he shall deliver the Rove of Wooll in barter to save himself harmlesse.

Answer.

Anſwer. Say, if 6 $\frac{2}{3}$s. which is the juſt price of the barrell of Beer, be ſold in barter for 8 ſhillings: for how much ſhall 10 ſhillings (which is the juſt price of the Rove of Wooll) be ſold in barter? Work by the Rule of Three direct, and you ſhall finde 24 s. Then for becauſe the firſt Merchant will gaine after 10 pounds upon the 100 pounds, he maketh his 100 pounds, 110 pounds. And therefore ſay by the Rule of Three, If the ſecond Merchant of 110 pounds doe make but 100 pounds, how much ſhall he make of 24 s? Multiply and divide, and you ſhall finde 21 s. 9 d. $\frac{9}{11}$ of a peny. And for ſo much ſhall he ſell the Rove of wool to be delivered in barter, to the end the firſt Merchant may give 10 in the 100.

Two Merchants will change their Commodities the one with the other. The one of them hath white Paper at 4 s. the Reame, to ſell for ready money. And in barter he will doe it away for 5 s. and yet he will gain moreover after the rate of 10 pounds upon the 100 pounds. And the other hath Mace at 14 s. 6 d. the pound weight to ſell in barter. Now I demand what the pound did coſt in ready money.

Anſwer. Say if 5 s. (which is the over price of the Paper in barter) become of 4 s. the juſt price of how much ſhall come 14 $\frac{1}{2}$ ſhillings, which is the ſurprize of the pound of Mace in barter? Multiply and divide, and you ſhall find 11 $\frac{3}{5}$ s. Then for becauſe the firſt Merchant of Paper will gain after 10 upon the 100, Say, if

100 doe give 110, what ſhall 21 $\frac{2}{3}$ ſhillings give? Worke and you ſhall finde 12s.-$9\frac{11}{13}$ d. and ſo much did the pound of Mace coſt in ready money.

The fourteenth Chapter intreateth of exchanging of money from one place to the other.

EXchange is no other thing, then to take or receive money in one City, to render or pay the value thereof in another City, or elſe to give money in one place, and receive the value thereof in another, at term of certain dayes, moneths, or fairs, according to the diverſity of the place.

But this practice chiefly conſiſteth in the knowledge of the Money or Coyns in divers places, of which for thy benefit (after a few examples given to the Introduction of this work) I will ſet down certain notes of the diverſity of the common and uſuall Coynes in moſt places in Chriſtendome for traffick.

And firſt I will begin at *Antwerp*, where they uſe to make their accounts by *Deniers de groſſe*, that is to ſay, pence *Flemmiſh*, whereof 12 doe make 1 s. *Flemmiſh*, and 20s. doe make one pound *de groſſe*.

Item,

Item, *A Merchant delivered at* Antwerp, 400 *pounds* Flemmiſh *to receive in* London 20s. *ſterling, for every* 23 s. —— 4 d. Flemmiſh: *The queſtion is now, how much ſterling money is to be received at* London *for* 400 *pounds* Flemmiſh?

Anſwer. Say by the Rule of Three, If 23 $\frac{1}{3}$ *Flemmiſh* give 20s. ſterling, what 400 pounds *Flemmiſh*? Worke, and you ſhall finde 342 l. —— 17 s. —— 1 $\frac{1}{7}$ pence, and ſo much ſterling ſhall I receive in *London* for the ſaid 400 pounds *Flemmiſh*.

Otherwiſe alſo wrought by Rules of Practice in taking the $\frac{1}{7}$ of the *Flemmiſh* mony delivered, and abating the ſame from the principall, the reſt is *Engliſh* money, as before.

400 li. —— 20s. —— 0 d.
57 —— 2 —— 10 $\frac{5}{7}$
342 —— 17 —— 1 $\frac{2}{7}$ ſterling.

A Merchant at London *delivered* 200 lib. *ſterling for* Antwerp, at 23 s. 4 d. Flemmiſh *the pounds ſterling: the queſtion is, how much he muſt receive at* Antwerp.

Anſwer. Say by the Rule of Three, if 1 pound ſterling give 25 s. 5 d. *Flemmiſh*, what 200 li. ſterling? Worke, and thou ſhalt finde 234 li. —— 3 s. —— 4 d. So many pounds *Flemmiſh* ſhall he receive at *Antwerp* for the ſaid 200 pounds ſterling.

Otherwiſe by Practice.

1——13——5——200

3s.-4 d. 33——6——8

1 d. ——16——8

maketh ſterling——————2, 4 li.—3s.—4d

In London 20 *pound ſterling is delivered by Exchange for* Antwerp, *at* 23s. 9 d. Flemmiſh *the pound ſterling: the queſtion is, at what rate the* Flemmiſh *money ought to be returned to gain foure pounds upon the* 100 *pound ſterling at* London.

Anſwer. Firſt, ſay by the Rule of Three direct: If 1 pound ſterling give $23\frac{1}{4}$ Flemmiſh, what 200 pounds ſterling? Multiply and divide, and you ſhall finde 237 pounds 10 ſhillings. The which to return to gaine 8 pounds ſterling in *London*, ſay by the backer Rule, If 200 pounds ſterling require the exchange 23 s. 9d. Flemmiſh, what the exchange to make 208 li. ſterling? Work by the Rule, and finde 22s. $10\frac{1}{6}$ d. Flemmiſh, the effect in the queſtion required.

If I take up money at Antwerp *after* 19s. 4d. Flemmiſh, *to pay for the ſame at* London, 20. *ſhillings ſterling, and when the day of payment is come, I am forced to return the ſame money again in* London, *to pay my Bill of Exchange: ſo that for* 20 *ſhillings which I take up here at* London, *I muſt pay* 19s. 6d. *at* Antwerp, *I demand whether*

ther I doe win or loſe, and how much in or upon the 100 *pounds of money?*

Anſwer. Say by the Rule of Three: If 19 $\frac{2}{3}$ give 19 $\frac{1}{3}$ what will 140 pounds give? Multiply and divide, and you ſhall finde 99 li. 2 $\frac{126}{132}$ s. which being abated from 100 pounds, there will remain 17 s. $\frac{11}{117}$, and ſo much I doe loſe upon the 100 pounds of money.

If I take up at London 20 *ſhillings ſterling to pay at* Antwerp 22s. 4d. *and when the day of payment is come, my Factor is conſtrained to take up money again at* Antwerp, *wherewith to pay the aforeſaid ſumme, and there he doth receive* 23 s. 4 d. Flemmiſh, *for the which I muſt pay* 20 s. *at* London: *The queſtion is now, whether I doe win or loſe, and how much upon the* 100 lib. *of money after that rate.*

Anſwer. Say by the Rule of Proportion, If 22 $\frac{1}{3}$ s give 23 $\frac{1}{3}$ s. what will 100 pounds give? Multiply and divide, and you ſhall finde 104 pounds 9 ſhillings $\frac{27}{67}$, from the which abate 100 pounds, and there will remain 4 pounds 9 ſhillings $\frac{27}{67}$, and ſo much is there gained upon the 100 pounds of money.

In Antwerp *is delivered* 200 *pounds* Flemmiſh *by exchange for* London, *at* 20 *ſhillings ſterling for every* 23 *ſhillings* 4d. Flemmiſh. *The queſtion is, at what rate the ſame is to be returned to gain* 10 *pounds upon the* 100 *pounds* Flemmiſh *in* Antwerp.

Answer. First, say by the Rule of Three, if $23\frac{1}{3}$ Flemmish give 20s. what shall 200 pounds gain? Worke, and you shall finde 171 pounds 8s.6d. Then say again by the Rule of Three direct, if 171 pounds 8s. $6\frac{6}{7}$s. sterling, give me 210 pounds Flemmish, what shall 20s. sterling give? Work, and you shall find 24s.6d. Flemmish. And at the same rate ought the same to be returned at *Antwerp*, to gain 10 pounds upon the 100 Flemmish.

A Merchant of Antwerp *delivereth* 234 lib. 3s. 4d. Flemmish, *to receive at* London 200 l. *sterling: The question is now, how the exchange goeth after this rate?*

Answer. Say by the Rule of Three direct, if 200 give 20. what $234\frac{2}{3}$? Multiply and divide, and you shall find 23s——5d. And for so much goeth the exchange.

Item, *the exchange from* London *into* France, *is not like as it is to* Flanders, *but it is delivered by the* French *Crown, which is worth* 50 Soulx Turnois *the piece.*

Whereupon also you must note, that in *France* they make their accounts by Franks, Soulx, and Deniers Turnois, whereof 12 Deniers make one Soulx Turnois, and 20 Soulx maketh one pound Turnois, which they call a Liure or Frank. But the Merchants to make their Accounts, doe use French Crowns, which is currant among them for 51 Soulx Turnois. But by exchange it is otherwise, for it is delivered but for 50 Soulx Turnois the Crown, or as the taker

up

up of the money can agree with the deliverer. And note that this △ Character repreſenteth the Crown by exchange, and is ever 50 Soulx Turnois or *French* money.

A Merchant delivereth at London 240 *pounds ſterling, after five ſhillings ſixe pence the Crown, to receive at* Paris 50 *Soulx Turnois for every Crowne. I demand how much* Turnois *or* French *money payeth the Bills for the ſaid* 240 *pounds ſterling.*

Anſwer. Say by the Rule of Three. If 5 ½ s. ſterling give me 50 s. Turnois, what ſhall 240. pounds ſterling give? Reduce the pounds into ſhillings, then multiply and divide, and you ſhall finde 2181 Liures, 16 Soulx. 4 Deniers, and $\frac{8}{11}$ Turnois, and ſo much payeth the Bills at *Paris*, for the 240 pound ſterling.

A Merchant delivereth at Roan, *or elſewhere in* France, 1430 *pounds or franks, the which franke or pound is* 20 *Soulx, or a pound* Turnois *to receive in* London 6s. 4 d. *ſterling for every* △ *of* 50 *Soulx* Turnois. *The queſtion is, how much ſterling money I ought to receive at* London *for my* 1430 *pound* Turnois.

Anſwer. Say, if 2 ½ pounds give me 6 $\frac{1}{2}$ s, what will 1430 give me? Worke, and you ſhall find 3622 $\frac{2}{3}$ ſhillings ſterling, which maketh 181 li. 2. 8d. and ſo much money is to be received at *London*, for the ſaid 1430 Liure Turnois, after 6s. 4d. for every △ of 50 Soulx.

In London *is delivered* 200 *pound ſterling by exchange for* Paris, *at* 5s. 9d. *the* △ *of* 50 *Soulx*

Turnois. *The queſtion is, at what price the ſaid* △ *is to be returned to gain 6 pounds upon the* 100 *pounds ſterling at* London.

Anſwer. Firſt, ſay (by the Rule of Three direct) if 5 $\frac{1}{4}$ s. ſterling give 50 Soulx Turnois; what ſhall 200 pound ſterling give? Work, and you ſhall find 1739 Franks or Liures, 2 $\frac{14}{23}$ Soulx. Then the which to return and gaine 6 pounds upon the hundred pounds in *London*, ſay by the Rule of Three direct, if 1739 Franks 2 $\frac{14}{11}$ Soulx yield 212 pound, what the △ of 50 Soulx? work and find 6s. 1 $\frac{1}{30}$ d. the effect required in the queſtion.

A Merchant delivered in London 160 *pounds ſterling, to receive in* Biskay *for every* 5 s. 6 d. *one Ducat of* 374 Marvides. *The queſtion is, how many* Marvides *ought I to receive at* Biskay?

Anſwer. Say, if 5 $\frac{1}{2}$s. ſterling give 374 Marvides: what ſhall 160 pounds ſterling give? Multiply and divide, and you ſhall find 217600 Marvides, and ſo many I ought to receive at *Biskay* for my 160 pounds ſterling.

A Merchant delivered in Baion, 4000 Marvides *to receive in* London 5 s. 8 d. *ſterling for every Ducat of* 374 Marvides. *The queſtion is now, how much ſterling money payeth the Bills of Exchange for the ſaid* 4000 Marvides?

Anſwer. Say, if 374 Marvides make one Ducat, what 4000 Marvides? Multiply and divide, and find, 106 $\frac{178}{187}$

Then ſay again, if 1 Ducat give 5 $\frac{2}{3}$ s. what giveth 106 $\frac{178}{187}$ Ducats? Work, and find 30 l. 6 s. $\frac{124}{111}$

Other-

Otherwiſe it is wrought more briefe at one working, as in the laſt queſtion before, in conſidering that 5 s. 8 d. containeth one Ducat, or 374 Marvides. Therefore ſay by the Rule of Three, if 374 Marvides give $5\frac{2}{3}$ s. what 4000 Marvides? Work, and you ſhall alſo find in your quotient 30 $\frac{14}{381}$ s. And ſo many pounds ſterling is to be received for the 40000 Marvides.

In London 200 *pounds delivered by exchange for* Vigo, 374 *Marvides the Ducat of* 5s. 10. d. *ſterling, maketh* 256457 $\frac{1}{7}$ *Marvides: the which to return and gain* 10 li. *upon the* 100 *pounds in* London, *ſay by the* Rule of Three direct, *if* 220 li. *require* 256457 $\frac{1}{7}$ *Marvides: what* 5 s. 10 d. *Work, and find* 340 *Marvides, the price of every Ducat in return, which is the effect in the queſtion required.*

Theſe may ſeem ſufficient for inſtructions.

NOtwithſtanding *for thy further aid and benefit, hereafter follow ſix ſpeciall and moſt briefe* Rules of Practice, *for* Engliſh, French, *and* Flemmiſh *money.*

teacheth:
1. How to turn Flem. to Engliſh ſterling.
2. How to turn Engliſh ſterling to Flem.
3. How to turn Flemmiſh to French.
4. How to turn French into Flemmiſh.
5. How to turn ſterling into French.
6. How to turn French into ſterling.

The

The fifteenth Chapter intreateth of the ſaid ſix Rules of brevity, and of valuation of *Engliſh*, *Flemmiſh*, and *French* money, and how each of them may eaſily bee brought to others value.

How briefly to reduce pounds, ſhillings, and pence Flemmiſh *into pounds, ſhillings, and pence* Engliſh *ſterling.*

1 Rule. IT is to be noted, that 7 pounds *Flemmiſh* maketh but 6 pounds ſterling: 7s. *Flemmiſh* maketh 6s. ſterling, and 7 d. *Flemmiſh* 6d. ſterling: ſo that 7 yieldeth but 6. Wherein is evident that then is loſt $\frac{1}{7}$, (if it may be ſo called) when it is reduced into *Engliſh* money: wherefore to know how much 233 li. 12s. 4d. *Flemmiſh* maketh *Engliſh*, you muſt ſubtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which reſteth after this ſubtraction, is the ſumme required: ſo that 233 li. 12s. 4 d. *Flemmiſh*, maketh 209 li. 5s. 8 $\frac{4}{9}$ d. ſterling.

Example

Example.	li.	s.	d.
	233	13	4
	7/17 33	7	7 2/7
	200	5	8 2/7 Ster.

Another Example.	li.	s.	d.
	311	0	0
	7/8 44	8	6 4/7
	266	11	5 3/7

To reduce pounds, ſhillings, and pence ſterling, into pounds, ſhillings, and pence Flemmiſh.

Note that a pound ſterling maketh 1 li. 15 s. 4 d. *Flemmiſh*: that is, 1 1/5 li. 1 s. ſterling maketh 1 1/5 s. *Flemmiſh*, and 1 d. ſterling maketh 1 1/5 d. *Flemmiſh*. So that there is gained (if it may be ſo called) 1/5 of the ſumme being thus reduced to *Flemmiſh*, for of 5/5 is made 6/5 which is one whole and 1/5. Then to know how much 237 li. 7 s. 6 d. ſterling maketh *Flemmiſh*, ſubtract from your ſterling the 1/6 of the whole ſumme, and adde it to the ſame ſumme, and it maketh 276 li. 18 s. 4 d. which is the ſumme required.

Example.	li.	s.	d.
	237	7	6 Ster.
	1/6 39	11	3
	276	18	9 Flem.

Another example.	li.	s.	d.
	337	0	0
	1/6 56	3	4
	393	3	4

To

Rule 3. *To reduce pounds, shillings, and pence Flemmish, into pounds, shillings, and pence French.*

Ye shall note, that the equality of Flemmish and French money is this, that is to say, the pound Flemmish, maketh 7 pound $\frac{2}{3}$ French, or Turnois. 1 s. Flemmish maketh 7 $\frac{1}{3}$ s. French, and a groat Flemmish, maketh 7 $\frac{1}{3}$ d. French.

Wherefore to know how much 143 li. 4 s. 9 d. Flemmish maketh French, ye must multiply the whole number twice by 6, beginning at pence, and so forward, and the Product of your second multiplication divide by 5, so the work is finished. Or multiply the said summe by 7, and take out of it $\frac{1}{3}$, adding it to the Product of your multiplication by 7. and that is your number required. So that as well by the one as by the other, 143 li. 4 s. 9 d. Flemmish, maketh 1031 li. 6 s. 2 $\frac{2}{3}$ d. French or Turnois.

Example.

li.	s.	d.	
143	4	9	Flem.
		6	
859	8	6	
		6	
5156	11	0	Fren.
$\frac{1}{5}$ 1031	6	2 $\frac{2}{5}$	Fren.

The same otherwise.

li.	s.	d.
134	4	9
		7
1002	13	3
$\frac{1}{5}$ 28	12	11 $\frac{2}{5}$
1031	6	2 $\frac{2}{3}$

Ano-

Another Example.

143 l. Flem.
6

858
6

$\frac{2}{5}$ 5148 French.

1029 li. $\frac{1}{5}$ *or* 12 s. French.

Or thus:

143
7

1001
$\frac{1}{3}$ 28——12

1029 li——12

To reduce pounds, ſhillings, and pence, French, *into pounds, ſhillings, and pence,* Flemmiſh. Rule 4.

Multiply 233 li——8s.——4d. French by 5, and divide the Product twice by 6, that is, the ſaid number by 6, and the Product or Quotient again by 6, and the quotient of this ſecond Diviſion is the thing required. So that 233 li.—8s.-4d. French, maketh 32 li.-8s.-$4\frac{1}{4}$d. Flemmiſh.

Example.

li.	s.	d.	
233	8	4	Fren.
		5	
1167	1	8	
$\frac{1}{6}$ 194	10	$3\frac{2}{3}$	
$\frac{1}{6}$ 32	8	$4\frac{1}{9}$	Flem.

Another Example.

li.	s.	d.	
758			French.
	5		
3765			
$\frac{1}{6}$ 627	10		
$\frac{1}{6}$ 104	11	8	Flem.

To

5 Rule. *To reduce pounds, ſhillings, and pence Sterling, into pounds, ſhillings, and pence, French, or Turnois.*

The pound Sterling maketh 8 li. 8 s. *French*, that is to ſay, $8\frac{2}{3}$ pounds: the ſhillings maketh $8\frac{2}{3}$ ſhillings, and the peny $8\frac{2}{3}$ d. *French*. Wherefore to know what 231 li. 13 s. 4d. Sterling maketh *French*, ye muſt multiply your whole ſumme by 42, that is, by 7. and the Product of it by 6. and divide this ſecond Product by 5. and that is the ſumme required.

Otherwiſe, multiply the ſumme Sterling by 8, and adde twice to the Product $\frac{1}{3}$, and it ſhall produce the ſumme required. So that both waies 231 li. 13 s. 4 d. Sterling maketh 1946 pound *French*, as here undes followeth.

Example. *The ſame other wiſe.*

Sterling.	li.	s.	d.	Ster.	li.	s.	d.
	231	13	4		231	13	4
			6				8
	1390	0	0		1853	6	8
			7		46	6	8
					46	6	8
	$\frac{1}{3}$ 9730	0	0				
					1946	0	0
French.	$\frac{1}{3}$ 1946	0	0	Fren.			

Another

Another example.

753
6
4518
7
31626
1/3 6325 —— 4 Fren.

The same. Sterling.

Ster. 753
8
6024
1/2 250 —— 12
1/2 150 —— 12
6325 —— 4 French.

To reduce pounds, shillings, and pence, French, into pounds, shillings, and pence, sterling.

To know how much 1256 li. 12 s. 6d. *French* maketh in sterling money: multiply the summe by 5, and divide the Product by 7 and 6 at twice, and the last Quotient shall be the thing required, that is to say, 1256 li. 12 s. 6d. maketh 149 pounds, 11 s. 11 4/7 d. sterling.

Example.

li.	s.	d.
1256	12	6
		5
6283	2	6
1/7 1047	3	9
1/6 149	11	11 4/7 Ster.

Another example.

French. li.	s.	d.
2531	0	0 *French.*
		5
12655		
1/6 2109	3	4
1/7 301	6	2 1/2 Ster.

Note,

Note, that when any money is given by exchange at *London* for *Roan* at 71 ½ d. or rather 71 $\frac{1}{7}$ for the Crown of 50 s. French, there is neither gain nor loſſe: for it is one money for another, accounting 8 li. 8 s. French, for one pound ſterling. So the giver loſeth the time of payment, which is about 15 dayes, and he that taketh it, hath the gain of the ſame.

They of *Roan*, that put forth or take mony by exchange for *London*, ought to have like conſideration.

Item, When any man giveth at *London* 64 pence $\frac{1}{3}$, or rather 65 $\frac{1}{7}$ d. to have at one of the Fairs of *Lions* a Crown *de Marc*, he that ſo giveth the money, loſeth the time, and he that taketh it, gaineth the ſame: for 62 pence $\frac{1}{7}$ is equall in value to 45 s. French. He that putteth or taketh mony at *Lions* for *London*, ought to conſider the ſame.

Item, when any deliver in *Antwerp* 75 pence, to receive at *Lions* a Crowne *de Marc*, he that putteth it forth, loſeth the time, and he that taketh it gaineth the ſame. For 75 groats Flemmiſh, is equall in value to 45 s. French.

Thus for this time I make an end of the practice of Exchange, and the inſtructions thereunto belonging, and according to my promiſe: yet further to gratifie ſuch as are deſirous to know the common Coynes uſed for trafficke among Merchants in theſe Cities following, a briefe declaration of their Moneyes, and the reckonings, and account of them.

The

The sixteenth Chapter containeth a Declaration of the valuation and diversity of Coyns of most places of Christendome for Traffick; And the manner of Exchange in those places from one City or Town to another: which known, is right necessary for Merchants, by means whereof they doe finde the gain or losse upon the Exchange.

ITem, for as much as the greatest diversity of money of Exchange is at *Lions*; therefore I will begin duely of the money of that place.

At *Lions* they use Franks, Soulx, and Deniers Turnois. A Frank maketh 20 Soulx, and one Soulx 12 Deniers; but the Merchants to keep their Bookes of accounts, doe use French crownes of the marke at 45 Soulx the piece, and doe divide it into 20 Soulx, one Soulx is 12 Deniers.

Item, a mark of gold maketh 65 Δ of the mark, which serveth for exchange, and divide it into 8 ounces, the ounce into 24 pence or deniers, the denier into 24 grains, and so the sum or whole by imagination or guesse.

Δ This mark standeth for a Crown.

Also at *Lions* there are four Fairs in a yeare, at the which they doe commonly Exchange, which are from 3 moneths to 3 moneths.

At *Genoa* they use the Soulx: one Ducat maketh 3 pound.

At *Naples* they use Ducats, Taries, and Grains; the Ducat maketh five Taries, and one Tary 20 Grains: but they take 6 Ducats which maketh 30 Taries for the ounce.

A Ducat maketh ten Carlins, and a Carlin ten grains, so that 2 Carlins make a Tary, and 100 Grains make a Ducat.

At *Rome* they use the Ducats of the Chamber; one Ducat is worth 12 Guillis, and one Guillis ten Soulx.

At *Venice* they use Ducats currant at 124 Soulx a piece, or 24 Demers, and one Denier maketh 33 Picolis.

At *Palerme* and *Messina* they write, 1 [illegible] ounce, tary, and grains, and one ounce is worth 6 Ducats of 30 Taries, and 1 Tary is 20 Grains, and 1 grain 6 Picolis, 1 Ducat is also worth 24 Carlins.

At *Millan* they use li. s. d. of Ducat Imperials, and △ of exchange is worth 4 li.

At *Lucques*, *Florence*, and *Ancone*, they use the △ of Gold: in Gold the *French* Crowns is worth 7 li. but at *Bolaigne* 3 li. 10 s.

At *Barcelone* they use the Soulx: the Ducat of Exchange is worth 22 Soulx.

At *Valence* and *Saragosse* they use the Liver, Soulx, and Denier: the *French* Crowne of exchange is worth 20 Soulx, and one Soulx is 12 Demers.

At the Fairs of *Castile* they use the Mar-

veides, the Ducat is worth 375 Marvedes.

At *Lisbone* they use the Rayes, one Ducat of Exchange is worth 400 Rayes.

At *Norenburg*, *Frankeford*, and *August* in *Germany*, they use the *Kranzers*, whereof 60 make a Floren.

At *Antwerp* they use li. s. d. de Gros, and they exchange into the Denier de Gros, to wit, our English peny.

At *London* they use the li. the s. and d. sterling, and they exchange in pence sterling.

The exchange of Lions *at sundry places.*

Item, at *Lions* there is exchange in three sorts, at the Cities and Townes following.

First, they deliver at Lions one Mark to have or receive at Naples almost 41 ½ Ducats, at Venice 70 ducats currant, at Rome 63 ducats of the Chamber, at Lucques and Florence 65 ℔ of Gold, at Millan 82 ℔.

And contrariwise, at the said Cities aforesaid they doe give so much of money, to have a mark at Lions.

Secondly, they give at Lisbone one of ℔ of Mark of 45 Soulx Turnois a piece, to have at Geans almost 68 Soulx, at Palerme and Messine almost 24 Carlins, at Barcelone 22 Soulx, at Valence or Saragosse 26 Soulx, at the Faire at Castile 350 Marvedes, at Lisbone 300 Rayes, at Antwerp 57 Deniers de Gros, and at London, 70 d. sterling.

And contrariwise, they give in the said Cities almost as much of their money to have a French Crown of the mark at *Lions*.

Thirdly, they do give at *Lions* 2 △ of the Sun to have almost 93 Krentzars at *Frankeford*, *Augsburg*, *Noremberge*, or other Cities in *Almain*.

Also at *Lions* onely they do pay, they change the $\frac{2}{3}$ in Gold, and $\frac{1}{3}$ in money, or else all in money, in giving $1\frac{1}{2}$ for the hundreth.

Changes at Naples *and other Towns.*

Item, at *Nayles* they give or deliver almost 112 Ducats, to receive at *Rome* 100 Ducats of the Chamber at the old value.

Through *Luegues* and *Florence* they deliver 100 Ducats Carlins, to receive there almost 86 △ of Gold.

Through *Palerm* and *Messine*, one Ducat of 5 Tary, to receive there almost 164 grains.

Through *Millain*, one Ducat to receive there almost 90 Soulx.

Through *Geans*, one ducat to receive there almost 65 Soulx. The whole summe to be paid within ten dayes after the sight of the Bill of Exchange.

Also at *Naples*, they deliver one ducat to receive in *Antwerp* almost 67 d. or Deniers *de Gros*, within two moneths. At *London* almost 60 d. sterling in three moneths. At *Barselone* almost 20 Soulx within two moneths.

At

At *Valence*, almoſt 18 Soulx within two months. At *Lisbone* 333 Rayes within 3 months: and at the Fair at *Caſtile* almoſt 340 Marvides, at the ſame Fair.

Change of Venice *to other places.*

At *Venice* they deliver 100 Ducats currant to receive in *Almain* almoſt 140 Florens at 60 Krentzers the piece.

At *Lucques* and *Florence* almoſt 109 ℔ of Gold in 10 dayes.

Likewiſe at *Venice* they deliver a Ducat currant to receive at *Palerme* and *Meſſine* almoſt 21 Carlins: at *Millan* almoſt 93 Soulx: at *Genes* almoſt 69 Soulx, the whole at 10 dayes end.

Of the Pair *or* Pari.

As touching the Exchange, it is neceſſary to underſtaud or know the *Pair*, which the *Italians* call *Pari*, which is no other thing then to make the money of the change of one City or Town, to or with the money of another, by means whereof they doe finde the gains or loſſe upon the Exchange.

Example.

Item, having received Letters of credit of one of *Antwerp*, that the Ducat of the Sunne is there worthy 23 Soulx: The queſtion is, what the ſame is worth at *London*, when the *Pair* of Exchange goeth for 23 ſhillings?

Anſwer. *Say, if 23 give but 20, what giveth? Worke, and finde* 81 11, ½ d. *and ſo much is the* ℔ *of the Sun worth at* London.

The seventeenth Chapter containeth also a Declaration of the diversity of the Weights and Measures of most places of Christendome for traffique. At the end of which discourse are two Tables, the one for Weight, and the other for Measure, proportionate and reduced to an equality of our *English* Measure and Weight, by the aide whereof, the ingenious may easily by the *Rule of Three*, convert the one into the other at pleasure &c.



In *London*, and to all *England* belonging, are used two kindes of Weights and Measures, as the *Troy* weight, and the *Haberdepoise*. From the *Troy* weight is derived the proportion and quantity of all kinde of dry and liquid Measures, as Pecks, Bushels, Quarters, &c. Wherewith is bought and sold all kinde of graine and other Commodities mete by the Bushel, and in liquid, Ale, Beere, Wine, Oyle, Butter, Honey, &c. Upon these grounds and Statutes is bread made, and sold by the *Troy* weight: and so is Gold, Silver,

Silver, Pearle, pretious Stones, and Jewels. The leaſt quantity of this *Troy* weight is a graine, twenty foure of theſe graines make a penny-weight, twenty penny-weights an Ounce, and twelve ounces a pound, two pounds of this of this weight maketh a quart. And ſo aſcending into bigger quantities, is produced the Meaſures whereby are ſold our other naturall ſuſtenance: *viz.* Ale or Beere, with all other neceſſary commodities, as Butter, Hony, Herings, Eeles, Sope, &c. All which laſt before rehearſed, though their Meaſures (wherein they are contained) bee framed and derived from the *Troy* weight, yet are they in traffique with divers Commodities, as Leade, Pitch, Flax, Wax, with all other commodities, both of this Realme, and of other forraine Countries whatſoever, bought and ſold by the *Haberdepoiſe* weight after ſixteen ounces to the pound, and 112 pound to the hundred weight. And to every hundred is allowed but 4 pound weight at the common-beam. From hence is alſo derived the weight of *Suffolke* Cheeſe, which containeth thirty two Cloves, 8 pound to a Clove, and weigheth in all 256 pounds. And alſo the Barrell of *Suffolke* Butter is, or ſhould be of like weight with the weight of Cheeſe, *viz.* 256 pounds. More 14 of theſe pounds make a Stone, and 26 ſtone containeth a Sacke of *Engliſh* Wooll. Forraine Wools, to wit, *French*, *Spaniſh*, and *Eſtriche*, is alſo ſold by the pound, or C. weight, but moſt

See further of theſe Weights and Meaſures in Reduction beginning, pag. 133.

commonly by the Rove, 25 pound to a Rove; other commodities of Tale, are bought and sold by the C. fivescore to the C. Except headed ware, to wit, Cattell, Nails, and Fish, which are sold after sixscore to the C. There are also two other sorts of measures, to wit, the Ell and the Yard. By the El is usually mete Linnen cloth, as Canvas, &c. And by the yard Silks, woollen clothes, &c.

Antwerp.

At *Antwerp* are also two sorts of weights, their Gold and Silver weight, and their common weight. Gold and Silver is weighed by the mark, the mark is 8 ounces, the ounce 20 Esterlings, and the Esterling 32, as our grains. The Goldsmiths divide that into smaller, but not the Merchants. The proof of Gold is made by Karects, whereof 24 maketh a marke of fine Gold, the Karect is 24 grains: the proof of the money is made by Deniers: 12 Deniers is 1 s. fine, that is, a mark of fine silver; the Denier also is divided into 24 grains, and the grain in 4 foure quarters.

Item, 100 marks in *Antwerp* Troy weight, maketh at *Lions* 103 marks, 2 ½ ounces, and 20 grains, 23 p. At *Noremberg* 103 marks 2 ½ ounces, 2 Quints, 3 Deniers; at *Frankeford* 105 marks: at *Ausburg* 104 marks, 3 ounces, 1 Quint: at *Venice* 103 marks, 1 ounce, 7 Deniers, 18 grains: at *London* 66 pounds.

The

The Mark of gold or silver at Antwerp, *Troy weight, which is* 8 *ounces, maketh* 7 ½ *ounces common weight, with which all other Merchandize is weighed. So that the Troy weight is greater then the common weight by* 6 ½ *in the* C. *By this weighe of* Troy, *they also weigh* Musk, Amber, Pearl, *&c. All silks are bought at* Antwerp, *by the* Burges Ell, *which is greater then the common measure, by which they retaile by* 2 *in the hundred. Their common Ell is* ¾ *of our yard, and* ⅘ *of our Ell.*

Lions.

At *Lions* is used 3 sorts of weight, whereof the first is the common Towne weight, with which they weigh all kind of Spicery, and divers other Merchandize. The second is called *Geneva* weight, which is 8 in the C. greater then the common weight, with which they weigh Silks, *&c.* The third is *French* weight, called commonly the Mark weight, and 100 pounds thereof maketh 106 ½ li. *Geneva*, and 114 ⅔ of their common weight: with which *French* weight is weighed all things that paid Custome or Toll.

At *Lions* is also used two sorts of Ells or Aulnes. The one wherewith they measure grosse clothes, as Canvasse, and such like. The other is called the *French* Ell or Aulne, with which they measure all other kind of Merchandize, whereof 7 common Towne Ells maketh 11 ordinary *French ells.*

Roan

Roan.

At *Roan*, 6½ Muides of Salt, being the measure of the place, make a C. at *Armvviden* in *Zeeland*, and the C. of *Bronage* measure of *Armviden*, maketh at *Roan* 11 Muides, 30 Muides make a Last of Corn, and 26 a Last of Oats, 100 pound weight there, maketh at *London* 114 ½, and 190 ¼ at *Antwerp*. And 100 Ells make at *London*, 115 ¼

Noremburge.

A 100 pound weight at *Noremburge* maketh at *London* 111 ⅘, at *Antwerpe* 107 ⅐, and 100 Ells at *Noremburge* make at *London* 75 ⅓, at *Antwerp* 95 ⅔ &c.

Lisbone.

The C. weight at *Lisbone* maketh 4 Roves, every Rove 32 pounds, so that their C. weight is 128 pounds, and their pound containeth 14 Ounces, and 100 pounds of their weight maketh at *London* 125 ⅛

Their Silke, Cloth of Gold, and Woollen is measured with a measure which they call a cubit, containing about ¾ of a Varre of *Castile*. Howbeit their common Measure is called a Varre, which maketh five Palms, and containeth 1 ¼ of a Varre of *Castile*, our Ell of *London* is equall with the Varre of *Lisbone*.

All

All kinde of Merchandize brought from *Flanders*, *Roan*, or *Brittain*, payeth at *Lisbone*, as a duty or cuſtome to the king, 20 in the C. which they call the tenth in Merchandize, and the other tenth in money.

Note alſo, that all kinde of Merchandize comming to *Lisbone* by land, payeth leſſe in cuſtome, then that that commeth by water.

Civill.

The Rove of *Civill* is 30 pound, 4 Roves make their C. weight, which is 120 pounds. The 100 pounds of *Civill* maketh at *London* 102 pounds. Their other common meaſure is a Vare, whereof 100 maketh at *London* 74 Ells, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* bee two ſorts of weight, the one called *la Groſſe*, the other *la Suttle*; with the groſſe is weighed all kinde of great wares, and with the ſmall all kinde of ſpicery, and ſuch like: 96 pounds of groſſe weight there, maketh at *London* 100 pound; and 100 pounds of ſpicery there, without any tare or allowance, make at *London* 94, and with tare 65.

Their owne common Meaſure are Braces, whereof 100 make at *London* 55 ½ Ells, at *Antwerp* 92 ½, &c.

Fla-

Florence.

At *Florence* the 100 lib. weight maketh at *Aquila*, for Saffron 110 and 145 pounds of *Florence*, make at *Roan* but 100 pounds the weight of *Florence*, and that of *Lucque* is all one.

Their other measures are Braces, whereof 100 maketh at *Antwerp*, *Burges* measure, 81 $\frac{1}{3}$ Els, 100 Braces there, make at *London* 49 Els, &c.

Lucque.

The *Lucque* Sattens commonly sold at *Lions* by weight, and 133 $\frac{1}{3}$ pounds maketh at *Lions* 100 pound, so that 1 pound $\frac{1}{3}$ maketh at *Lions* but one pound.

Their other measures are Braces, whereof 100 of them make at *London* 50 Ells; at *Antwerp* 83 $\frac{1}{3}$ Ells, &c.

Aquila.

At *Aquila*, their 100 pounds maketh at *London* 71 $\frac{1}{4}$, their 136 $\frac{2}{3}$ pounds of Saffron maketh at *Geneva* but 100, and 11 lib of *Geneva*, maketh 15 l. at *Aquila*.

Valentia.

At *Valentia* be two sorts of weights, a great and a small. The C. weight or great weight con-

containeth four Roves, the Rove 36 li. ſo the C. great weight is 144 li. and the C. weight ſmall containeth but 120 pounds, and is alſo parted into four Roves, which is 30 pounds to a Rove. By the ſmall is ſold the ſcarlet grain, with all other kind of Spicery, and by the great is ſold wooll, with all ſuch like groſſe wares. The 1 ⅓ pounds of Silk at *Valentia*, maketh at *Lions* one pound *Geneva* weight. The charge of great Merchandize at *Valentia* containeth 432 pound, and in ſmal wares 360 pounds. The weight here and at *Barſellone* is all one. Their 100 pound weight maketh at *London* 78 pound, at *Antwerp* 75.

Danſicke.

At *Danſick* or *Spruceland* the rule is, that whoſoever buyeth any merchandize there, buyeth it by the ſhip-pound, which is 320 li. 20 Liſpounds make a ſhip-pound, and the Liſpound containeth 16 pound, which ſhip-pound of *Danſick* maketh at *Antwerp* 266 ⅔ li. Their 100 li. weight maketh at *London* 86, *&c.*

Their other common meaſures are Ells, whereof 100 maketh at *London* 72 ¼, and at *Antwerp* 120 ½ Ells.

Toulhouſe.

At *Toulhouſe* 6 cabes of Woad maketh a charge, two ciſterns of corn-meaſure, and all kind

kinde of grain maketh a charge, the Lister weigheth 160 l. weight of that place. Their 100 in weight, maketh at *London* but 91 ¼ pound.

Geant.

At *Geneva* or *Geans*, 100 li. of their weight maketh at *London* 71 ¼, and at *Antwerp* 68 ¼. 100 li. weight at *Genua*, maketh at *Venice*, to wit, Suttle 106 li.

Their other common Measures are *Palmes*, whereof 100 make at *London* 110 ½ Ells, and at *Antwerp* 34 ½.

The rest are supplied in two Tables, which hereafter followeth: whereby the ingenious may gather his desire.

The Table of the agreement of the weights of divers Countries, the one with the other, being reduced to an equality, as followeth.

112 pounds weight at *London* is

Antwerp	107 ½	*Venice* gross	105
Frankeford	099	-qui ght	
Colen and *Ausburgh*	102 ¼	*Venice* subtle weight	166 ½
Norenburg.	100 ½		
Roan	098	*Aquino*	167
Paris	102 ½	*Vienna*	089 ⅛
Lions	118 ½	*Ruslam*	134 ¼
Deep	100 ¼	*Leipsig*	101 ¼
Geneva	090 ½	*Dansick*	129 ¼
Towlouse	133 ¼	*Lubeck*	097 ⅛
Rochell	124 ¼	*Barcellon*	144 ½
Marsellis	124 ½	*Lisbone*	099
Sivil, &c.	109 ¼	*Genua*	95 ¼

The other Table of agreement of Measures of divers Countries reduced unto an equality, by the aid whereof you may with the use of the Rule of Three, convert either more or lesse of any one Measure unto the other.

Antwerp	100	
Norimburg	104 1/2	
Frankeford	125	
Leipsig	125	*Elle.*
Presław	125	
Dansick	183	
Vienna in *Austria*	87	
Lions in *France*	60	
Paris in *France*	57	*Aulnes.*
Roan in *Normandy*	52	
Lisbone	60	
Sivil in *Spain*	81	
Castile in *Spain*	81	*Varres.*
Methera Isles	62	
Venice	108	
Lucques	120	
Florence	122 1/2	*Braces.*
Millan	138 1/2	
Rome	90	*Canes.*
Geane	888 1/2	*Palmes.*

60 Ells, or 79 Yards at *London* make at

The

The eighteenth Chapter treateth of Sports, and Paſtimes, done by number.



IF you would know the number that any man doth think or imagine in his minde, as though you could divine, bid them triple it, or put twice ſo much more to it as it is, which done, aske him whether it bee even or odde; if he ſay odde, bid him take one to it, to make it even, and for that one, keepe one in your minde. Now after he hath taken one to it to make it even, bid him give away half, and keep the other half for himſelf, which when he hath done, bid him triple that half, and again, after he hath tripled it; aske him whether it be even or odde: if hee ſay odde, then bid him take one to make it even again, and for that laſt one, keep two in your minde: now after hee hath made his number even, bid him caſt away the one halfe, and keep the other ſtill, from which halfe that he keepeth, cauſe him ſubtilly to put away or give you nine out of his number, as oft as he can, and for each 9 that he giveth you, keep 4 in mind, and thereunto joyn the 3 which I bad you keep, and you ſhall have your deſire.

Exam-

Example.

Imagine he thought 7, the triple whereof is 21, and becauſe it is odde, he is to take 1 to make it euen, which firſt 1 giuen, is for you to keep in mind. Then the halfe of his 22 being caſt away, he reſerveth ſtil 11, which after you have bidden triple, it maketh 33: then in giving of him one again to make it euen, upon that laſt 1 reſerue 2 in your minde, then his halfe of 34 maketh 17; from whence he can give you 9 but once. Therefore that yeilding to you 4, and the 3 that you keep, make 7 your deſire.

Another kind of Diuination, to tell your friend how many pence of ſingle pieces, reckoning them one with another, he hath in his purſe, or ſhould think in his mind.

Which to doe, firſt bid him double the pieces he hath in his purſe, or the number he thinketh (if he participate his number or ſecrecie unto ſome one friend that ſitteth by him that can but multiplie, and adde never ſo little: if their number be great, then ſhall they worke as you bid them ſo much the ſurer.)

Now after he hath doubled his number, bid him adde thereunto 5 more; which done, bid him multiply that his number by 5 alſo; which done, bid him tell you the juſt ſumme of his laſt multiplication, which ſumme the giver thinketh it nothing available, becauſe it is ſo great above his pretended imagination: yet thereby ſhall you preſently with the help of ſubtraction tell his propoſed number.

The Rule is this.

Imagine he thought 17, *double* 17, *and it maketh* 34, *whereunto if you adde* 5, *it maketh* 39: *which multiplyed by* 5, *as here is practiſed, it yielded* 195, *which* 195 *is the ſumme delivered you in the worke: then for a generall Rule you ſhall evermore cut off the laſt figure toward your right hand, with a daſh of your pen, as here is performed, as a figure nothing available unto your work, and then rebate* 2 *from your firſt figure, after* 5 *is cut off, and the reſt ſhall evermore be your deſire, as by this example doth appeare.*

17	
2	
34	
5	
39	
5	
195	
2	
17	

Another of a Ring.

If in any company you are diſpoſed to make them merry by manner of divining, in delivering a Ring unto any one of them, which after you have delivered it unto them, that you will abſent your ſelfe from them, and they to deviſe after you are gone, which of them ſhall have the keeping thereof, and that you at your return will tell them what perſon hath it, upon what hand, upon what finger, and what joynt: Which to doe, cauſe the perſons to ſit down all in a row, and to keep likewiſe an order of their fingers; now, after ye are gone

out

out from them to ſome other place, ſay unto one of the lookers on, that he double the numbers of him that hath the Ring and unto the double bid him adde 5, and then cauſe him to multiply the Addition by 5, and unto the product bid him adde the number of the finger of the perſon that hath the Ring. And laſtly, to end the work, beyond that numbers towards his right hand, let him ſet down a figure ſignifying upon which of the joynts he hath the Ring, as if it be upon the ſecond joynt, let him put down 2. Then demand of him what number he keepeth, from the which you ſhall abate 250, and you ſhall have three figures remaining at the leaſt. The firſt toward your left hand, ſhall ſignifie the number of the perſon which hath the Ring, the ſecond or middle number ſhall declare the number of the finger: and the laſt figure towards your right hand ſhall betoken the number of the joynt.

Example.

Imagine the ſeventh perſon is determined to keep the Ring upon the fifth finger, and the third joynt: firſt double 7, it maketh 14, thereto adde 5, it maketh 19, which multiplyed by 5, yeeldeth 95, unto which 95, adde the number of the finger, and it maketh 100; and beyond 100 toward the right hand, I ſet down 3 the number of the joynt, all maketh 1003, which is the number

that is to be delivered you, from which abating 205, there resteth 753, which prefigureth unto you the seventh person, the fift figure, and the third joynt.

But note, that when you have made your subtraction, if there doe remain 0, in the place of tens, that is to say, in the second place, you must then abate 1, from the figure which is in the place of the hundreds, that is to wit, from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tenth finger, as if there should remain 803, you must say, that the seventh person upon his tenth finger, and upon his third joynt, hath the Ring.

Another of three Dice.

If a man doe cast 3 Dice, you may know the points of one of every of them. For if you cause him to double the points of one Die, and to the double to adde 5, and the same summe to multiply by 5, & unto the product adde the points of one of the other Dice; and behind the number towards the right hand, to put the figure which signifieth the points of the last Die, and then aske what number he keepeth, from which abate 250, and there will remaine 3 figures, which doe note unto you the points of every Die.

Another of things hidden.

If three divers things are to be hidden of three

three divers perſons and you to divine, which of the three perſons hath the three divers things, doe thus: imagine the three things to be repreſented, *A*, *B*, *C*. Then ſecondly, keep well in your mind which of the perſons you mean to be the firſt, ſecond, and third. Then take 24 Counters or Stones, and your three things, and give *A* to the party whom you imagine to be your firſt man, and therewithall give him one of your 24 Counters in his hand, and *B* unto your ſecond man, and therewithall 2 Counters, and *C*, unto your third man, and therewithall 3 Counters: and leave the reſt, which are 18, ſtill among them: which done ſeparate your ſelfe from them, and afterwards bid them change the things among them as they ſhall think good: which done, after they are agreed, bid him that hath ſuch a thing, as before you have repreſented by *A*, for every Counter that he hath in his hand, to take up as many more. And for him that hath *B*. for every one in his hand to take up two. And for him that hath *C*, for every one in his hand to take up 4, and the reſt of them to be ſtill upon the board. Theſe 3 things, and the three perſons being fully printed in your minde, come to the Table, and you ſhall evermore find out of theſe 6 numbers, 1, 2, 3, 4, 5, 6, or 7. If therefore one remaine ſtill upon the board, then have they made no exchange, but keep them ſtill as they were delivered unto them. So that the firſt man hath *A*, the ſecond

B, and the third man C. But if 2 remain, then the first man hath B, your second man A, and your third man C. The rest of the work and the order thereof are here apparent by the Table following.



Another divination of a number upon the Casting of two Dice.

First let the Caster cast both the Dice, and mark well the number: then let him take up one of them, it maketh no matter which, and look what number it hath in the bottom, and adde all together: then cast the Die again, and keep in his mind what all together maketh: then let the Dice stand, and bring seven with you, and thereunto adde the rest of the pits that you see upon the upper side of the Dice, and so many did the caster cast in all.

FINIS.

An

An Appendix concerning the Resolution of the *Square* and *Cube* in Numbers, to the finding of their side, by *Ro. Hartwel.*

A Figurate Number is a number made by the multiplication of one number or more by another. A figurate number what.

The sides of a figurative number, are the numbers by whose multiplication it is made. The sides of a figurate number what.

A Figurate number is two-fold, { *Plain.* / *Solid.*

And it is { *Of one Multiplication.* / *Or consequently of many.* } { *Plain.* / *Solid.* }

And in each { *Both Æquilater.* / *And Inequilater.* }

A figurate number made of one multiplication, by two sides or numbers multiplyed together, is called a plain figurate number.

For every number made by the mutuall multiplication of two numbers, may be called a *Plaine*, because it bringeth forth a right-angled parallelogramme, according to his unities disposed in *length* and *breadth*, the *sides* whereof are the two multiplying numbers. As the number 20, made by the mutuall multiplication of 4 and 5 is called a *Plaine*, and A plain figurate number.

the *ſides* thereof are 4 and 3 as here.

Becauſe the unities thereof diſpoſed in *length* and *breadth*, as the *ſides* do expreſſe, do bring forth an *inequaliter Parallelogram*, for that the numbers, or *ſides* are inequall.

By like reaſon 36 made by multiplication of 6 by 6, is called an *Æqualiter plain*, for the *ſides* thereof 6 and 6 are equall.

Moreover one and the ſame *plain number* may have many *ſides*, as the *plain number* 24, hath ſides 4 and ſix: 3 and eight: 2 and 12. For it is produced from the mutuall multiplication of theſe numbers: whereupon for the invention of the ſides, to wit, in *inæqualiter Plains*, it is needfull to give one of the *ſides* by which the *plain* it ſelfe divided, the other *ſide* is made known. As the *plain* 48 being divided by the *ſide* 8, the *quotient* 6 is the remaining *ſide*. Notwithſtanding another reſolution and inquiſition doth happen in the *ſides* of the *Æquilater plains*.

An Equilater Plain or quadrat what.

An Æquilater plain is a number made by two equall ſides, or by any number multiplyed by it ſelfe. It is vulgarly called a ſquare or quadrat; by the Arabians Zenſus, *it is commonly expreſſed by this note z, by us q.*

A quadrat or *ſquare* in *Geometry* is called a right lined *plain figure*, made by four equall right lines, and ſo many right Angles, & every one of the lines is called the *ſide* of the *quadrat*,

drat, as this *figure a b c d* whose *side* is *a b*, or *b c*, as also *c d*, and *a d*.



To the similitude hereof, that number is called a *Quadrate*, which is made by the multiplication of two equall numbers, or of one in it self, of which manner 36 is made, by 6 multiplyed in it selfe, or by the mutuall multiplication of 6 & 6. For if 36 unites be placed in plain forme, it bringeth forth a perfect *Geometricall Quadrat*, having in every side sixe unities as here.

* * * * * *
* * * * * *
* * * * * *
* * * * * *
* * * * * *
* * * * * *

The number whereof the Quadrat is produced by multiplication in it selfe, is called the side or root of the Quadrat.

The side or root of a number what.

Concerning the extraction of the Quadrat or square Root.

THerefore to find the *Quadrat Root*, or the *side* of any *Quadrat number*, is to search a number, which brought or multiplyed in it self, maketh the number propounded: concerning the finding whereof, as it is requisite that the *sides* (being lesser then 10) of the *squares* under an hundred should be gathered by the Table of Multiplication: so the *sides* of the

grea-

greater squares are to be sought out by *Art.* 1. first, the *squares* whose *sides* are simple numbers, are here set down as you see.

The roots.	1	2	3	4	5	6	7	8	9
The squares.	1	4	9	16	25	36	49	64	81

The knowledge of a square is by finding out his side expressed by a whole number.

Although the finding out of the *side* of a *square* be applyed to each number given, as to a *square*, yet *square numbers* only have a *side* to be expressed by a certain number of unites, or by rationall numbers, the other are to be expressed but only in power. The *sides* are commonly called *Roots* by a *Metaphoricall phrase.*

The Root or side of a square is to be found by the Theorem following.

If the odde degrees of a *square number* being marked from the right toward the left hand with points, you subduct from the number given, the particular *square* of the last *period*, setting the *side* thereof alone by it selfe.

2. Then going on, if you divide the remainder (if there be any) with the figure going before it, by the double of the side set alone by it selfe.

3. And multiply the *quotient*, found out (being placed by the side, which was first set alone by it selfe, and also before the doubled number

on

on the right hand) by both the numbers (namely by the double number, and the Figure ſet by it ſelfe) being counted as one *diviſor*, ſubducting the products from the given number, and then renue this laſt work of diviſion ſo many times as there are pricks remaining, the *ſide* of the *ſquare* ſhall be found out.

This artificiall devier is taken out of the 4 P. 2 of Euclide, where by demonſtration it is proved, that if a right line be cut into two ſegments, how ſoever the ſquare of the whole line is equall to the ſquares of the ſegments, and the two right-angled figures made of the ſegments as in the figure ſhewed, that is the two Diagonals, bk, and b f, are the ſquares of the ſegments, ab, and bc, & the complements hk, and fg, are the right-angled figures made by multiplying the line ab, by bc,

To extract the ſquare root



The ſelfe-ſame parts are to be found in any *ſquare number*. As for *example*, let the *number* be 169, whoſe *ſide* is 13. This *ſide* being divided into two pieces, 10 and 3, multiplying each piece by it ſelfe once, namely, 10 by 10, and 3 by 3: then multiply one by another, as 10 by 3, and 3 by 10; ſo ſhall you have foure *plain numbers*, whereof two are *ſquare*, as here you ſee.

The firſt example.

There-

Therefore as the *square* 169 is made by adding together of theſe foure *plain numbers*, ſo by ſubducting them ſeverally is reſolved.

	10	3
	10	3
	100	
	30	
	30	
	9	

169

First therefore I mark each odde place with points, becauſe the *particular ſquares* are to be found in the odde places. Then for ſo much as the unity ſtanding under the firſt point next the left hand, and repreſenting the laſt *period*, is both a *ſquare* and the *ſide* of a *ſquare*, that *figure* therefore being ſet alone in the *quotient*, and being ſubducted from the unity ſtanding over the point, there remaineth nothing.

This unity ſet alone by it ſelf in the *quotient*, ſhall ſignify 10, when another *figure* is ſet by it, repreſenting the ſide of ſome other particular *ſquare*. Whereupon I ſay, that the greater *Diagonall* $k\,g$; is now ſubducted from the whole *ſquare*, and the ſide of it $k\,i$, or $a\,b$, (for they are equall one to another) and alſo the *ſide* of the *complement* is found out.

This is the firſt ſtep to this Reſolution.

Moreover, I double the *figure* found out, becauſe being doubled, it is the ſide of both the *complements* taken jointly together, namely, $k\,i$, and $g\,i$. Then ſetting the doubled number under 6, I divide 6 (which in this place is as much

much as 60, and repreſenteth both the *complements*) by 2, the *quotient* is 3, repreſenting the other *ſide* remaining of the *complement*; namely, *bf* for *bc*, which number I ſet in the *quotient* and count it for the *ſegment remaining* of the right line given. Wherefore becauſe this number 3 is the ſide of the remaining *Diagonall*, that is to ſay, of the leſſer *ſquare bf*, therefore being ſet by the *diviſor* on the right hand, and multiplyed by it ſelfe, and alſo by the *diviſor*, it bringeth forth three *plain numbers*, namely, the *ſquare bf*, and the two *complements ai*, and *il*, which being ſubducted from the numbers ſtanding over them, there remaineth nothing.

The example is thus.

169 (12	*Which is all one, as if you had put downe the numbers found out in this manner.*	169
223		100 *The greater Diagonall.*
3		60 *The complements twofold.*
69		9 *The leſſer Diagonall*
		169

The ſubtilty of this invention is illuſtrated by many examples. The ſecond example.

Let the *ſquare* given be 1764. This number being marked with two points, telleth us, that the ſide thereof is to be written with two Figures.

First,

First therefore beginning at the point on the left hand, I seek the side of the last period, namely, 17. But for so much as it is no square number, I take 4 the side of the next lesser square, which I set alone by it self in the quotient, and then multiply it by it self, the product is 16, which being subducted from 17, there resteth 1. Moreover I double the side found out, the product is 8, I place this doubled number under 6, and by it divide 16 standing above it, the *quotient* is 2, which must be set by 4. This *quotient* 2 must be set before the *Divisor* 8, on the right hand under the point, and then must it be multiplyed both by it selfe, and into 8, the product is 164, which being subducted from the figures standing over them, there remaineth nothing: whereby I gather that the number given is a just *square*.

The Example standeth thus.

2

1764 (42

168

2

164

1764 *The Collection.*

The same manner of working is to be followed in greater square numbers given, saving that the former part of the worke is to be used but

once,

once, but the latter part is to be followed ſo many times as there are points remaining excepting the laſt.

As in 5 47 56, I ſay, that the *ſide* of the *ſquare* next unto 5 is 2: therefore 2 being ſet in the *quotient*, and multiplyed by it ſelf, makes 4, and taken from 5, the remainder is 1. Moreover I double the *quotient* the product is 4, which I ſet under the next *figure* toward the right hand, and thereby divide 14, the *quotient* is 3, which three being ſet both in the *quotient*, and alſo before the *Diviſor* toward the right hand, I multiply both the numbers by it, the *product* is 129: this being ſubducted from 147 ſtanding above it, the remainder is 18. But becauſe there is yet one point remaining, with which I have not medled; I therefore again double all the whole *quotient*, for in this caſe I muſt take 23 for the ſide of one former *ſquare*, and generally in great numbers, when I light upon more particular *ſquares* then two, I muſt eſteeme them but as two, and take the ſides which are firſt found out, but as the *ſides* of one only *ſquare*. Therefore twice 23 is 46: by this I divide 185, the number to be ſet in the *quotient* is 4, which number alſo muſt be ſet before the *Diviſor* on the right hand: then muſt 463 be multiplyed by 4: the *product* is 1856, this *product* being ſubtracted from the numbers ſtanding over it, there remaineth nothing. The example ſtandeth thus. The third example.

Note.

The

128

54756 (234

44364

129

4

1836

54756 *The Collection.*

See also the Example following.

10942864 (3308.

Therefore out of this invention is this *corollarie.*

4 Example of a surd number.

The number whose side cannot be expressed by whole numbers, is not a square number.

Such are all *prime numbers*, and (the *squares* themselves excepted) all other *compound numbers*. For if in them you desire to find out the *square side*, you shall labour in vain, because they are not *squares*, for to the whole numbers arising in the *quotient*, there will be some *fraction* adjoyned, whereby it commeth to passe, that the number of the *side* is not to be expressed by a true number, and it is commonly called a *surd number.*

Notwithstanding, if you adjoyne to the *side* found out, the number remaining, taking his *denomination* from the double of the *side augmented*

mented by an unity, you ſhall find the next ſide that may be like to the *ſide of a ſquare.*

As if from 40 you take the neereſt *ſquare*, to wit, 36, the remainder is 4. Here therefore the ſide ſought for of the *ſquare* exceedeth not the *ſide* found out by an unity, but either by one, or more parts of ſome whole number: wherefore I double 6, the ſide found out, and adde an unity to it being doubled, the totall is 13; this number I ſet under 4 the remainder, and ſay that the ſide of 40 demanded as neere as may be, is $6\frac{4}{13}$: the *Denominator* of the *Fraction* being added to the greateſt *ſquare* in the number given, namely unto 36, maketh the next greateſt *ſquare* above it, namely, 49, whoſe ſide is 7. But this ſurd ſide, to wit, $6\frac{4}{13}$, multiplyed by it ſelfe, maketh $39\frac{112}{169}$, which are not juſt equall to 40, the given number.

Judge the like concerning the reſt which are not *ſquares.*

Thus much concerning plain figurate numbers, but eſpecially ſuch as are ſquare numbers.

Concerning ſolid figurate Numbers.

A *Solid figurate Number is made of two multiplications by three numbers, or ſides, multiplyed together, admitting length, breadth, and thickneſſe.*

A ſolid figurate number.

Therefore every *number* made by the mutual multiplication of three numbers, may be called a *ſolid*, becauſe it bringeth forth a *right angled Parallelipipedon*, diſpoſed according to his unities in *length*, *breadth*, and *thickneſſe*, the *ſides* whereof are the three multiplying *numbers*. As the *number* 30 made by the mutuall multiplication of 2, 3, and 5, is called an *Inequilater ſolid number*, and the ſides thereof are 2, 3, and 5; becauſe the unities thereof diſpoſed by a certain diſtance one from another, in *length*, *breadth*, and *depth*: as the ſides doe expreſſe, do bring forth in reſemblance an *Inequilater Parallelipipedon*, for that the *numbers* or ſides are inequall.

By like reaſon 216, made by multiplication of 6 by 6, and the product thereof by 6, is called an *Equilater ſolid*, for the ſides thereof 66, and 6 are equall.

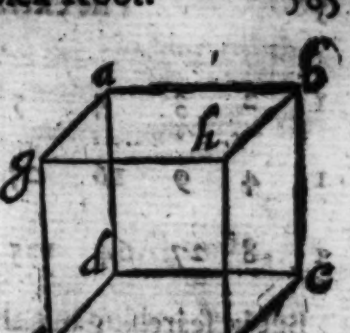
An Equilater Solid or Cube.

An Equilater, is a number made by three equall ſides, or by any number multiplyed by it ſelfe, and that product againe by the foreſaid number. And it is called an Equilater and Equiangled Parallelipipedon or Cube, and is commonly repreſented by us thus C.

A Cube in Geometry is a right-angled Parallelipipedon having ſix equall *ſurfaces*: and 8

ſolid

ſolid angles, and 12 ſides, as this figure *a. b. c. d. e. f. g. h.* whoſe ſide is *a b*, or *a d*, alſo *b c*, or *c d*, either *c e*, or *e f*, likewiſe *c h*, or *h g*, alſo *g f*, or *d f*, or *d a*, and *g a*.



The number whereof the Cube is produced by Multiplication in it ſelfe twice is called the ſide or root of the Cube; which being found out in whole numbers, the Cube is known.

The ſide or root of Cube.

Concerning the extraction of the Cubick Root.

THerefore every *Cube* in numbers hath ſuch a ſide as may be expreſſed in whole numbers, but in magnitudes it is not alwayes ſo, as indeed in magnitudes there are many things not to by expreſſed in *whole number*. Now for as much as the ſide of any *Cube* under 1000, is a ſimple figure, it is neceſſary, before we undertake to find out the ſide of any great number, to know what *Cube* is made of each ſimple figure, and what is the ſide of any *Cube* leſſer then 1000, as I have here ſet them down.



Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	226	343	512	729

But in ſearchiug out the *ſides* of greater *Cubes* we are to proceed as the *theorem* following teacheth us.

If you diſtinguiſh with *points* as it were into periods, the given *Cube*, beginning at the firſt figure on the right hand, and omitting each two figures continually; and firſt of all ſubduct the *particular Cube* of the laſt *period* from the given number, ſetting the *ſide* thereof in the *quotient*: and then ſet triple of the *quotient* under the *figure* next following the former *point*, on the right hand, and the *ſquare* of the *quotient* being tripled beneath it one *degree* more toward the left hand: and afterward divide the number above written by the triple of the *ſquare*, ſetting the *quotient* by it ſelf, and then multiply the *diviſor* by the *quotient* found out, and the tripled *ſquare* by the *ſquare* of the *quotient*, and the *quotient cubically*, ſubducting the *products* (ſo orderly added together, that each *figure* may anſwer the *numbers* whereof it was multiplied) from the *number* given, and renue this laſt manner of diviſion ſo many times as there are *points* remayning, the *ſide* of the *Cube* ſhall be found out.

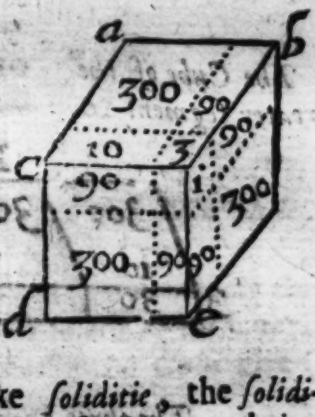
This

This artificiall device is drawne out of that *theorem*, which *Ramus* made, imitating that of *Euclide* concerning *ſquare numbers* in this manner.

If a right line be cut into two ſegments, the Cube of the whole line ſhall be equall to the Cubes of the ſegments, and the two ſolid figures comprehended three times under the ſquare of his ſegment, and the ſegment remayning.

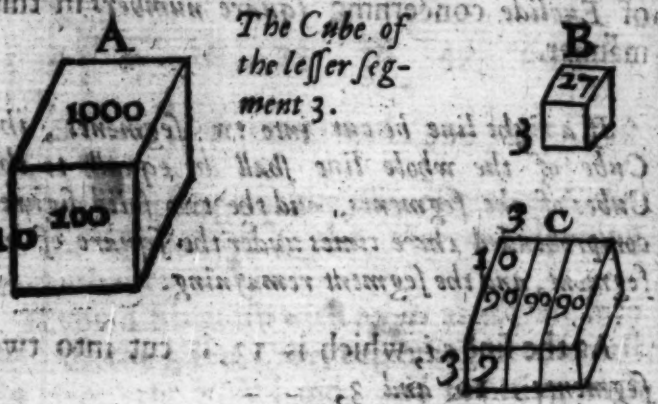
The extraction of the Cubick ſide or root.

As the line *ci*, which is 13, is cut into two *ſegments*, 10 *and* 3, therefore the *Cube* of the whole line, namely 2197, is equall unto the *Cubes* of the *Segments*, namely unto 100, and 27 alſo. to *the two-fold Solids* or *Parallelipipedons thrice* taken,



whereof three have like *ſoliditie*, the *ſoliditie* of each of the three leſſer is 90, being made of the *Square of the Segment* 3, that is to ſay of 9, multiplyed by the other *Segment* 10. Theſe three *Parallelipipedons*, joyntly taken together, make 270. But of the three greater *Parallelipipedons* each containeth 300, being made of 100, the *Square* of the greater *Segment* 10, multiplyed by the *leſſer Segment* 3,

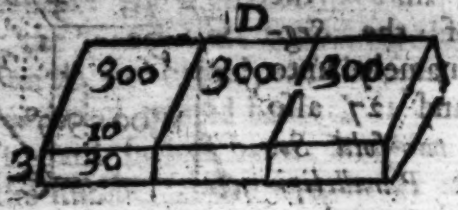
and they being taken joyntly together, make 968.



The Cube of the leſſer ſegment 3.

The Cube of the greater ſegment 10.

The 3 *leſſer Parallelipipedons.*



The three greater Parallelipipedons.

The *Cube* therefore hath eight *particular ſolids in number*, which are made of the parts of the number given, namely of 10 and 3 in this manner. Firſt, let there be foure *plain numbers* made, each part being multiplied by it ſelf, and one by another.

10 3
20 3

9
30
30
100

If againe I multiply the *Plaines* by the ſame parts, there will ariſe 8 *ſolids*, as you ſee here,

9	9	
30	30	
30	30	
100	100	
3	10	
27	90	*All theſe being added*
90	300	*together, are equall to*
90	300	*the* Cube *of the whole,*
300	1000	*to wit,* 2197.

The firſt example to extract the Cubick root.

Therefore the ſame way that is kept in making the Cube, *is alſo to be followed in reſolving the* Cube.

As for example, I marke the *Cube* given points in this manner, 2197.

Then I ſubduct the particular *Cube* of the number ſet under the laſt point: but for ſo much as that number is no *Cube*, I take the neereſt to it, namely, an unity, which alſo I ſet in the *quotient*. This unity in the number given, is 100, but in the *quotient* it is but 20, the unity ſubducted from 2, the remainder is 1, which muſt be written over the number gi-

given. So that the greater *Cube A*, is to be ſuppoſed to be ſubducted from the number given.

This is the firſt ſtep of this work.

1
2197 (1
1

After I triple the *quotient* found out (that is to ſay, I multiply it by 3) this triple repreſenteth the three *ſides* (joyntly taken together) of the three leſſer *ſolids* marked with *C*, I place the tripled number under 9. Again, I multiply the *quotient* ſquare wiſe, and triple the *product*, which maketh likewiſe 3. This *product* reſembleth the three *ſquare ſides* (taken joyntly together) of the three *greater ſolids*, marked with *D*, I place the *product* on a degree lower toward the left hand underneath 1. With it I divide 11, which written above it, the quotient is 3. This *ſegment* or *quotient* 3, being multiplyed by 3, the *Diviſor* maketh 9, which in reſpect of the place wherein it ſtandeth, is 900, and reprpſenteth the three greater *ſolids* marked with *D*, taken joyntly together. Furthermore the ſame *quotient* being multiplyed ſquare wiſe, maketh 9, and multiplyed afterward by the *tripled number* ſtanding under 9, it maketh 27, which in reſpect of the place wherin it ſtandeth, is 270, and repreſenteth the 3 *leſſer ſolids* marked with *C*. Laſt of all, the

ſame

ſame *quotient* multiplyed *cubically*, breedeth the leſſer *Cube B*. Theſe 3 *products* therefore being added together, and the totall ſubducted from the *numbers* ſtanding over it, there remayneth nothing, which importeth the given number is a *Cube*.

The example is as you ſee.

2197 (13	2197 (13
1 3	1000 *The greater Cube.*
3	3
9	3
27	900 *The 3 greater Parallelipipe.*
27	270 *The leſſer Parallelipipedons,*
	27 *The leſſer Cube.*
2197	2197

The matter may be explained by many examples. Let the ſide of the given *Cube* 16387064, be ſought out, contrive it therefore (as it were) into certain periods with points. Then firſt of all, ſearch out the *ſide* of the *Cube* next to the left hand: But for as much as 16 is no *Cube*, take 2 the *ſide* of the next *Cube* under it, that is to ſay, of 8, and ſet in the *quotient*, and ſubduct 8 the *Cube* thereof from 16, there remaineth 8. The firſt work is not to be renued throughout the whole *number*, but the *rules* following muſt be repeated as often as there are *points* remayning. The ſecond example of the Cubick root.

The

The first step to find out the root, is in this manner.

8

16387064 (2

8

Moreover, triple the *quotient* now found out and the *product* is 6, which is to be placed under 8, namely, under the figure following the next prick toward the right hand. Then multiply the *quotient* by this tripled *number* (or which is all to one purpose, square the *quotient*, and then triple the *product*) it maketh 12, set that number in a lower place one degree neer the left hand, & make it the *divisor*: divide 83 by 12, observing this rule in choosing your *quotient*, that it be no greater, then that the numbers afterward produced by multiplication may not exceed the numbers standing over it. So that here you shall take 1 in 8, but 5 times. Afterward by this number 5, multiply the *divisor* 12, and by the square of 5, multiply the tripled number 6, and last of all multiply 5 cubically: so shall you produce three numbers, namely, 60, 150, 125, to be described in such sort as you see. These numbers added together, and subducted from 8387, the remainder is 762.

The second step to step to find out the root, in this manner.

8762

8762.

16387064 (25

6

12

60

150

1225

7625

And becauſe there is yet one point remayning, this laſt manner of Diviſion muſt be wrought again.

Firſt, therefore I triple the *quotient*, the product is 75. which muſt be ſo placed, that the firſt figure thereof, namely 5, may ſtand under 6, the ſecond under the 0. Again, multiply the *quotient* by this *tripled number* (or which is all one, ſquare the quotient and triple the product) it maketh 1875, which muſt be the *Diviſor*, whoſe firſt figure namely 5, muſt be placed under 7, the *laſt figure* of the *tripled number*. I finde that 2 may be contained in 7, many times, but I can take it but 4 times, I ſet 4 in the quotient and multiply the *Diviſor*, by 4, the product is 7500, afterward I ſquare 4, it maketh 16 which I multiply by the tripled number 75, the product is 1200. Laſt of all I multiply 4 ſolidically, it maketh 64, theſe products added all toge-

together, make 762064, which number being ſubducted from the *Cube* given, there remayneth nothing, whereby I gather that the number given is exactly *cubicall*.

The third ſtep to finde out the ſide is in this manner.

762
16387064 (254

75
1875

7500
1200
64

762064

The third example of the Cubick root.

Behold alſo the example following.

61412500000 (850

Another manner of working.

Hitherto the Princely high-way to find out the ſide of the Cube *hath been declared.*

But there are moreover certain other wayes alſo bending thereto, and leaning to the ſame principles, whereof this is one.

The ſecond form. 1.

Having found out in the Table of ſimple *Cubes*, the firſt figure repreſenting the ſide of the *cube* contained in the number ſtanding under the firſt *point* on the left hand, ſet it in the *quotient*

quotient, & ſubduct the particular *Cube* of that figure as you did before: then *ſquare* that *figure*, and triple that *ſquare*, the *product* ſhal be the *Diviſor*, the firſt *figure* whereof ſhall be ſet under that *figure* which is on the right hand next of all to the point (now examined) before going.

2. See how many times the *Diviſor* is contained in the number written over it, and multiply the *Diviſor* in the *quotient*, and ſubduct the *product* from the *dividend*: yet here you muſt take heed, that you take not a greater *quotient* then that the *products* made afterward thereby may be ſubducted from the *number* given.

3. The ſubduction being done, triple the firſt *figure* which was ſet in the *quotient*, and adde to the triple the laſt *number* which was ſet in the *quotient* on the right hand of the *product*.

4. This totall multiplyed by the ſquare of the *figure* laſt found out, and ſet down the *product* ſo, that the firſt *figure* thereof toward the right hand may ſtand under the *point* next before going on the ſame hand, and finally, ſubduct the ſame from the *number* given.

The fourth example of the Cubick root.

As in 804357. The particular *Cube*, namely, 729 being taken from the number ſtanding under the laſt *period* upon the left hand, there remaineth 75357, the ſide of that particular *Cube* being 9, I ſet in the *quotient*. Then I *ſquare* that *ſide*, it maketh 81, and triple the *ſquare*, the *product* 243 is my *Diviſor*, which I ſet under the given number, ſo that 3 may ſtand under 3 with

with this *Diviſor*, divide the number ſtanding over it; you ſhall find 2 to be contained in 7 three times. Therefore I ſet 3 in the quotient, and multiply the *Diviſor* by it, the *product* is 729, which being ſubducted from 753, the remainder 24.

The Induction is thus

2
754.
104357 (93
243
729

Moreover I triple 9, the product is 27, by which on the right hand I ſet 3 the quotient laſt found out, the totall is 273.

This *number* I multiply by 9 the ſquare of the quotient laſt found out, the product ſhall be 2457, which being ſubducted from the ſuperiour number, there remaineth nothing.

The Induction is thus:

24
804357 (93
732
9
2457

Another

Another manner.

THe ſelfe-ſame work may be diſpatched another way, a little differing from the former in this manner.

The figure in the *quotient*, being found out by ſubducting the particular *Cube*, and alſo the ſecond figure in the quotient being found by diviſion, let the totall quotient be tripled, and let the tripled number be multiplyed by the former figure in the quotient. Then let the product be multiplyed againe, by the latter figure found out, and let a *cipher* be ſet on the right hand of that *product*. Laſt of all, let the *Cube*, of the latter figure found out, be added to this *product*, and let the totall ſumme be ſubducted from the number given. As in 373248. The third forme.

The firſt induction is in this manner.

30

373248 (7

343

Moreover I *ſquare* the ſide found out, it maketh 49, and triple the ſquare, the *product* is 147, which ſhall be the *diviſor*, by this I divide 302, the number written over it, the *quotient* is 2. Now I triple the *totall quotient* 72, it maketh 216, and multiply this triple by 7, the former *figure* in the *quotient*, the product is 1512. I multiply this *product* alſo by 2, the latter *figure* of the *quotient*, and ſet a *cipher* on the right hand of it, ſo as it maketh 30240, unto this number laſt of all I adde 8, the *Cube* of The fifth example.

of the latter *figure* found out, the totalis 30248, which being ſubducted from the *figure* above it, there remaineth nothing.

The ſecond Induction is thus.

30

373248 (72

147

30248

All the *points* of the *number* given being examined, if any thing remaine, it ſignifieth the *number* given is no *Cube*: wherefore the true *ſide* of it cannot be exactly given in *numbers*. Yet if it pleaſe you to ſift out the neereſt *ſide* that may be, by the firſt kinde of reduction of *mixt numbers*, you ſhall reduce the *number* given unto a *cubicall* fraction of a greater *denomination*, and afterward ſeeke out the *cubicall ſide* of that *fraction*.

To finde the neereſt Cubick root in a ſurd number.

For example ſake, becauſe 120 is no *Cube*, therefore let it be reduced into ſixty *cubicall* parts, after this manner. Multiply 60 *cubically* in it ſelf, it maketh 216000, by this being taken for the *denominator* of the *fraction*, multiply 120 the *number* given, the *product* is 2592000 whoſe *cubicall ſide* is $\frac{221}{60}$ that is $4\frac{11}{15}$ the neereſt to the true *ſide* that can be.

For the extraction of all ſorts of roots, the table of Logarithmes ſet forth by Mr. Briggs *are moſt excellent, and ready.*

FINIS.

A Table of Board and Timber measure more perfect then ever hath been made; shewing also the Squares between 4 and 37 from quarter to quarter, calculated by Robert Hartwell.

Board measure.	Inches & quarters	Squares	Timber measure	Board measure.	Inches & quarters.	Squares	Timber measure.
36.0.0	4	16	08.0.2	16.0.0	9	81	11.3.3
33.8.8	1	18	06.0.0	15.5.6	1	84	10.3.3
32.0.0	2	2	86.4.0	15.1.6	2	90	19.2.0
30.3.1	3	2	78.5.4	14.7.7	3	95	18.1.8
28.8.0	5	20	69.1.2	14.4.0	10	100	17.2.8
27.4.3	1	27	64.0.0	14.0.5	1	105	16.4.6
26.1.8	2	35	57.6.0	13.7.5	2	110	15.7.1
25.0.4	3	38	52.3.6	13.3.9	3	115	15.0.2
24.0.0	6	36	48.0.0	13.0.9	11	121	14.2.8
23.0.4	1	39	44.3.9	12.8.0	1	126	13.7.1
22.1.5	2	43	41.1.4	12.5.2	2	132	13.0.9
21.3.3	3	45	38.4.1	12.2.7	3	138	12.5.2
20.5.7	7	42	35.2.6	12.0.0	1	144	12.0.0
19.8.6	1	52	33.2.3	11.7.5	1	150	11.5.1
19.2.0	2	56	30.8.6	11.5.2	2	156	11.0.7
18.5.8	3	60	28.8.0	11.2.9	3	162	10.6.6
18.0.0	8	64	27.0.0	11.0.7	13	169	10.2.2
17.4.6	1	68	25.4.1	10.8.7	1	175	9.8.7
16.9.4	2	73	24.0.0	10.6.7	2	182	9.4.6
16.4.6	3	76	22.7.5	10.4.7	3	189	9.1.4

Board measure	Inches & quarters.	Square.	Timber measure	Board measure	Inches & quarters.	Square.	Timber measure
10.2.8	14	169	8.8.1	6.8.6	21	441	3.2.2
10.1.1	1	203	8.5.1	6.7.7	1	451	3.8.2
9.9.3	2	210	8.2.3	6.6.6	2	462	3.7.3
9.7.6	3	217	7.9.6	6.6.1	3	473	3.6.5
9.6.0	15	225	7.6.8	6.5.4	22	484	3.5.7
9.4.5	1	232	7.4.3	6.4.7	1	495	3.4.9
9.2.9	2	240	7.2.0	6.4.0	2	506	3.4.1
9.1.4	3	248	6.9.7	6.3.3	3	517	3.3.3
9.0.0	16	256	6.7.5	6.2.6	23	529	3.2.7
8.8.6	1	267	6.5.4	6.1.9	1	540	3.2.0
8.7.3	2	272	6.3.5	6.1.2	2	552	3.1.3
8.6.0	3	280	6.1.6	6.0.6	3	564	3.0.6
8.4.7	17	289	5.9.8	6.0.0	24	576	3.0.0
8.3.5	1	297	5.8.1	5.9.4	1	588	2.9.4
8.2.3	2	306	5.6.4	5.8.8	2	600	2.8.8
8.1.1	3	315	5.4.8	5.8.2	3	612	2.8.2
8.0.0	18	324	5.3.3	5.7.6	25	625	2.7.6
7.8.9	1	333	5.1.8	5.7.0	1	637	2.7.1
7.7.8	2	342	5.0.5	5.6.5	2	650	2.6.5
7.6.8	3	351	4.9.2	5.5.8	3	662	2.6.0
7.5.8	19	361	4.7.8	5.5.4	26	676	2.5.5
7.4.8	1	370	4.6.7	5.4.8	1	689	2.7.0
7.3.9	2	380	4.5.5	5.4.3	2	702	2.4.7
7.2.9	3	390	4.4.3	5.3.8	3	715	2.4.1
7.2.0	20	400	4.3.2	5.3.3	27	729	2.3.8
7.1.1	1	410	4.2.1	5.2.8	1	742	2.3.2
6.0.2	2	420	4.1.1	5.2.3	2	756	2.2.7
6.9.3	3	431	4.0.1	5.1.8	3	767	1.2.4

Board measure	Inches & quarters.	Squares.	Timber measure	Board measure	Inches & quarters.	Squares.	Timber measure
5. 1. 4	28	784	2. 2. 0	4. 3. 6	33	1089	15. 9
5. 0. 9	1	798	2. 1. 6	4. 3. 3	1	1104	1. 5. 6
5. 0. 5	2	812	2. 1. 9	4. 3. 0	2	1120	1. 5. 4
5. 0. 0	3	826	2. 0. 9	4. 2. 7	3	1136	1. 5. 2
4. 9. 6	29	841	2. 0. 5	4. 2. 3	3	1156	1. 4. 9
4. 9. 2	1	855	2. 0. 2	4. 2. 0	1	1174	1. 4. 7
4. 8. 8	2	871	1. 9. 8	4. 1. 7	2	1190	1. 4. 5
4. 8. 4	3	885	1. 9. 5	4. 1. 4	3	1210	1. 4. 3
4. 8. 0	30	900	1. 9. 2	4. 1. 1	35	1225	1. 4. 1
4. 7. 6	1	915	1. 8. 9	4. 0. 8	1	1237	1. 4. 0
4. 7. 2	2	930	1. 8. 6	4. 0. 5	2	1247	1. 3. 7
4. 6. 8	3	945	1. 8. 3	4. 0. 3	3	1280	1. 3. 5
4. 6. 4	31	961	1. 7. 9	4. 0. 0	36	1296	1. 3. 3
4. 6. 1	1	974	1. 7. 7	3. 9. 7	1	1313	1. 3. 1
4. 5. 7	2	987	1. 7. 5	3. 9. 4	2	1331	1. 2. 9
4. 5. 3	3	1000	1. 7. 2	3. 9. 1	3	1350	1. 2. 8
4. 5. 0	32	1024	1. 6. 9	3. 8. 9	37	1369	1. 2. 6
4. 4. 6	1	1040	1. 6. 6	3. 8. 7	1	1388	1. 2. 4
4. 4. 2	2	1056	1. 6. 3	3. 8. 4	2	1416	1. 2. 2
4. 4. 0	3	1072	1. 6. 1	3. 8. 2	3	1425	1. 2. 0

The use of this former Table.

IF upon a *Scale* or *Ruler* you divide one *inch* into ten *equall parts or primes*, and againe by *diagonals*, and *parallell-lines*, you ſubdivide each of them into ten *equall parts or ſeconds*, with your *compaſſes*, you may take a more exact running meaſure for *board and timber*, then by any other meanes whatſoever, and ſo place the ſame, or this *Table* if you will, upon any *Ruler*.

Alſo by meanes of the *columnes of Squares*, you may readily finde a *ſquare* equall to any *Parallelipipedon*, or piece of *timber*, which is thicker then it is broad. As for example, ſuppoſe a piece of *timber* to be ten *inches thicke*, and 9 *inches broad*: if I multiply thoſe *ſides* one by another, they will produce 290, then ſeeking the *columne of ſquares* for 290, which I finde not, but I finde 280 the neereſt *number* to 290, to ſtand againſt 17: therefore I ſay 17 *inches fere*, will make a *ſquare* equall to ſuch an unlike *ſquared piece*, then looking in the *columne of timber meaſure* againſt 17, you ſhall find that 5 *inches*, 9 *primes*, or $\frac{9}{10}$ and 8 *ſeconds*,

or

or $\frac{11}{100}$ of an *inch in length*, of that piece will make a *foot of timber*.

Likewise for *board measure*, you may finde how much in *length* of *breadth* of *board* must be in one *foot*.

By the like meanes, suppose *for example* that a *board*, appointed to be measured, is 15 *inches* $\frac{1}{4}$ *broad*, if I desire to know how much in *length* thereof will make a foot; I seeke in the *columnes* that stand under *unites* and *quarters*, for 15 $\frac{1}{4}$, and also against the same in the *columne* under the title of *board measure*, where I finde 9 *inches* 1 *prime*, or *tenth* of an *inch*, and 4 *seconds*, or *hundreds* of an *inch* will make a *foot* at that *breadth*: The like may be practised for any other *breadth of board* whatsoever.

Certain *Tables* ſhewing the Intereſt of any *ſumme of mony* whatſoever unto 40 yeares; how much *Annuities* reſpited or forborn commeth unto. And for *buying* or *ſelling* of *Annuities* for the ſaid time; and alſo the ſame in *reverſion* after any *number of yeeres* unto 30. What they may be worth in preſent ready mony, by *R. C.* and now diligently corrected and amended by *Robert Hartwell.*

Definition of Intereſt.

1 PRincipall *is the ſumme from which the* Intereſt *is reckoned.*

2 Intereſt *is the ſumme reckoned for the lending or forbearance of the* Principall *for any termes or time.*

3 Intereſt ſimple *is that which is counted from the* Principall onely.

4 Intereſt compound *is that which is counted for the* Principall, *together with the* Arrerage.

5 Intereſt profitable *is that which is added to the* Principall.

6 Intereſt Damageable *is that which is to be ſubtracted from the* Principall.

The uſe	1 li.		is	2s.
	10s.			12d.
per annum	5 s.			6d.
of	2 s.	6 d.		3d.
	12.			1d, $\frac{1}{3}$ of a peny.

A Table shewing what 1 li. with interest, and interest upon interest after 10 in the 100 comes to every yeer under 41 yeers. As followeth.

yeers.	li.	s.	d.
1	1	2	0
2	1	4	2
3	1	6	7
4	1	9	3
5	1	12	2
6	1	15	5
7	1	18	11
8	2	2	10
9	2	7	1
10	2	11	10
11	2	17	1
12	3	3	2
13	3	9	0
14	3	15	11
15	4	3	6
16	4	11	10
17	5	1	1
18	5	11	2
19	6	2	3
20	6	14	6

li.	s.	d.	yeers.
7	8	0	21
8	2	9	22
8	19	1	23
9	16	11	24
10	16	8	25
11	18	4	26
13	3	2	27
14	8	5	28
15	17	3	29
17	8	11	30
19	3	10	31
21	2	3	32
23	4	9	33
25	10	11	34
28	2	6	35
30	18	0	36
34	0	0	37
39	8	1	38
41	2	10	39
45	5	2	40

By the former Table, if you desire to know what 1 li. commeth to with *interest*, and *interest* upon *interest* after 10 in the 100, for any *number* of years unto 40. Look in the *row*, or *margent* (over which is written *years*) and against it on the right hand close unto it, in the *row or margent of pounds, shillings, and pence*, (which is titled thus, li, s. d.) you shall finde your desire.

Example.

I would know what 1 li. *with interest, and interest upon interest commeth to in* 7 *years?*

I looke in the *row* of years for the *number* 7. And against it on the right hand I finde 1 li. 18 s. 11 d. Also what it commeth unto in 13 years. I seeke among the years for 13, and against it I finde 3 l. 9 s.

Again, for 21 years. I look for 21 among the years, and I finde 7 li. 8 s. 0 d. But if you would know for a greater sum then 1 li. Then multiply your summe by that summe of 1 li. in the Table for any of those years, and you shall easily finde it. As thus, I would know what 10 li. commeth to for 7 years with interest, &c. I see that 1 li. commeth to 1 li. 18 s. 11 d. in that time. Then say I that 10 li. must bee 10 times as much in that space, which is 19 li. 9 s. 2 d. Also of 10 li. in 13 years, I see that 1 li. in that time commeth unto 3 li. 9 s. Then must 10 li. be 10 times as much in that space, which

is

is 34 li. 10 s. Alſo what 10 li. commeth to in 21 years. I finde firſt that 1 li. in that ſpace commeth to 7 li. 8 s. Then I ſay 10 muſt be 10 times as much, which is 74 li. Laſtly, I would know what 100 li. commeth to in 7 years, I ſee it muſt bee 100 times as much as 1 li. commeth to in that ſpace, which is 194 li. 11 s. 8 d. Hereby you ſee the common ſaying is not true, that 100 li. doth double it ſelfe in 7 years, for it wants thereof 5 li. 8 s. 4 d. But in 8 years 100 l. commeth to 210 li. 8 s. 4 d. which you ſee is more then double it ſelfe by 10 li. 8 s. 4 d. And in this ſort may any that can but caſt with Counters, or indeed by memory finde the increaſe of any ſumme whatſoever for any of the number of years in the foreſaid Table, after they have found what 1 li. commeth unto for that time, as before is ſpecified.

A Table ſhewing if 1 li. *annuity to endure for any number of years, under* 41, *be all reſpited or forborne, untill the laſt payment grow due, and then all be received together; with intereſt, and intereſt upon intereſt after* 10 *in the* 100 per annum,

annum, *what they will amount unto by any of the said number of yeers. As followeth.*

yeers	li.	s.	d.	li.	s.	d.	yeers.
1	1	0	0	64	0	0	21
2	2	1	0	71	8	0	22
3	3	6	1	79	10	10	23
4	4	10	10	18	9	11	24
5	6	2	1	98	6	11	25
6	7	14	3	109	3	7	26
7	9	9	8	121	1	11	27
8	11	8	8	134	4	8	28
9	13	11	7	148	12	7	29
10	15	18	8	164	9	10	30
11	18	10	7	181	18	10	31
12	21	7	8	201	2	9	32
13	24	10	5	221	5	0	33
14	27	19	5	245	9	6	34
15	31	15	6	271	0	5	35
16	35	18	11	299	2	6	36
17	40	10	10	330	0	9	37
18	45	11	11	364	0	10	38
19	51	3	2	401	8	11	39
20	57	0	6	442	11	10	40

By this Table you may know what any Annuity being reſpited or forborn for any number of yeers unto 41. with intereſt upon intereſt, after ten in the 100 will come unto: firſt ſeeking in the *Table*, what 1 li. will come unto, in that time, and that being found to multiply it by the ſumme you deſire to know.

Example.

Firſt, I would know what 1 li. *Annuity being forborn or reſpited for* 14 *yeers commeth unto.*

I look in this laſt Table (which is for purpoſe) and I find 27 li. 19 s. 5 d.

Again, what 1 li. *Annuity* reſpited for 21 *yeers* commeth to? I look in the ſaid Table for 21 *yeers*, and find 64 li. Alſo the like for 1 li. for 30 *yeers* reſpited. I look, and find it to be 164 li. 9 s. 10 d. as by the ſaid Table may appeare. Now for greater *Annuities*, as 30 li. *per annum*, reſpited or forborn, what it amounteth to in 16 *yeers*. I ſeek firſt for 1 li. in this laſt *Table* before for 16 *yeers*, and againſt it I find 35 li. 18 s. 11 d. Then ſay I, that 30 li. *per annum* being reſpited for that time, will come to 30 times as much, which is 1078 li. 7 s. 6 d. Alſo if there be an *Annuity* of 45 li. due and unpayed for 12 *yeers*, I look in the ſaid Table what 1 li. commeth to, 12 yeers being reſpited, and I find it is 21 li. 7 s. 8 d. Then I conclude that

5 li.

5 li. must be 45 times as much, which is 962 li. 5 s.

Lastly, I have an Annuity of 50 li. per annum, which hath been behind for 16 years, and must be answered unto me with interest, and interest upon interest, all at one payment, what shall or ought I to receive in all, at the 16 years end?

I seek what 1 li. comes unto in that time (as before taught) and I finde 35 li. 18 s. 11 d. Then must my 50 li. *per annum* forborne for that time, come to 50 times as much, which is 1797 li. 6 s. 10 d. And thus may you find any other *summe* great or small, for any *number of years* contained in the aforesaid *Table*, without the helpe of *Arithmetick*, if you can but use your Counters, or [illegible] memory count well.

A Table shewing if 1 li. Annuity (to indure for any number of years unto it) were to be sold for present ready money, how much ought that ready money to be, reckoning 10 per 100 per annum

annum abating interest, and interest upon interest. As followeth.

yeers.	li.	s.	d.
1	0	18	2
2	1	14	8
3	2	9	8
4	3	3	4
5	3	15	9
6	4	7	1
7	4	18	4
8	5	6	8
9	5	15	2
10	6	2	10
11	6	9	9
12	6	16	8
13	7	2	0
14	7	7	4
15	7	12	1
16	7	16	5
17	8	0	5
18	8	4	0
19	8	7	3
20	8	10	3

li.	s.	d.	yeers.
8	12	11	21
8	15	5	22
8	17	7	23
8	19	8	24
9	1	6	25
9	3	0	26
9	4	8	27
9	6	1	28
9	7	4	29
9	8	6	30
9	9	7	31
9	10	6	32
9	11	4	33
9	12	2	34
9	12	10	35
9	13	6	36
9	14	1	37
9	14	7	38
9	15	1	39
9	15	6	40

This Table before laſt ſpecified is very neceſſary and commodious for all *Gentlemen* or others, that ſhall have cauſe to buy or ſell *Annuities* or ſuch like, for by this they ſhall know what they doe, whether they demand, or take too little or too much, after the rate of ten in the 100, by which proportion all theſe theſe *Tables* are ruled.

As for example, I am to buy an Annuity of 16 li. per annum, *for* 12 *yeers, and am demanded for it ready mony* 120li. *I would know, if I give this rate, whether I give too much or too little, according to the proportion of* 10 *in the* 100 per annum, &c.

I look in the *Table* laſt before what 1 li. is worth for 12 *yeers*, and I find againſt 12 this *ſumme* 6li. 16s. 3d. Now I ſay that 16li. *Annuity* for that time, and after that proportion commeth to 16 times as much, which is 100 li. So that I ſee the party demanded of me 11 li. too much after the rate of 10 in the 100 *per annum*, and therefore I muſt draw him to a lower price or leave it.

Again, I am offered an Annuity of 20li. per annum *of* 14 *yeers for* 130li. *I would know if I give it whether I give too much or too little, according to the proportion aforeſaid.*

I ſeek firſt what 1li. *Annuity* is worth for 14 *yeers*, and I find in the ſaid laſt *Table* 7 li. 7s. 4d. Then ſay that the *Annuity* of 20 li. *per annum*, will come to 20 times as much, and will be worth 147li. 6s. 8d. according to the

the proportion before mentioned: and is more then his demand by 17 li. 6s. 8 d. So that I ſee if I accept of it, I ſhall have a good bargaine. And thus may you know readily by looking in your Table, and finding what 1 li. is worth for any time therein contained, how much any greater ſumme will come unto, if you multiply it by that ſumme of 1 li. as before is ſufficiently ſhewed.

But ſuppoſe this I have 300 *li. ready mony, and would beſtow the ſame for a valuable Annuity anſwerable thereunto according to the proportion aforeſaid. I would know what Annuity to endure* 21 *yeers this* 300 li. *will buy?*

I look in the former Table what 1 li. Annuity will coſt for that time, and I find 8 li. 12 s. 11 d. Then I ſay by the *Rule of proportion.* If 8 li. 12 s. 11 d. will buy 1 li. Annuity for 21 yeeres: what Annuity ſhall 300 li. buy or be worth for that time? I reduce the *ſummes* to the leaſt denomination (which is pence) and I find 34 li. 10s. 9d. And after this manner (by the help of this *rule*) may you find all other ſummes for any time contained in the foreſaid laſt *Table.*

A Table shewing what 1 l. in reversion for any number of years under 31 is worth in ready money, the buyer staying until the thing be fallen in hand.

years	l.	s.	d.	l.	s.	d.	years.
1	0	18	2	0	4	4	16
2	0	16	6	0	3	11	17
3	0	15	0	0	3	7	18
4	0	13	7	0	3	3	19
5	0	12	5	0	2	11	20
6	0	11	3	0	2	8	21
7	0	10	3	0	2	5	22
8	0	9	3	0	2	2	23
9	0	8	5	0	2	0	24
10	0	7	8	0	1	10	25
11	0	7	1	0	1	8	26
12	0	6	4	0	1	6	27
13	0	5	9	0	1	4	28
14	0	5	3	0	1	2	29
15	0	4	9	0	1	1	30

This laſt *Table* differeth, and is contrary to the other three before mentioned: For whereas the others increaſed more and more according to the *number of yeers* ſpecified, this doth grow and diminiſh leſſe and leſſe, as the *number of yeers* increaſeth. As for example.

There is a Tenement, the fee ſimple whereof after 7 yeers will be worth 40 li. what am I to give for it in ready mony, now ſtaying untill it fall in hand?

To know this I look in this laſt Table for 7 years, and againſt it I find 10s. 3d. So that a thing that after 7 years will be worth 1 li. is worth now in ready mony but 10 s. 3 d. Then ſay I, that the foreſaid Tenement (which after 7 years will be worth 40 li.) is now worth 40 times 10s. 3d. which is 20li. 10s.

Again, there is a Farm which after 9 yeers will be worth the Fee-ſimple 420 li. what is it now worth in ready mony, ſtaying untill it fall in hand?

I look in the ſaid Table what 1 li. is worth in reverſion after 9 years, and I find 8s. 5d. Then ſay I, that the *Farm* of 420 li. ſo long in Reverſion, will be now worth in ready mony, 420 times as much, which is 176li. 15s.

Laſtly, there is a Lordſhip to be ſold, the Fee-ſimple whereof after 14 yeers will be worth 7500

li. I *would know what the ſame is now worth in ready mony for the Reverſion.*

I look in this laſt *Table* for 14 yeeres, and againſt it I find 5s. 3d. ſo much 1 li. is worth in reverſion after 14 yeers. Then ſay I, that 7500 li. is worth no more in reverſion for that time then 7500 times 5s. 3d. which is 1968 li. 15s. And after this manner may you finde out any other ſumme whatſoever. And though ſome men of their own experience can ayme (as they think) neer enough the mark to ſerve their own turns: yet I dare undertake they ſhall never ſo exactly doe it, nor juſtify what they doe, as if they did it by Art.

FINIS.

New *Tables of Interest* at 8 *per centum per annum*, exactly calculated for 30 yeers by *Robert Hartwell*, with necessary *questions* for the use of them.

The first Table expressing the increase of one pound principall, put out and forborn for any number of yeers under 31, *at* 8 per centum, per annum.

yeers	li.	s.	d.	q.	li.	s.	d.	q.	yeers
1	1	1	7	0	3	8	6	0	16
2	1	3	3	3	3	14	0	0	17
3	1	5	2	1	3	19	6	0	18
4	1	7	2	2	4	6	3	3	19
5	1	9	4	3	4	13	2	2	20
6	1	11	8	3	5	0	8	0	21
7	1	14	3	1	5	8	8	3	22
8	1	17	0	0	5	17	0	0	23
9	1	19	11	3	6	6	9	3	24
10	2	3	2	0	6	16	1	2	25
11	2	6	7	2	7	7	11	0	26
12	2	10	4	1	7	19	9	0	27
13	2	14	4	2	8	12	6	2	28
14	2	18	8	3	9	6	4	0	29
15	3	3	5	1	10	1	3	0	30

The deſcription and uſe of the Tables of Intereſt at 8 per annum, being profitable.

The firſt of them.

THeſe Tables conſiſt of foure *Columns*, in the firſt and fourth whereof is written over the head, yeeres, and under the firſt number of yeeres deſcending from 1 to 15, likewiſe in the fourth the number of yeeres deſcending from 16 to 30. And againſt every yeere in the ſecond Columne, toward the right hand the pounds, ſhillings, pence and farthings, which one pound, or 20s. principall will amount unto, being put forth and forborn for the number of yeeres ſet againſt it; (but the pounds, ſhillings, pence, &c. in the third Columne belongeth to the yeeres ſet in the laſt Columne.)

1 Example.

Let it be required what one pound or 20 ſhillings, being put forth and forborn for 12 yeers, ariſeth to at 8 per centum, per annum, *intereſt upon intereſt.*

Seek in the firſt Columne under the title of yeeres, for 12 the number of yeers propoſed in the queſtion, and right againſt it toward the right hand in the ſecond Columne, you ſhall find 2li——10s——4d——1q, which is the principall and increaſe thereof due for the time required.

2 Example.

If 100 li. *be put forth for* 17 *yeeres according*

19

to the ſame intereſt, I demand what it will amount to in that time?

Look in the Column under the title of yeers for 17, and right againſt it towards the left hand in the Table is found 3li.——14s——0d.——0q, which is the increaſe of 1li. by which you may thus gather the increaſe of 100li. or any other *ſumme*; a hundred times 3li. is 300 li. then 100 times 14 *ſhillings* is 70. li. both which added together doe make 370li——s——0d. which is the *increaſe* of 100li. put forth and forborn 17 *yeeres*, the ſolution to the queſtion.

	li	s	d.	q
	300	0	0	0
	70	0	0	0
	370	0	0	0

3 Example.

Suppoſe 60*l. be put forth for* 19 *yeeres according to that rate, what will it increaſe to in that time?*

Seek 19 under the *title* of yeers, and againſt it toward the left hand is found 4li——6s——3d——3q. now ſay 60 times 4li. is 240, li. & 60 times 6 *ſhillings* is 360 *ſhillings*, or 18li. and 60 times 3d. is 180 d. or 15 *ſhillings*, and 60 times 3 *farthings* is 3 *ſhillings* 9d. all which added together make 258li. 18 s. 9d. the increaſe thereof demanded.

	li	s	d	q
	240	0	0	0
	18	0	0	0
	0	15	0	0
		3	9	0
	258	18	9	0

The second Table *shewing what one pound Annuity or yeerly rent is worth at the end of any number of yeers under* 31, *being forborn, at* 8 per centum, per annum.

yeers.	li.	s.	d. q.	li.	s.	d. q	yeers.
1	1	0	0 0	30	6	5 3	16
2	2	1	7 0	33	15	0 0	17
3	3	4	6 0	37	9	0 0	18
4	4	10	1 1	41	8	11 0	19
5	5	12	2 3	45	15	2 3	20
6	7	6	8 0	50	8	5 2	21
7	8	18	5 1	55	9	1 2	22
8	10	12	8 3	60	17	10 1	23
9	12	9	9 0	66	15	3 2	24
10	14	9	8 3	73	2	1 1	25
11	16	12	10 3	79	19	1 0	26
12	18	19	0 2	87	7	0 0	27
13	21	9	10 3	95	6	1 2	28
14	24	4	3 2	113	19	3 3	29
15	27	3	0 2	118	5	7 3	30

The

The uſe of the ſecond Table, (*whoſe diſpoſition is altogether like the former according to the title thereof, being profitable.*

1 Example.

There is a Leaſe worth 28 li. per annum, *to endure* 14 *years, I demand what it will riſe unto at the end of thoſe yeares, being all forborne with the intereſt upon intereſt at the rate preſcribed in this* Table.

Look in the third Table for 14 yeares, againſt which toward the right hand, you ſhall find 24li——4ſ——3d——2q. Now multiply 28 li. by 24, there ariſeth 672li. then 28li. by 4s. yeeldeth

	li.	s	d	q
112s. or 5 li. 2 s.				
Again 28li. by 3d.	672	0	0	0
produceth 84d. or	5	12	0	0
7s. finally, 28. by		7	0	0
2 farthings yeeld-		1	2	0
eth 56 farthings or				
1s. 2d. All which ad-	678	0	2	0
ded together make				

678li. 0s. 2d. to be received at the end of 14 yeers, the ſame rent or annuity being reſpited.

2 Example.

If 60 li. *yeerly rent or annuity be forborne* 20 *years: I demand how much it will increaſe at the end of the ſaid term?*

In the Table I find that 1 pound in 20 yeers will ariſe to 45li---15s--2d--3q. therefore 60li. in the like terme will yeeld 60 times as much; which I will reckon thus: 60 times 45 li. is 2700 li, 60 times 15 s. is 900 s. or 45l. 60 times 2 d. is 120d. or 10s. laſt of all, 60 times 3q is 180 farthings, or 3s. — 9d. all which together amount unto 2745 li.---13s.---9d. the value thereof to be received at the end of the term.

	li	ſ	d	q
	2700	0	0	0
	45	0	0	0
	10	0	0	0
	3	0	9	0
	2745	13	9	0

3 Example.

The yeerly rent of 1li. 13s. 4. *being behind and unpaid the ſpace of* 7 *yeers: in the end of which term the Tenant is compelled to pay the ſame with the intereſt thereof according to the above named rate; I demand what the payment ought to be.*

The increaſe of 1li. *yeerely rent* anſwering to 7 *yeeres*, is 8li. 18s. 5d. 1q. which for 6li. *rent* is to be taken 6 times, which ariſeth to 55 li. 10s. 7d. 1q. now becauſe 13s. 4d. is two third parts of 1li. therefore I take ⅔ of 8li. 18s 5d. 1q. which is the increaſe of 1l. for born for 7 *yeers*, that is 5li. 18s. 11d. 2q. which together make 59li. 9s. 6d. 3q. the *ſumme* to be received, as was required.

	li	ſ	d	q
	59	9	6	3
	5	18	11	1

The third Table declaring what one pound due at the end of any number of yeers under 31 is worth ready mony at 8 per centum, per annum.

yeers	li.	s.	d.	q.	li.	s.	d.	q.	yeers
1	0	18	7	0	0	0	10	0	16
2	0	17	6	2	0	0	5	2	17
3	0	16	5	3	0	0	2	3	18
4	0	15	4	3	0	0	1	0	19
5	0	14	3	3	0	0	0	2	20
6	0	13	3	0	0	0	0	1	21
7	0	12	2	1	0	0	0	0	22
8	0	11	1	2	0	0	0	0	23
9	0	10	0	3	0	0	0	0	24
10	0	9	0	0	0	0	0	0	25
11	0	8	0	0	0	0	0	0	26
12	0	7	11	1	0	0	0	0	27
13	0	7	4	0	0	0	0	0	28
14	0	6	9	2	0	0	0	0	29
15	0	6	3	3	0	0	0	0	30

This third Table *is diſpoſed as the firſt, the uſe according to the Title thereof, being damageable.*

1 Example.

Suppoſe there is 750 li. *due to be payed at the end of* 9 *yeeres, the Creditour would ſell this debt for preſent mony, what ought that mony to be at the rate deſcribed in the* Table?

Seek in this third Table for 9 yeers at the left ſide of the Table, and right againſt it toward the right hand, you ſhall find 10 ſhillings which multiplyed or taken 750 times, yeeldeth 7500 ſhillings, which is 375 li. the value of that debt in preſent mony.

2 Example.

There is a Leaſe worth 500 li. *after the end of* 7 *yeers; what is it worth preſent mony, according to the rate deſcribed in the table ſtaying till it fall?*

I ſeek in the Table for the 7 yeers, and right againſt it I find 11s — 8d 3; now I multiply 5500 by 11, it yeeldeth 5500 ſhillings, or 275 li. then 500 times 8d. maketh 4000d, which is 16 li. 13s. 4d. which added together is 291 li. 13s. 4d. the value of the Leaſe to be paid before it fall in hand.

li	ſ	d	q
275	0	0	0
16	13	4	0
291	13	4	0

The

The fourth Table *expressing what one pound yeerly rent or annuity for any number of yeeres not exceeding* 30 *is worth ready money at* 8 per centum per annum.

yeers	li.	s.	d.	q.	li.	s.	d.	q.	yeers
1	0	18	6	0	8	17	0	1	16
2	1	15	7	3	9	2	5	0	17
3	2	11	6	2	9	7	5	1	18
4	3	6	2	3	9	12	0	3	19
5	3	19	10	1	10	16	4	1	20
6	4	12	5	1	10	0	4	0	21
7	5	4	1	2	10	4	0	0	22
8	5	14	11	0	10	7	5	0	23
9	6	4	11	1	10	10	6	2	24
10	6	14	2	1	10	13	5	3	25
11	7	2	9	1	10	16	2	1	26
12	7	10	8	2	10	18	8	1	27
13	7	18	0	3	11	1	0	0	28
14	8	4	10	2	11	3	2	0	29
15	8	11	2	1	11	5	1	3	30

The

The fourth Table is disposed altogether as the former, and the use thereof in like sort being manageable.

1. Example.

There is an annuity or rent of 20s. per annum *to endure* 25 *yeers, it is required what it is worth ready mony?*

Look in the Table for 25 *yeeres*, and right against it you shall find 10li. 13s-5d. 3q. which is the solution.

2. Example.

What is the Lease of certaine Land valued at 140 li. per annum, *to begin presently and endure* 18 *yeers, worth ready mony?*

Search in the *Table* for 18 *yeeres*, the *term* named in the *question*, and right against it toward the left hand you shall finde 9li-7s-5d-1q. which expresseth that *one pound rent* to be bought for that *terme* is worth so much; therefore that summe 140 times is the value required. Now 140 times 9li. is 1260, and 140 times 7s. is 980s. or 49li; likewise 140 times, 5d. is 700d. or 2li-18s-4d. and 140 *farthings* is 2s-11d. all which added together make 1312 li-1s-3d. for the value of the said *Lease* paying no *rent*.

li	s	d
1260	0	0
49	0	0
2	18	4
	2	11
1312	1	3

3 Ex-

3 Example.

A Lease taken for 21 yeers at 13li 6s. 8d. per annum, which after 5 yeers expired, the Tenant is desirous to give a fine, and bring the rent down to 8li. per annum, *for the rest of the term, the demand is what fine is to be payed?*

Subtract 5 *yeers* from 21, the remain 16, is the time unexpired; likewise from the present *rent* abate 8li, the rest will be 5li-6s-8d. now the drift of the *question* is, what 5li-6s-9d. *yeerly rent* or *annuity* to indure 16 yeeres is worth present mony.

The value of 1 li. *rent* or *annuity* answering to 16 *yeers* is, 8li-17s-0d. 1q. Now 5 times 8li. is 40li. and 5 times 17s. 4li-5s. and 5 times one farthing, is 1d-1q. and because 6s-8d. is $\frac{1}{3}$ of 1li. I take $\frac{1}{3}$ of 8li-17s-0d 1q, which is 2li-19s 0d. all which added together, make 4li-4s-1d-q. which is the *fine* that ought to be paid to bring the *rent* to 8l. *per annum.*

li.	s	d	q
40	0	0	0
4	5	0	0
2	19	1	1
47	4	1	1

The

The fifth Table declaring what yeerly rent or annuity of one pound ready mony will purchase for any number of yeers under 31 at 8 per centum, per annum.

yeers	li.	s.	d.	q.	li.	s.	d.	q.	yeers.
1	0	1	7	0	0	29	3		16
2	0	14	0	2	0	28	3		17
3	0	9	8	2	0	28	0		18
4	0	7	6	0	0	27	0		19
5	0	6	3	0	0	26	2		20
6	0	5	4	3	0	25	3		21
7	0	4	9	2	0	25	1		22
8	0	4	4	0	0	24	3		23
9	0	4	0	0	0	24	1		24
10	0	3	8	2	0	24	0		25
11	0	3	6	0	0	23	3		26
12	0	3	3	3	0	23	1		27
13	0	3	1	3	0	23	0		28
14	0	3	0	1	0	22	3		29
15	0	2	11	0	0	22	2		30

In

In the fifth Table *the* Numbers *and* Columns *are all diſpoſed as the former* Tables, *and needeth no further explanation but onely Examples.*

1 Example.

The Table *declareth at firſt ſight what yearly rent or annuity one pound ready mony will purchaſe for any terme in the* Table *expreſſed.*

But if the *ready mony* be above one pound, then if any *value* or *rent* ſet down in this Table, be multiplyed by the *number* belonging to the *yeers in queſtion*, the *product* will ſhew what *yeerly rent* or *annuity* that *ready mony* will purchaſe for the *time propoſed.*

2 Example.

A certain man hath 750li. to purchaſe an Annuity to endure 27 yeers, ſo as it may yeeld him the like profit, as if it were put out according to the rate in the Table *expreſſed, it is required what that annuity ought to be?*

Becauſe the *annuity* is to endure 27 yeeres, ſeek out the value or rent ſet againſt 27 yeers, in this fift Table, which is 2s--3d--1q. now this

being the *Annuity* of 10s. ready money will purchase for that terme, it must be multiplyed by 750 li. as followeth, because 2s. is the tenth part of 20s. therfore take the tenth part of 750 li. which is 75 li. which set first down, then 750 time 3d. is 9l. 7s. 6d. which set under the former, last of all 750 *farthings* is 15s. 7d. ob. All which added together, produce 85 li. 3s. 1d. ob. the *yeerely Annuity* requited.

li	s	d	q
75	0	0	0
9	7	6	0
15	7		2
85	3	1	2

Deo soli laus, omnis honor & gloria tribuatur.

AMEN.

FINIS.

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to Dods Jeanie the 24